TUTORAGGIO FISICA III

(1)

TUTORAGGI : 1, 2, 3

ESERCIZI : 1 - 24

ES 15 PUNTO 6

+4 DA RIVEDERE

IBL spa
Industria Battitura Lamiere

1 $\vec{E}, \vec{B}, \vec{k}$

$\hat{k} = \frac{\vec{k}}{|\vec{k}|}$

$\vec{E} = E\hat{E}$

$\vec{B} = \frac{1}{v}(\hat{k} \times \hat{E}) \frac{|\vec{E}|}{|\vec{B}|}$

2 $l_0 = 1\ cm$, $t = 10\ K$, $\phi = \pi/2$ IN P

$\vec{r} = (\mu_x + \mu_y + \mu_z)\ cm$

UNITÀ // A $(\mu_x + \mu_y = \mu_z)$

TROVA ϕ' a $r' = (2\mu_x + 2\mu_y + \mu_z)\ cm$

$\vec{e} = (\mu_x + \mu_y - \mu_z)$ $\vec{r} = (1, 1, 1)$

$\hat{e} = \frac{1}{\sqrt{3}}(1, 1, -1)$ $\vec{r}' = (2, 2, 1)$

$\kappa = \frac{2\pi}{\lambda} = 6.28 \cdot 10^2\ \frac{rad}{m}$ $\lambda = 0.01\ m$

$\vec{\kappa} = \kappa \hat{e}$

$\phi = \frac{\pi}{2} = \vec{\kappa} \cdot \vec{r} - \omega t \rightarrow \omega = \left(\frac{\kappa}{\sqrt{3}} - \frac{\pi}{2}\right)\frac{1}{t}$

$\phi' = \vec{\kappa} \cdot \vec{r}' - \left(\frac{\kappa}{\sqrt{3}} - \frac{\pi}{2}\right) = \frac{\kappa}{\sqrt{3}} 3 - \frac{\kappa}{\sqrt{3}} - \frac{\pi}{2} = \frac{2\kappa}{\sqrt{3}} - \frac{\pi}{2}$

κ DA PRENDERE IN CUI $\rightarrow |\vec{r}| = cm$

3 $\vec{E} = (0, 0, E_0 \sin(\kappa(y + ct)))$

$\vec{B} = (-B_0 \sin(\kappa(y + ct)), 0, 0)$

$\kappa = 9.93 \cdot 10^2\ \frac{rad}{m}$ $E_0 = 450\ V/m$ $B_0 > 0$

$\hat{v} = \hat{E} \wedge \hat{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{vmatrix} = -\hat{j}$

$|\vec{I}| = I = c\varepsilon_0 E_0^2$

$\kappa \rightarrow \frac{1}{\lambda} = \frac{c}{\nu} \rightarrow \lambda = \frac{2\pi}{\kappa} \rightarrow \nu = \frac{c\kappa}{2\pi}$

$B_0 = \frac{E_0}{c}$

POL LINEARE

4 $\omega = 6 \cdot 10^4\ \frac{rad}{s}$ NEL VUOTO LUNGO Y

POL LINEARE $\vec{E}$ CON $\theta = 45°$ CON YZ

$E_0 = 1.4 \cdot 10^3\ V/m$

$E_{0y} = 989.9\ V/m = E_{0z}$

$\omega \rightarrow \nu = \frac{\omega}{2\pi} \rightarrow \lambda = \frac{c}{\nu} \rightarrow \kappa = \frac{2\pi}{\lambda}$

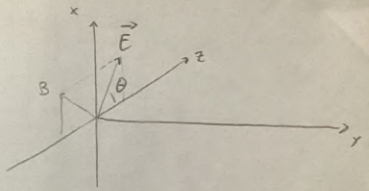
$\vec{E}_y = E_{0y} \sin(\kappa y - \omega t)$

$\vec{E}_z = E_{0z} \sin(\kappa y - \omega t)$

$\hat{m} = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right)$

4) $\omega = 6\cdot10^{14}$ VUOTO $\hat{y}$ LINEARE $\theta = 45°$ su yz $E_0 = 1{,}4\cdot10^3$

$k = \frac{\omega}{c}$ $\quad \vec{k} = \frac{\omega}{c}\hat{y}$



$\vec{E} = E_0 \sin(\vec{k}\cdot\vec{y} - \omega t)$

$\vec{E} = \frac{\sqrt{2}}{2}E_0(\hat{u}_x + \hat{u}_z)\sin(\vec{k}\cdot\vec{y} - \omega t)$

$= 989{,}9(u_x + u_z)\sin(\frac{\omega}{c}\cdot\vec{y} - \omega t)$

$= 989{,}9(u_x + u_z)\cos(2\cdot10^6 y - 6\cdot10^{14}t + \phi)$ V/m

$\hat{B} = \hat{c}\times\hat{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{vmatrix} = \left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right)$ $\quad B_0 = \frac{E_0}{c} = 4{,}6\cdot10^{-6}$ T

cc $B \perp E \Rightarrow \theta_B = 45°$ RISPETTO x

$\vec{B} = B_0(\frac{\hat{u}_x}{\sqrt{2}} - \frac{\hat{u}_z}{\sqrt{2}})\sin(2\cdot10^6 y - 6\cdot10^{14}t + \phi)$ T

$= 3{,}25\cdot10^{-6}(u_x - u_z)\sin(\quad)$ T

$\vec{I} = \frac{\vec{E}\wedge\vec{B}}{\mu_0} = \frac{|E_0||B_0|}{\mu_0}\hat{u}_c\wedge\hat{u}_B = 5124{,}8\cos\alpha \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \end{vmatrix} = \begin{pmatrix} 0 \\ -(-\frac{1}{2}-\frac{1}{2}) \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = I_0\hat{u}_y \Rightarrow I_0 = 5124{,}8$ W/m²

$\Rightarrow \vec{I} = 5{,}1\cdot10^3\hat{u}_y\cos^2(2\cdot10^6 y - 6\cdot10^{14}t)$ W/m²

$\hat{m}' = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right)$

$\vec{E} = \frac{\sqrt{2}}{2} E (\hat{\mu}_x + \hat{\mu}_z)$

$\vec{B} = \frac{\sqrt{2}}{2} B (-\hat{\mu}_x + \hat{\mu}_z)$

$\frac{\sqrt{2}}{2} E_0 = 989,96 \ V/m$

$B_0 = \frac{\frac{\sqrt{2}}{2} E_0}{c} = 3,29 \cdot 10^{-6} \ T$

$\vec{I} = \frac{\vec{E} \vec{B}}{\mu_0} (\hat{m} \times \hat{m}') = \frac{E_0 B_0}{\mu_0} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 0 & 1 \\ -1 & 0 & 1 \end{vmatrix}$

$= \frac{E_0 B_0}{\mu_0} (0, 1, 0) = 5,2 \cdot 10^3 \ (\hat{\mu}_y) \ W/m^2$

(5) $d = 8 \ m$ ISOTROPA $I = 1,24 \ W/m^2$

$\Rightarrow P = \int_S I \cdot d\vec{S} = 4\pi d^2 I = 997,27 \ W$

SO $I = \frac{E_0^2}{2Z} \rightarrow E_0 = \sqrt{2ZI} = 30,5 \ V/m$

$B = \frac{E}{c} = 1,019 \cdot 10^{-7} \ T$

$B_{EFF} = \frac{B'}{\sqrt{2}} = 1 \ \mu T = 10^{-6} \ T \Rightarrow B' = 1,4 \cdot 10^{-6} \ T$

$\Rightarrow E' = B' c = 424,26 \ V/m \Rightarrow I' = \frac{E'^2}{2Z} = 238,7 \ W/m^2$

$\Rightarrow P = 4\pi d^2 I' \Rightarrow d = \sqrt{\frac{P}{4\pi I}} = 0,87 \ m$

(6) $d = 1,3 \cdot 10^9 \ al$ $\quad \varepsilon$ PARI $3 M_\odot$

$M_\odot = 2 \cdot 10^{30} \ Kg$ $\quad \Delta t = 0,15 \ s$

$\Rightarrow d = 1,23 \cdot 10^{22} \ Km$

$E = 3 M_\odot c^2 = 5,4 \cdot 10^{47} \ J$

$P = \frac{E}{\Delta t} = 3,6 \cdot 10^{46} \ W$

$P = 4\pi d^2 I \Rightarrow I = \frac{P}{4\pi d^2} = 1893,57 \ W/m^2$

SAPENDO $\nu = 150 \ Hz$ E $c = \nu \lambda$

$\lambda = \frac{c}{\nu} = 2 \cdot 10^6 \ m$

SECONDO ME È PIÙ CORRETTO → IN SFERICHE

1 NEL VUOTO $\vec{E} = \vec{\mu}_z E_0 \sin(kx)\cos(\omega t)$

$E_0 = 10\ V/m$ $\quad k = 10^{-3}\pi\ rad/m$

$\omega = kc = 3\cdot 10^5 \pi\ rad/s$

NODI (MIN): $x = m\frac{1}{2}$ $\quad m \in \mathbb{N}$

VENTRI (MAX): $x = (2m+1)\frac{1}{4}$

$\vec{E}$ È UNA ONDA STAZIONARIA ED NON DEI MASSIMI E MINIMI ED È COMPLESSIVAMENTE LA SOVRAPPOSIZIONE DI UN'ONDA PROGRESSIVA ED UNA REGRESSIVA

$\vec{v} = c\hat{\mu}_x$

$B_0 = \frac{E_0}{c} = 3{,}3\cdot 10^{-8}\ T$

VALE $\vec{B} = \hat{v}\times\hat{E} = \hat{\mu}_x \times \hat{\mu}_z = -\hat{\mu}_y$

QUINDI $\vec{B} = B\hat{\mu}_y$

MA SO CHE ONDE STAZIONARIE $\vec{E}$ E $\vec{B}$ SONO SFASATI DI $\pi/2$

$\vec{B}(x,t) = -B_0\hat{\mu}_y \sin(kx+\pi/2)\cos(\omega t+\pi/2)$

$= B_0\mu_y \cos(kx)\sin(\omega t)$

$\vec{I} = \frac{B_0 E_0}{\mu}\sin(kx)\cos(kx)\sin(\omega t)\cos(\omega t)(-\hat{\mu}_x)$

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3 $\lambda = 3{,}18\ mm$ $\quad \varepsilon_r = 2$ $\quad \mu_r = 1$

$\vec{E} = 10\cos(\omega t+\alpha)\hat{\mu}_z\ V/m$ $\quad$ e $\quad \vec{B} = B\hat{\mu}_x$

$\lambda = \frac{v}{\nu} \Rightarrow \nu = \frac{v}{\lambda} = \frac{c}{\lambda\sqrt{\varepsilon_r\mu_r}} = 6{,}67\cdot 10^{7}\ 1/s$

$E_0 = 10\ V/m$ $\quad B_0 = \frac{E_0}{c}\sqrt{\varepsilon_r\mu_r} = 4{,}7\cdot 10^{-8}\ T$

$k = \frac{2\pi}{\lambda} = 1{,}97\ rad/m$

$\omega = kv = 4{,}19\cdot 10^{8}\ rad/s$

$\vec{I} = \frac{E_0 B_0}{\mu}\cos^2(\omega t+\alpha)(\hat{\mu}_z \times \hat{\mu}_x)$

$= 3{,}7\cdot 10^{-2}\cos^2(\omega t+\alpha)(+\hat{\mu}_y)\ W/m^2$

$\langle \vec{I} \rangle = \frac{1}{2}\frac{E_0 B_0}{\mu} = 0{,}018\ W/m^2$

9 NON STAZIONARIE

$$E = \begin{cases} \text{UNIF} & r \le R \\ 0 & r > R \end{cases}$$

$$\text{NOTA} \qquad B \equiv B\left(r, \frac{\partial E}{\partial t}\right)$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \varepsilon_0 \mu_0 \frac{\partial E}{\partial t}$$

SCRIVO IN CILINDRICHE

$$\vec{\nabla} \times \vec{B} = \left[\frac{1}{r}\frac{\partial B_z}{\partial \theta} - \frac{\partial B_\theta}{\partial z}\right]\hat{r} + \left[\frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r}\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial (r B_\theta)}{\partial r} - \frac{\partial B_r}{\partial \theta}\right]\hat{z}$$

MA SE E UNIF $r \le R$ È DIRETTO SOLO LUNGO $\hat{z}$

$$\Rightarrow B_z = 0 \qquad e \qquad \vec{B} = \vec{B}_r + \vec{B}_\theta = \vec{B}(r)$$

USO $\vec{\nabla} \cdot \vec{B} = \frac{1}{r}\frac{\partial}{\partial r}(r B_r) + \frac{1}{r}\frac{\partial}{\partial \theta} B_\theta + \frac{\partial}{\partial z} B_z$

$$= \frac{1}{r}\frac{\partial}{\partial r}(r B_r) + \frac{1}{r}\frac{\partial}{\partial \theta} B_\theta = 0$$

$$\frac{\partial}{\partial r}(r B_r) + \underbrace{\frac{\partial}{\partial \theta} B_\theta}_{=0} = 0 \Rightarrow \frac{\partial}{\partial r}(r B_r) = 0 \Rightarrow B_r = \cos t$$

E CHE IN PIÙ $= 0$ $\Rightarrow B(r) = \vec{B}_\theta(r)$

$$\nabla \times B = \frac{1}{r}\frac{\partial}{\partial \theta}(r B_\theta)\hat{u}_z = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

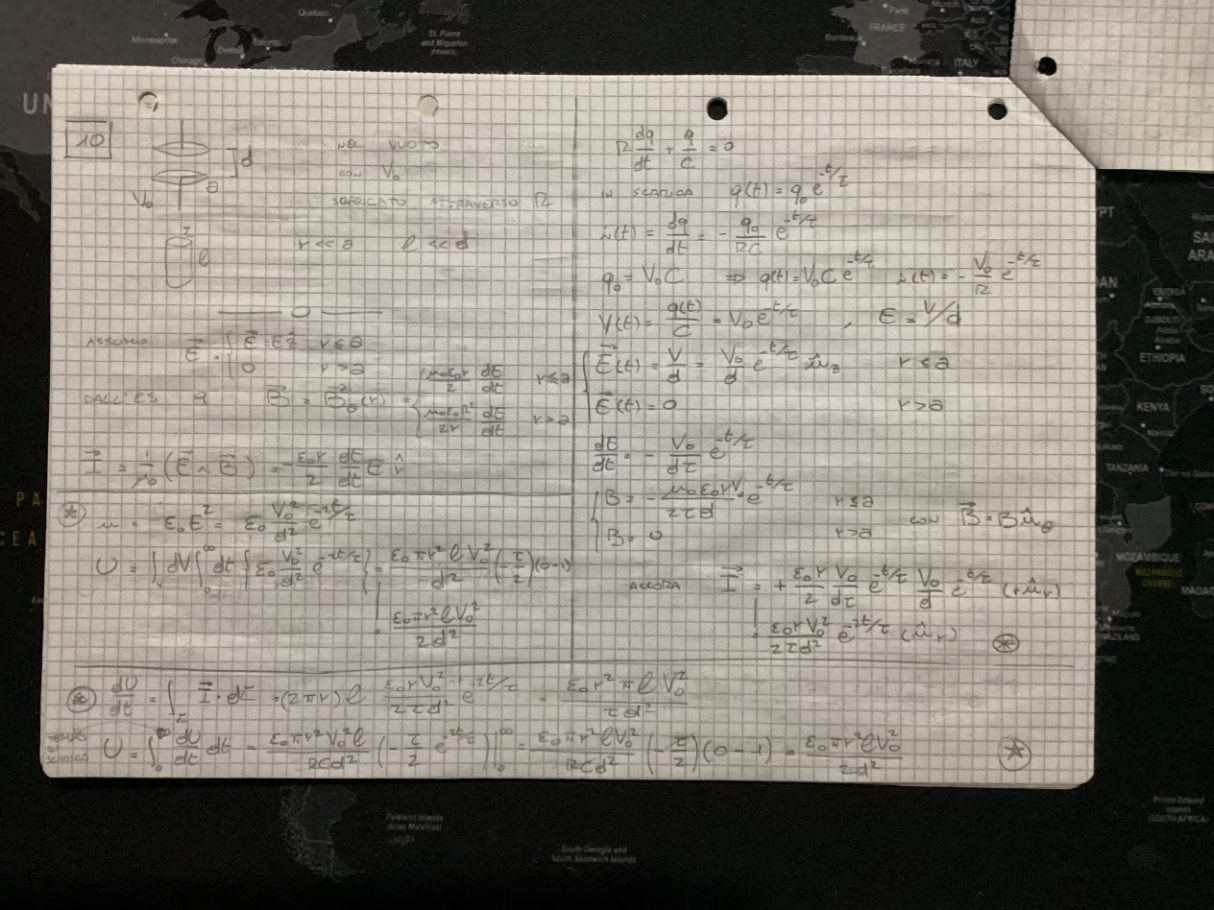
$$\oint_\Sigma \vec{B} \times \vec{ds} = \int_\Sigma \vec{B} \cdot \vec{ds} = \mu_0 \varepsilon_0 \int_\Sigma \frac{\partial E}{\partial t}$$

$$\vec{B}(r)\, 2\pi r = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} \pi r^2 \qquad r \le R$$

$$\Rightarrow \vec{B} = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} \frac{r}{2}$$

$$\vec{B}\, 2\pi r = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} 2\pi \qquad r > R$$

$$\Rightarrow \vec{B} = \mu_0 \varepsilon_0 \frac{R^2}{2} \frac{\partial E}{\partial t}$$



11

$$I(t) = \frac{2}{T} I_1 t = \begin{cases} I & t = 1 \\ 0 & t = 0 \end{cases}$$

$$\vec{B}(t) = \mu_0 I(t) \frac{N}{w} \hat{y} = \mu_0 I_1 \frac{2}{T} t \frac{N}{w} \hat{y}$$

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \int_0^a \mu_0 I_1 \frac{2}{T} t \frac{N}{w} a \, dx = \mu_0 I_1 \frac{2}{T} t \frac{N}{w} a^2$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\mu_0 I_1 \frac{2}{T} \frac{N}{w} a^2 = -\mu_0 I_1 \frac{2}{T} \frac{N}{w} \frac{\lambda}{2} = -\mu_0 I_1 \frac{N}{T w} \lambda$$

$$\vec{E} = \frac{\mathcal{E}}{2\pi r} \hat{\theta} = -\frac{\mu_0 I_1 N}{2\pi T w} \frac{\lambda}{r} \hat{\theta}$$

$$\vec{F} = q\vec{E} = -\frac{\mu_0 I_1 N}{2\pi T w} \frac{q\lambda}{r} \hat{\theta}$$

$$\vec{F} = \int d\vec{F} = \int_0^{2\pi} \frac{\mu_0 I_1 N}{2\pi T w} \frac{q\lambda}{r} r \, d\theta \, \hat{\theta} = -\frac{\mu_0 I_1 N}{T w} q\lambda \hat{\theta}$$

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$$\vec{B} = \frac{\mu_0 I}{2\pi r} \hat{\theta}$$

$$\vec{\mu} = I A \hat{z} = I \pi r^2 \hat{z}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B} = I \pi r^2 \frac{\mu_0 I}{2\pi r} \hat{r} \times \hat{\theta} = \frac{\mu_0 I^2 r}{2} \hat{z}$$

$$\vec{\tau} = \frac{d\vec{L}}{dt} \Rightarrow \int_0^t \vec{\tau} \, dt = \vec{L}(t) - \vec{L}(0)$$

$$\vec{L}(t) = \int_0^t \frac{\mu_0 I^2 r}{2} \hat{z} \, dt = \frac{\mu_0 I^2 r}{2} t \hat{z}$$

$$\vec{L} = I \vec{\omega} = I \omega \hat{z}$$

$$\omega = \frac{\mu_0 I r}{2} t$$

$$\vec{v} = \omega r \hat{\theta} = \frac{\mu_0 I r}{2} t \hat{\theta}$$

$$\vec{p} = m \vec{v} = \frac{\mu_0 I m r}{2} t \hat{\theta}$$

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{\mu_0 I m r}{2} \hat{\theta}$$

$$\vec{F} = q\vec{v} \times \vec{B} = q \frac{\mu_0 I r}{2} t \hat{\theta} \times \frac{\mu_0 I}{2\pi r} \hat{r} = -\frac{\mu_0^2 I^2 q}{4\pi} t \hat{z}$$

$$\frac{\mu_0 I m r}{2} = -\frac{\mu_0^2 I^2 q}{4\pi} t$$

$$\frac{m}{q} = -\frac{\mu_0 I r}{2\pi} t$$

$$\frac{m}{q} = -\frac{\mu_0 I r}{2\pi} \frac{2}{T} = -\frac{\mu_0 I r}{\pi T}$$

$$\frac{m}{q

13 $\lambda = 632\ nm$ asse ott. $= \hat{u}_z$

$d = 0\ mm$ ε APPROX CON ONDA PIANA

$\mathcal{E}_0 = 50\ kV/m$, VUOTO

a) A $t=0$ $\quad \vec{E} = E\,\hat{u}_y \Rightarrow \vec{B} = B\,\hat{u}_x$

$k = \frac{2\pi}{\lambda}$, $\omega = kc$, $\nu = \frac{c}{\lambda}$

$k = 9{,}9 \cdot 10^6\ \frac{rad}{m}$; $\omega = 2{,}970 \cdot 10^{15}\ \frac{rad}{s}$ $\nu = 4{,}7 \cdot 10^{14}\ Hz$

HP POLARIZZAZIONE $\Rightarrow$ COMPONENTI SFASATE DI $\pi/2$

$\vec{E}(z,t) = \mathcal{E}_0(\pm\hat{u}_x \sin\alpha + \hat{u}_y \cos\alpha)$

$\alpha = kz - \omega t$

$\Rightarrow t=0,\ z=0 \Rightarrow \vec{E} = \mathcal{E}_0 \cos\alpha\ \hat{u}_y$

$\vec{B}(z,t) = \frac{\mathcal{E}_0}{c}(\mp\cos\alpha\,\hat{u}_x \pm \sin\alpha\,\hat{u}_y)$

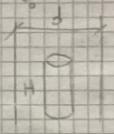
b) $\vec{I} = \frac{\vec{E}\times\vec{B}}{\mu_0} = \frac{\mathcal{E}_0^2}{\mu_0 c}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \pm\sin\alpha & \cos\alpha & 0 \\ \cos\alpha & \pm\sin\alpha & 0 \end{vmatrix}$

$= \frac{\mathcal{E}_0^2}{\mu_0 c}(\sin^2\alpha + \cos^2\alpha)\,\hat{u}_z = \frac{\mathcal{E}_0^2}{\mu_0 c}\hat{u}_z$

c) $|\vec{I}| = \frac{\mathcal{E}_0^2}{\mu_0 c} = 0{,}6\,\frac{\mu W}{m^2}$

$P = \int_0^d \frac{dV}{dt}\,|\vec{I}| = \frac{I}{4}\pi d^2 = 130{,}3\ \mu W$

d)



$\rho = 30\ kg/m^3$

$d' < d$

PERFETTAMENTE RIFLETTENTE

$F_p = mg = \rho\,\pi\frac{d'^2}{4} H g$

$P_{RAD} = 2\,\frac{I}{c}\cos^2\Theta = 2\,\frac{I}{c} = \frac{2\mathcal{E}_0^2}{\mu_0 c^2}$

$F_{RAD} = \frac{2\mathcal{E}_0^2}{\mu_0 c^2}\,\pi\frac{d'^2}{4}$

$F_p = F_{RAD} \Rightarrow \rho\,\pi\frac{d'^2}{4} H g = \frac{2\mathcal{E}_0^2}{\mu_0 c^2}\,\pi\frac{d'^2}{4}$

$\Rightarrow D.\ H = \frac{2\mathcal{E}_0^2}{\mu_0 c^2}\,\frac{1}{\rho g} = 1{,}5 \cdot 10^{-4}\ m = 0{,}15\ mm$

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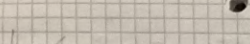
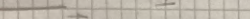


I

RIFLESSIONE $\begin{cases} P = \frac{I}{3C} & \text{ASS.} \\ P = \frac{2I}{3C} & \text{RIFL.} \end{cases}$



ASS RIFL

2 RIFL

IN GENERALE $P = \beta \frac{I}{C} \cos^2\theta \quad , \beta = 1, 2$

HA ARRIVA DA $\theta \in [0, \pi/2] \quad \varphi \in [0, 2\pi]$

$I = \alpha \frac{}{}$ ANGOLO SOLIDO

$$I = \int_0^{2\pi} d\varphi \int_0^{\pi/2} \alpha \cos\theta \, d\theta = \alpha 2\pi (-0+1) = \alpha 2\pi$$

$$\Rightarrow \alpha = \frac{I}{2\pi} \qquad \Rightarrow dI = \frac{I}{2\pi} d\Omega$$

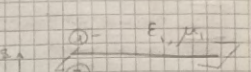
$$\Rightarrow dP = \beta \frac{dI}{C} \cos^2\theta \Rightarrow P = \frac{\beta}{C} \int_0^{2\pi} \frac{d\varphi}{2\pi} \int_0^{\pi/2} \cos^2\theta \cos\theta \, d\theta$$

$$\Rightarrow P = \frac{\beta}{C} \frac{I}{3}$$

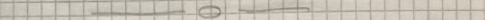
ASSORBENTE $P = \frac{I}{3C} \quad , \beta = 1$

RIFLETTENTE $P = \frac{2I}{3C} \quad , \beta = 2$

16 MEZZI OMOGENEI



VALGONO LE EQ. DI CONTINUITÀ



$$\mu_2 B_{1n} = \mu_1 B_{2n} \Rightarrow B_{\perp 1} = B_{\perp 2}$$

$$E_{\parallel 1} = E_{\parallel 2} \qquad \varepsilon_1 E_{\perp 1} = \varepsilon_2 E_{\perp 2}$$

PRESA UNA DIREZIONE GENERICA DI PROP. SCOMPONGO I CAMPI IN $\parallel$ E $\perp$

$$\vec{I}_2 = \frac{\vec{E} \times \vec{B}}{\mu_2} = \frac{1}{\mu_2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ E_{2\perp} & B_{2\parallel} & 0 \end{vmatrix} = \frac{E_{2\perp} B_{2\parallel}}{\mu_2} \hat{k}$$

$$\vec{I}_1 = \frac{\vec{E}_{1\parallel} B_{1\parallel} \hat{k}}{\mu_1} = \vec{I}_{1\perp}$$

$$= \vec{I}_{2\perp}$$

IPOTIZZANDO $\vec{E} = E \hat{\mu}_x \qquad \vec{B} = B \hat{\mu}_y$

$$\vec{I}_1 = \vec{I}_2 \Rightarrow \frac{E_{2\perp} B_{2\parallel}}{\mu_1} \hat{k} = \frac{E_{2\perp} \mu_1 B_{2\parallel}}{\mu_1 \mu_2} \hat{k} \quad \frac{E_{2\perp} B_{2\parallel}}{\mu_2} \hat{k} = \vec{I}_{2\perp}$$

6 onde piane polarizzate circolarmente

$\vec{F}_1$ sinistrorsa

$\vec{F}_2$ destrorsa

a) hanno stessa A_0 e ω ma $\vec{F}_2$ ha $\varphi + \frac{\pi}{2}$ rispetto $\vec{F}_1$ cioè $\frac{\pi}{2}$

b) hanno stessa ν e $\vec{F}_1$ ha $I_1 = I$ e $I_2 = 4I$

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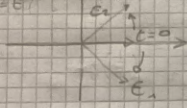
a) scrivo le due polarizzate circolarmente $\Rightarrow$ componenti sfasate di $\pi/2$

$\vec{E}_1 = \hat{u}_x A \cos(kz - \omega t) + \hat{u}_y A \sin(kz - \omega t)$

$\vec{E}_2 = \hat{u}_x A \cos(kz - \omega t + \phi) = \hat{u}_y A \sin(kz - \omega t + \phi)$

$z = 0 = t$

$\omega t = \pi/4$



$\Rightarrow \vec{E}_1$ sinistrorsa

$\vec{E}_2$ destrorsa

$\vec{E} = \vec{E}_1 + \vec{E}_2 = \hat{u}_x A\left(\cos(kz - \omega t) + \cos(kz - \omega t + \phi)\right) + \hat{u}_y A\left(\sin(kz - \omega t) - \sin(kz - \omega t + \phi)\right)$

$= \hat{u}_x A\left(2\cos\left(\frac{2kz - 2\omega t + \phi}{2}\right)\cos\left(\frac{\phi}{2}\right)\right) + \hat{u}_y A\left(2\cos\left(\frac{2kz - 2\omega t + \phi}{2}\right)\sin\left(\frac{\phi}{2}\right)\right)$

17 $\vec{B}(x,y,z,t) = 2 \cdot 10^{-6}(\hat{u}_x + 2\hat{u}_y)\cos\left(\underbrace{(2\pi \cdot x + 2z) \cdot 10^9}_{\alpha} - 6 \cdot 10^8 t\right)$

a) deve valere $\Box \vec{B} = 0$

$\nabla^2 \vec{B} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\vec{B} = 0$

$\frac{\partial}{\partial x}\left(-4 \cdot 10^{-9}\sin\alpha\right) + \frac{\partial}{\partial y}\left(2 \cdot 10^{-9}\sin\alpha\right) + \frac{\partial}{\partial z}\left(-4 \cdot 10^{-9}\sin\alpha\right) -$

$- \varepsilon\mu\frac{\partial}{\partial t}\left(+12 \cdot 10^{-1}\sin\alpha\right) = 0$

$-8 \cdot 10^{-12}\cos\alpha + 2 \cdot 10^{-12}\cos\alpha - 8 \cdot 10^{-12}\cos\alpha + \varepsilon\mu\, 72 \cdot 10^{8}\cos\alpha = 0$

$-8 \cdot 10^{-12} + 2 \cdot 10^{-12} - 8 \cdot 10^{-12} + \varepsilon\mu\, 72 \cdot 10^{8} = 0$

$\varepsilon\mu = \frac{18 \cdot 10^{-12}}{72 \cdot 10^{8}} \rightarrow \frac{1}{v^2} = \frac{9 \cdot 10^{-16}}{36}$

$\rightarrow v = \sqrt{\frac{36}{9}\,10^{16}} = 2 \cdot 10^{8}$ m/s

vedo $\omega = 6 \cdot 10^{8}$ rad/s

$\nu = \frac{\omega}{2\pi} = 9{,}55 \cdot 10^{7}$ Hz

$\lambda = \frac{v}{\nu} = 2{,}094 \cdot 10^{1}$ m

$\Rightarrow \mathcal{E} = 2A\left(\mu_x \cos\frac{\phi}{2} + \mu_y \sin\frac{\phi}{2}\right)\cos\left(\frac{2kz - 2\omega t + \phi}{2}\right)$

ALLORA $\tan\delta = \frac{\mathcal{E}_x}{\mathcal{E}_y} = \frac{2A\cos\frac{\phi}{2}}{2A\sin\frac{\phi}{2}} = \cot\frac{\phi}{2}$

$\Rightarrow$ POLARIZZAZIONE CIRCOLARE

b) $I_1 = A_1^2 \Rightarrow A_1 = \sqrt{I_1}$

$I_2 = 4I_1 = A_2^2 \Rightarrow A_2 = 2\sqrt{I_1}$

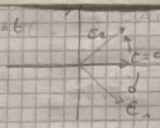
$\mathcal{E}_1 = A_1\hat{\mu}_x \cos(\xi) + \hat{\mu}_y A_2 \sin(\xi)$

$\mathcal{E}_2 = \mu_x A_2 \cos(\xi) - \hat{\mu}_y A_2 \sin\xi$, $kz - \omega t$

SOMMANDO ARRIVI COMUNQUE AL RAPPORTO, MA STAVOLTA NON SI ELIDONO LE AMPIEZZE

$\Rightarrow$ POLARIZZAZIONE ELLITTICA

$z = 0 = t$; $\omega t = \phi/2$



$\Rightarrow \mathcal{E}_1$ SINISTRORSA

$\mathcal{E}_2$ DESTRORSA

$\vec{\mathcal{E}} = \vec{\mathcal{E}}_1 + \vec{\mathcal{E}}_2 = \hat{\mu}_x A\left(\cos(kz + \omega t) + \cos(kz - \omega t + \phi)\right) + \hat{\mu}_y A\left(\sin(kz - \omega t) - \sin(kz - \omega t + \phi)\right)$

$= \mu_x A\left(2\cos\left(\frac{2kz - 2\omega t + \phi}{2}\right)\cos\left(\frac{\phi}{2}\right)\right) + \mu_y A\left(2\cos\left(\frac{2kz - 2\omega t + \phi}{2}\right)\sin\frac{\phi}{2}\right)$

$\vec{E}(x,y,z,t) = 2\cdot 10^{-6}\left(\hat{\mu}_x + 2\hat{\mu}_y\right)\cos\left((2\pi\cdot\ldots\cdot z\ldots\cdot 10^{7} - 6\cdot 10^{7}t\right)$

DEVE VALERE $\Box\vec{E} = 0$

$\nabla^2\vec{E} - \varepsilon\mu\frac{\partial^2}{\partial t^2}\vec{E} = 0$

$\left(-4\cdot 10^{-3}\sin(\alpha)\right) + \frac{\partial}{\partial z}\left(2\cdot 10^{-3}\sin(\alpha)\right) + \frac{\partial}{\partial z}\left(-4\cdot 10^{-3}\sin(\alpha)\right) -$

$- \varepsilon\mu\frac{\partial}{\partial t}\left(+12\cdot 10^{-4}\sin(\alpha)\right) = 0$

$\cdot 10^{-12}\cos\alpha - 2\cdot 10^{-12}\cos\alpha - 8\cdot 10^{-12}\cos\alpha + \varepsilon\mu\, 72\cdot 10^{6}\cos\alpha = 0$

$\cdot 10^{-12} + 2\cdot 10^{-12} + 8\cdot 10^{-12} = \varepsilon\mu\, 72\cdot 10^{6}$

$\mu = \frac{18\cdot 10^{-12}}{72\cdot 10^{6}} \rightarrow \frac{1}{v^2} = \frac{9\cdot 10^{-6}}{36}$

$\rightarrow v = \sqrt{\frac{36}{9}\,10^{16}} = 2\cdot 10^{8}\ m/s$

VERO $\omega = 6\cdot 10^{7}\ \frac{rad}{s}$

$\nu = \frac{\omega}{2\pi} = 9{,}55\cdot 10^{6}\ Hz$

$\lambda = \frac{v}{\nu} = 2{,}094\cdot 10^{1}\ m$

D) $\vec{B}(x,y,z,t) = B_0(\mu_x + 2\mu_y)\cos(\omega t)$

$\vec{k}\cdot\vec{r} = (2x - y + 2z)\cdot 10^{-3}$

$\Downarrow$

$(2,-1,2)10^{-3}\cdot(x,y,z)$

$\Rightarrow \vec{v}$ DIREZIONE GENERICA DI PROPAGAZIONE

$\hat{k} = \frac{1}{3}(2,-1,2)$

$\hat{B} = \frac{\vec{B}}{|\vec{B}|} = \frac{1}{\sqrt{5}}(\hat{\mu}_x + 2\hat{\mu}_y)$

$\hat{E} = \hat{B}\times\hat{k} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1/\sqrt{5} & 2/\sqrt{5} & 0 \\ 2/3 & -1/3 & 2/3 \end{vmatrix} = \begin{pmatrix} 4/3\sqrt{5} \\ -2/3\sqrt{5} \\ -5/3\sqrt{5} \end{pmatrix} = \hat{m}$

$\vec{E} = |E_0|\hat{v}\left(\frac{4}{3\sqrt{5}}\hat{\mu}_x - \frac{2\hat{\mu}_y}{3\sqrt{5}} - \frac{5\hat{\mu}_z}{3\sqrt{5}}\right)\cos(\omega t)$

$|B_0||\vec{v}| = \sqrt{5}|B_0|^2\sqrt{\frac{\varepsilon}{\mu}}v^2 = \sqrt{\frac{\varepsilon}{\mu}}|B_0|v$

$= 894.4\ V/m$

$\vec{E} = 894.4\,\hat{m}\cos(\omega t)$

$\vec{H} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4/3\sqrt{5} & -2/3\sqrt{5} & -5/3\sqrt{5} \\ 1/\sqrt{5} & 2/\sqrt{5} & 0 \end{vmatrix} = \begin{pmatrix} 10/3\sqrt{5} = 2/3 \\ -5/3\sqrt{5} = -1/3 \\ 9/3\sqrt{5} = 2/3 \end{pmatrix}$

$\vec{I} = \frac{|E_0||B_0|}{\mu_0} = (\frac{2}{3}\mu_x - \frac{\mu_y}{3} + \frac{2}{3}\mu_z)\cos^2(\omega t)$

$\vec{I} = 3,18\cdot 10^3\left(\frac{2}{3}\mu_x - \frac{\mu_y}{3} + \frac{2}{3}\mu_z\right)\cos^2(\omega t)$

$\vec{E} = \begin{cases} \vec{E}_x = (533,05)\cos(\omega t) \\ \vec{E}_y = (-266,54)\cos(\omega t) \\ \vec{E}_z = (-666,35)\cos(\omega t) \end{cases}$

$\forall x, y?\quad \tan\varphi = \frac{E_z}{E_y} \neq t$

$\Rightarrow$ POLARIZZAZIONE LINEARE

c) $\langle\vec{I}\rangle = 3,18\cdot 10^3\sqrt{\frac{4+1+4}{9}}\int_0^{\frac{T}{2}}\cos^2(\omega t) = \frac{3,18\cdot 10^3}{2}$

$= 1,59\cdot 10^3\ W/m^2$

18 $\nu = 600\ kHz \qquad n \simeq 9 \qquad \hat{\mu}_z$

POL. LINEARE con $\vec{E} = E\,\hat{\mu}_x$

$\langle\vec{I}\rangle = 30\ W/m^2$

— O —

$n = \frac{c}{v} \Rightarrow v = \frac{c}{n} = 3,3\cdot 10^7\ m/s$

$\lambda = \frac{v}{\nu} = 55,6\ mm$

$\langle\vec{I}\rangle = \frac{E_{EFF}^2}{Z} \longrightarrow E_{EFF} = \sqrt{\langle I\rangle Z} = 35,39\ V/m$

$\varepsilon_r = 81,07\ ;\ \varepsilon = 7,17\cdot 10^{-10}\ ;\ \mu = 1,25\cdot 10^{-6} \qquad = 1,059\cdot 10^{-6}\ T$

$\langle\vec{I}\rangle = H_{EFF}^2 Z \longrightarrow H_{EFF} = \sqrt{\langle I\rangle/Z} \Rightarrow B_{EFF} = \mu\sqrt{I/Z} =$

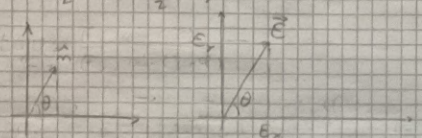
19

$\vec{E} = E_x \hat{u}_x \sin(kz - \omega t) + E_y \hat{u}_y \sin(kz - \omega t + \varphi)$

$E_x = \mathcal{E} = 50\ V/m$

a) POL LINEARE CON $\vec{E}$ $\nparallel$ a $\hat{n}$

$\hat{n} = \frac{1}{2}\hat{u}_x + \frac{\sqrt{3}}{2}\hat{u}_y$



$n_x = \cos\theta = \frac{1}{2} \rightarrow \theta = \pi/3$

$n_y = \sin\theta = \sqrt{3}/2$

quindi $\begin{cases} E_x = E\cos\theta \\ E_y = E\sin\theta \end{cases} \rightarrow \begin{cases} E = \dfrac{E_x}{\cos\theta} \\ E_y = E\tan\theta \end{cases}$

allora $E_y = \sqrt{3}\,\mathcal{E} = 10^3\sqrt{3}\ V/m$

se ho pol lineare $\Rightarrow \varphi = 0$

$\langle \vec{I} \rangle = c\varepsilon_0 \frac{E^2}{2}$ $\quad E = \sqrt{E_x^2 + E_y^2} = 119{,}98\ V/m$

$= 19{,}11\ W/m^2$

b) pol circolare $\Rightarrow \varphi = \pm\frac{\pi}{2}$, $E_x = E_y = \mathcal{E}$

$\mathcal{E}\ \mathrm{tg}\,\psi = \mathrm{tg}(\omega t)$

$\langle \vec{I} \rangle = \frac{1}{2}\varepsilon_0 c E^2 = \frac{\varepsilon_0 c}{2}(\sqrt{2}\mathcal{E})^2 = 9{,}6\ W/m^2$

c) $E_x = E_y$, $\varphi = \pi/4$

φ non è caso parz. collare

$\Rightarrow$ polarizzazione ellittica

$\langle \vec{I} \rangle = \frac{\varepsilon_0 c}{2}(\sqrt{2}\mathcal{E})^2 = 9{,}6\ W/m^2$

20 vuoto $\quad \delta = S_0 \cos^2(x + y + \sqrt{2}z - \omega t)$

$S_0 = 50\ W/m^2$

a) trovo campi del tipo

$E = E_0 \cos(x + y + \sqrt{2}z - \omega t)$

quindi $\vec{k}\cdot\vec{r} = (x + y + \sqrt{2}z)$

$\Rightarrow \vec{k} = (1, 1, \sqrt{2})$, $\vec{r} = (x, y, z)$

$|\vec{k}| = 2$

$\omega = kv = 6\cdot 10^8\ \frac{rad}{s}$

$\nu = \frac{\omega}{2\pi} = 9{,}55\cdot 10^7\ Hz$

ma $I_0 = c\varepsilon_0 E_0^2 \rightarrow E_0 = \sqrt{\frac{I_0}{c\varepsilon_0}} = 137{,}23\ V/m$

$\hat{n} = (1, -1, 0)$ $\quad \hat{n} = \frac{1}{\sqrt{2}}(1, -1, 0) \Rightarrow$ inclinato su xy di $\theta = \pi/3$

$$E_x = \frac{\sqrt{2}}{2} E_0 = 9\mathrm{T},02\ V/m = E_y$$

$$\vec{E}(x,y,z,t) = 9\mathrm{T},02\,(\mu_x + \mu_y)\cos(x + y + \sqrt{2}z - \omega t)$$

$$B_0 = \frac{E_0}{c} = 4,5\mathrm{T} \cdot 10^{-\mathrm{T}}\ T$$

$$\vec{k} = (1,1,\sqrt{2}) \longrightarrow \hat{k} = \frac{1}{2}(\mu_x + \mu_y + \sqrt{2}\mu_z)$$

$$\hat{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1/2 & 1/2 & \sqrt{2}/2 \\ \sqrt{2} & -\sqrt{2} & 0 \end{vmatrix} = \left(\tfrac{1}{2}, \tfrac{1}{2}, -\tfrac{1}{\sqrt{2}}\right)$$

$$\vec{B}(x,y,z,t) = 4,5\mathrm{T} \cdot 10^{-\mathrm{T}} \left(\frac{\mu_x}{2} + \frac{\mu_y}{2} - \frac{\mu_z}{\sqrt{2}}\right) \cos(x + y + \sqrt{2}z + \omega t)$$

$$2,29 \cdot 10^{-\mathrm{T}}(\mu_x + \mu_y - \sqrt{2}\mu_z)$$

$$\vec{B}_0 \cdot \begin{pmatrix} 1/2 \\ 1/2 \\ -\sqrt{2} \end{pmatrix} = \vec{B}_0 \cdot \begin{pmatrix} 1 \\ 1 \\ -\sqrt{2} \end{pmatrix} \frac{1}{2} = 2,29 \cdot 10^{-\mathrm{T}} \begin{pmatrix} 1 \\ 1 \\ -\sqrt{2} \end{pmatrix}$$

Es 1

a) SO CHE LA QTÀ DI MOTO TRASPORTATA DA $\vec{p} = \frac{\vec{F}}{v}$ NEL $\left(\frac{\Delta E = I t}{\Delta A = F_n t}\right)$ DI θ

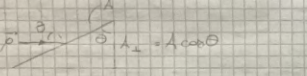
NEL CASO DI SUPERFICIE PERFETTAMENTE ASSORBENTE

$\vec{p}_{in} = \vec{p}_{ass}$ ONDA $\quad \vec{p}_r = 0$

$\vec{p}_{in} = \vec{p}_{ass}$ SUP $\quad \vec{p}_R = \vec{p}_{in}$

CONSERVANDO LA QTÀ DI MOTO ASSUMENDO L'INTERAZIONE COME UN URTO

DISEGNO

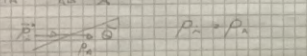


IN UN Δt LA $\vec{p}$ CHE ATTRAVERSA A CON UN ANGOLO θ È LA STESSA CHE ATTRAVERSO $A_\perp$

ALLORA

$$\frac{\Delta \vec{p}}{\Delta t} = \frac{I}{c} A_\perp = \frac{I}{c} A\cos\theta$$

COMUNQUE LA QUANTITÀ DI MOTO INCIDENTE VIENE COMPLETAMENTE TRASFERITA AD A



$\vec{p}_{in} = \vec{p}_R$

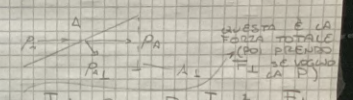
SU CHE $|\vec{p}_i| = |\vec{p}_R| = \frac{I}{c}$

SE INFINE SU UNA SUPERFICIE A (CHE SE $A_\perp = A\cos\theta$)

$$\frac{\Delta \vec{p}_R}{\Delta t} = \frac{I}{c} A\cos^2\theta = \vec{F}_R$$

CHE SARÀ DOVUTA SOLO LA COMPONENTE DELLA QTÀ DI MOTO IN UN Δt SULLA SUPERFICIE ORTOGONALE AD A

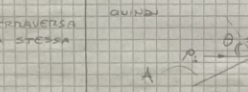
CONSIDERO LA $A_\perp$ INCIDENTE NORMALMENTE SU $A_\perp$



COSÌ SE $\vec{F}_A = \frac{I}{c} A\cos\theta \Rightarrow P = \frac{I}{c}\cos\theta = \frac{\vec{F}_\perp}{A}$

b) SUPERFICIE RIFLETTENTE (PERFETTAMENTE)

QUINDI



SO CHE SI DEVE CONSERVARE LA QTÀ DI MOTO

$\vec{p}_i = \vec{p}_r + \vec{p}_a$

SO CHE LA SUPERFICIE È PERFETTAMENTE RIFLETTENTE QUINDI

$|\vec{p}_i| = |\vec{p}_r|$ E VERO $\begin{cases} p_i\cos\theta = -p_r\cos\theta \\ p_i\sin\theta = p_R\sin\theta \end{cases}$

(a) CALCOLO LA p_A SULLA SUPERFICIE $A_\perp$ (COME PRIMA PARTE ES. GUIDATO)

$\Delta p_A = \frac{I}{c} A\cos\theta$

LA COMPONENTE CHE CONTRIBUISCE A $\vec{F}_{RAD}$ È QUELLA $\perp$

$\Delta p_{A\perp} = \frac{I}{c} A\cos^2\theta$

QUINDI

$F_A = \frac{\Delta p_{A\perp}}{\Delta t} = \frac{I}{c} A\cos^2\theta$

$P_{RAD} = \frac{F_A}{A} = \frac{I}{c}\cos^2\theta$ (*)

(*) OPPURE DA

$P_A = \frac{I}{c}$

ATTRAVERSO A SARÀ

$\frac{\Delta p_A}{\Delta t} = \frac{I}{c} A\cos\theta = F_A$

LA PER LA PRESSIONE SARÀ

$P_{RAD} = \frac{F_{A\perp}}{A} = \frac{I}{c}\cos^2\theta$

QUINDI VALE $\begin{cases} p_{\perp R} = -p_{\perp R} \\ p_{\parallel R} = p_{\parallel R} \end{cases}$

E NON ASSORBENDO NULLA $\vec{p}_A = \vec{p}_{A\perp}$

COSÌ HO

$$\vec{p}_{\perp I} + \vec{p}_{\parallel I} = \vec{p}_{\perp R} + \vec{p}_{\parallel R} + \vec{p}_{A\perp}$$

$$\Rightarrow \vec{p}_{A\perp} = 2\vec{p}_{\perp I} = 2\frac{I}{c}\cos\theta$$

QUINDI AVRÒ IN UN INTERVALLO DI TEMPO Δt E SU UNA SUPERFICIE A (O PENSO SU UNA SUPERFICIE $A_\perp = A\cos\theta$ SU CUI INCIDE $\perp$)

$$\frac{\Delta \vec{p}_{A\perp}}{\Delta t} = 2\frac{I}{c}\cos^2\theta \, A = F_A$$

QUINDI AVRÒ

$$P_{RAD} = 2\frac{I}{c}\cos^2\theta = \frac{F_A}{A}$$

IDEA È COME SE AVESSI "RADDRIZZARE" L'ONDA CON UN $\cos\theta$ E POI "RADDRIZZARE" LA SUPERFICIE CON UN $\cos\theta$

(VEDI DISEGNO MAZZOLDI P. 503)

C) ORA HO SFERA R

CASO PERFETTAMENTE ASSORBENTE

QUINDI $|\vec{p}_A| = |\vec{p}_{A\perp}|$

$p_\perp = \frac{I}{c}$ SU $dA_\perp = dA\cos\theta$ → RAGIONAMENTI DI PRIMA

$$\frac{\Delta p_\perp}{\Delta t} = dF = \frac{I}{c} dA\cos\theta$$

$$F = \frac{I}{c}\int_0^{2\pi} d\varphi \int_0^{\pi/2} \cos\theta \sin\theta R^2 \, d\theta = 2\pi\frac{I}{c}R^2\frac{1}{2} = \frac{I}{c}R^2\pi$$

CASO PERFETTAMENTE RIFLETTENTE

CON RAGIONAMENTI SUGLI COMPONENTI SI PUÒ TROVARE COME NEL PUNTO PRIMA

$p_{A\perp} = 2p_{I\perp}$ SU $dA_\perp = dA\cos\theta$

$p_A = p_{A\perp} = 2\frac{I}{c}\cos\theta$ SU dA IN Δt

$\frac{\Delta p_A}{\Delta t} = dF_\perp = 2\frac{I}{c} dA\cos^2\theta$ MA LA SFERA ESSENDO SIMMETRICA POSSO SCOMPORRE IN COMPONENTI. $dF_\perp = dF$ E ANNULLARE

$dF_\perp = dF\cos\left(\frac{\pi}{2} - \theta\right) = dF\sin\theta$

è NO SOLO IL CONTRIBUTO DI

$$dF_x = 2\frac{I}{c}\,d\ell\cos^2\theta = dF\cos\theta$$

INTEGRANDO

$$F = 2\frac{I}{c}\int_0^{2\pi} d\varphi \int_0^{\pi/2} d\theta \sin\theta\cos\theta\, R^2$$

$$= 2\frac{I}{c}\,2\pi R^2\,\frac{1}{4} = \frac{I}{c}\pi R^2$$

22 $\frac{I}{c} = 2{,}2\ \mathrm{cal/cm^2}$

ONDA PIANA INCIDENTE $\perp$ ALLA TERRA

(IPOTESI: TERRA TOTALMENTE ASSORBENTE)

$$I = 2{,}2\ \frac{1}{60}\ \mathrm{cal}\ \frac{1}{\pi}\ \mathrm{cm^2} = \frac{2{,}2}{60}\cdot 4{,}186\ \frac{\mathrm{W/cm^2}}{\mathrm{cm^2}}$$

$$= \frac{2{,}2\cdot 4{,}186}{60}\cdot 10^4\ \mathrm{W/m^2} = 1{,}5344\cdot 10^3\ \mathrm{W/m^2}$$

SO $\langle I\rangle = \frac{E^2}{2}\frac{1}{Z} \rightarrow E_0 = \sqrt{2Z\,I} = 1{,}075\cdot 10^3\ \mathrm{V/m}$

$\langle I\rangle = \frac{H_0^2}{2}Z \rightarrow H_0 = \sqrt{\frac{2I}{Z}} \rightarrow B_0 = \mu_0\sqrt{\frac{2I}{Z}} = 3{,}58\cdot 10^{-3}\ \mathrm{T}$

$$P_{RAD} = \frac{I}{c}\pi R^2 = 5{,}11\cdot 10^{-6}\ \mathrm{N/m^2}$$

23 MATERIALE DEFINITIVAMENTE ASSORBENTE

$R_\oplus, m_\oplus$

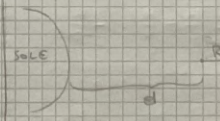
$P_\odot = 3{,}90\cdot 10^{26}\ \mathrm{W}$ $\quad M_\odot = 2\cdot 10^{30}\ \mathrm{Kg}$

SO CHE $P_{RAD} = \frac{I}{c}\pi R^2$ PER UNITÀ DI $\mathrm{m^2}$ DI $\mathcal{S}$

CALCOLO $I_\odot = \frac{P_\odot}{4\pi(R+d)^2}$

$$F_{GRAV} = \frac{m M_\odot}{(R+d)^2}G$$

DOVE $R+d$ = (RAGGIO VELA) + (DISTANZA VELA - ⊙)



ALLORA

$$F_{RAD} = P_{RAD}\cdot A = \frac{I_\odot}{c}\pi R^2$$

RISULTATO DIMOSTRATO DAL PUNTO C.) DELL'ES 21

QUINDI HO

$$F_G = \frac{m M_\odot}{(R+d)^2}G \qquad F_{RAD} = \frac{P_\odot R^2}{4c(R+d)^2}$$

AFFINCHÉ SI ALLONTANI VOGLIO $F_{RAD} > F_G$

$$\frac{P_\odot R^2}{4c(R+d)^2} > \frac{m M_\odot G}{(R+d)^2} \rightarrow \frac{m}{R^2} < \frac{P_\odot}{4cM_\odot G} = 2{,}47\cdot 10^{-3}\ \frac{\mathrm{Kg}}{\mathrm{m^2}}$$

24 $e = 1{,}602 \cdot 10^{-19}\ C$ $\quad m_e = 9{,}109 \cdot 10^{-31}\ kg$

$r_0 = 0{,}53 \cdot 10^{-10}\ m$

$m_p \gg m_e$

a) ORBITA CIRCOLARE $\Rightarrow F_c = F_{coul}$

$$\frac{e^2}{4\pi\varepsilon_0}\frac{1}{r^2} = m\frac{v^2}{r} \Rightarrow v = \sqrt{\frac{K}{rm}}$$

CHE SARÀ LA VELOCITÀ TANGENZIALE

$$a_m = \frac{v^2}{r} = \frac{K}{r^2 m}$$

$$E = \frac{1}{2}mv^2 + \frac{K}{r} = \frac{1}{2}\frac{K}{r} - \frac{K}{r} = -\frac{K}{2r}$$

PER LA POTENZA USO LA FORMULA DI LARMORE AVENDO UN DIPOLO OSCILLANTE

$$P = \frac{e^2 a^2}{6\pi\varepsilon_0 c^3} = \frac{e^2}{6\pi\varepsilon_0 c^3}\frac{K^2}{r^4 m^2} = \frac{e^2}{4\pi\varepsilon_0}\frac{4K^2}{3r^4 m^2 c^3}$$

$$= \frac{2K^2}{3r^4 m^2 c^3}$$

b) USO LA E $\quad \frac{dr}{dt} = \frac{dr}{dE}\frac{dE}{dt}$

MA $r(E) = -\frac{K}{2E}$ $\quad \frac{dr}{dE} = \frac{K}{2E^2}$

$$\frac{dr}{dt} = \frac{K}{2}\frac{4r^2}{\frac{e^4}{15\pi^2\varepsilon_0^2}} = \frac{e^2}{8\pi\varepsilon_0}\frac{16\cdot 4r^2\pi^2\varepsilon_0^2}{e^4}$$

$$= \frac{8r^2\pi\varepsilon_0}{e^2}$$

MA SO ANCHE CHE $P = -\frac{dE}{dt}$ → LA STA PERDENDO

QUINDI

$$\frac{dr}{dt} = \frac{dr}{dE}\frac{dE}{dt} = \frac{-8r^2\pi\varepsilon_0}{e^2}P = \frac{-8r^2\pi\varepsilon_0}{e^2}\frac{2K^3}{3r^4m^2c^3}$$

$$= \frac{-16K^3\pi\varepsilon_0}{3e^2m^2c^3r^2} = \frac{-4K^3}{3m^2c^3r^2}\frac{4\pi\varepsilon_0}{e^2} = \frac{-4K^2}{3m^2c^3r^2}$$

c) CON $r(0) = r_0$

$$\frac{dr}{dt} = -\frac{4K^2}{3m^2c^3r^2} \rightarrow dt\,r^2 = -\frac{4K^2}{3m^2c^3}dt$$

$$\frac{r^3}{3}\Big|_{r_0}^{r} = -\frac{4K^2}{3m^2c^3}t\Big|_0^t \rightarrow \frac{r^3}{3} - \frac{r_0^3}{3} = -\frac{4K^2t}{3m^2c^3}$$

$$r(t) = \left(r_0^3 - \frac{4K^2t}{m^2c^3}\right)^{1/3}$$

d) $r(t) = 0 \rightarrow t = \frac{r_0^3 m^2 c^3}{4K^2} = 1{,}576 \cdot 10^{-11}\ s$

$K = 2{,}3 \cdot 10^{-28}$

QUINDI PER LA FISICA CLASSICA UN ELETTRONE NON È STABILE MA CADE VELOCISSIMAMENTE SUL PROTONE