Esercizi - Fondamenti di Teoria Quantistica dei campi

APPUNTI PIENI DI ERRORI, SVISTE, BUCHI E DUBBI. SICURAMENTE GLI APPUNTI IN LATEX CONTENGONO MENO ERRORI E BUCHI, MA NON GARANTISCO NULLA SU SVISTE E, SOPRATTUTTO, DUBBI.

Scattering

Relazioni Utili

Spinori

$$\begin{cases} \sum_s u_\alpha(\vec{p})\, \bar{u}_\beta(\vec{p}) = (\not{p} + m)_{\alpha\beta} \\ \sum_s v_\alpha(\vec{p})\, \bar{v}_\beta(p) = (\not{p} - m)_{\alpha\beta} \end{cases}$$

Variabili di Handelstam

$$\begin{cases} s = (p_1 + p_2)^2 = (p_3 + p_4)^2 \\ t = (p_1 - p_3)^2 = (p_2 - p_4)^2 \\ u = (p_1 - p_4)^2 = (p_2 - p_3)^2 \end{cases} \xRightarrow[(m \ll p)]{\text{alte energie}} \begin{cases} p_1 p_2 = p_3 p_4 = \frac{s}{2} \\ p_1 p_3 = p_2 p_4 = -\frac{t}{2} \\ p_1 p_4 = p_2 p_3 = -\frac{u}{2} \end{cases}$$

Sezione d'urto

$$\left.\frac{d\sigma}{d\Omega}\right|_{cm} = \frac{1}{64\pi^2 s} \frac{|p_1'|}{|p_1|} |\mathcal{M}|^2$$

Polarizzazioni

$$\sum_\lambda \varepsilon_\mu^{(\lambda)}(p)\, \varepsilon_\nu^{(\lambda)*}(p) = -\eta_{\mu\nu}$$

MATRICI γ

$$\mathrm{Tr}\{\gamma^\mu \gamma^\nu\} = 4\eta^{\mu\nu}$$

$$\mathrm{Tr}\{\gamma^\alpha \gamma^\mu \gamma^\beta \gamma^\nu\} = 4\left(\eta^{\alpha\mu}\eta^{\beta\nu} + \eta^{\alpha\nu}\eta^{\beta\mu} - \eta^{\alpha\beta}\eta^{\mu\nu}\right)$$

$$\mathrm{Tr}\{\gamma_\rho \gamma^\nu \gamma_\sigma \gamma^\mu \gamma_\lambda \gamma_\nu \gamma_\tau \gamma_\mu\} = -32\,\eta_{\sigma\tau}\eta_{\lambda\rho}$$

REGOLE DI FEYNMAN

ELENCO REGOLE DI FEYNMAN PER LA TEORIA DI YUKAWA E LA QED.

YUKAWA: (SPAZIO DEGLI IMPULSI)

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}m^2\phi^2 + \bar{\psi}(i\partial\!\!\!/ - M)\psi - g\phi\bar{\psi}\psi$$

- AD OGNI LINEA <u>INTERNA</u>:

$$\tilde{\Delta}(p) = \frac{i}{p^2 - m^2 + i\varepsilon} \qquad (\phi^+\phi)$$

$$\tilde{S}_{\alpha\beta}(p) = i\left(p\!\!\!/ - M + i\varepsilon\right)^{-1}_{\alpha\beta} \qquad (\psi\bar{\psi})$$

- AD OGNI <u>VERTICE</u>:

$$-ig(2\pi)^4\,\delta_{\alpha\beta}\,\delta^4\left(\sum p_i\right)$$

- DIVIDIAMO PER IL FATTORE DI SIMMETRIA.

QED : (SPAZIO DEGLI IMPULSI)

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\xi}(\partial A)^2 + \bar{\psi}(i\not{D} - m)\psi$$

$$\mathcal{L}_{INT} = -eA_\mu \bar{\psi}\gamma^\mu \psi$$

- AD OGNI LINEA <u>INTERNA</u> :

$$\tilde{\Delta}_{\mu\nu}(p) = -\frac{i}{k^2}\eta_{\mu\nu} \qquad (\xi = 1)$$

$$\tilde{S}_{\alpha\beta}(p) = i(\not{p} - M + i\varepsilon)^{-1}_{\alpha\beta} \qquad (\psi\bar{\psi})$$

- AD OGNI <u>VERTICE</u> :

$$-ie(2\pi)^4 \gamma^\mu_{\alpha\beta}\delta^4(\Sigma p_i)$$

- DIVIDIAMO PER IL FATTORE DI SIMMETRIA.

AMPIEZZE DI PROBABILITÀ $\mathcal{M}$:

PER IL CALCOLO DI $\mathcal{M}$ BASTA APPLICARE LE REGOLE PER IL CALCOLO DELLA FUNZIONE DI GREEN $\tilde{G}$, MA RICORDANDO :

- AMPUTARE LE GAMBE ESTERNE
- INSERIRE LE POLARIZZAZIONI : (ℓ = LEPTONE)

$$\phi \sim 1 \qquad (\phi)$$

$$\left[\begin{array}{ll} \ell^-_{IN} \sim u_\alpha(\vec{p}) & (\bar{\psi}) \\ \ell^-_{OUT} \sim \bar{u}_\alpha(\vec{p}) & (\psi) \\ \ell^+_{IN} \sim \bar{v}_\alpha(\vec{p}) & (\psi) \\ \ell^+_{OUT} \sim v_\alpha(\vec{p}) & (\bar{\psi}) \end{array}\right.$$

$$\left[\begin{array}{ll} \gamma_{IN} \sim \varepsilon^*_\mu(\lambda) & (A_\mu) \\ \gamma_{OUT} \sim \varepsilon_\mu(\lambda) & (A_\mu) \end{array}\right.$$

TEORIA DI YUKAWA

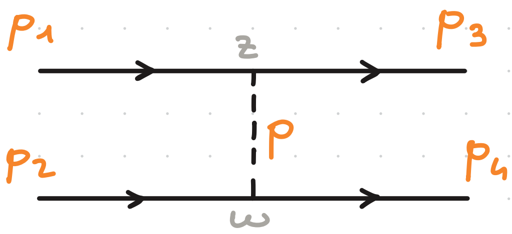
RIFERIMENTI : [SREDNICKI] : CAP. 45 (P. 282 - 290) (REGOLE E SEGNI RELATIVI)
CAP. 46 + 48 (P. 291 - 293 +
+ P. 298 - 302) (CONTI E RISULTATI)
NEGLI ESERCIZI

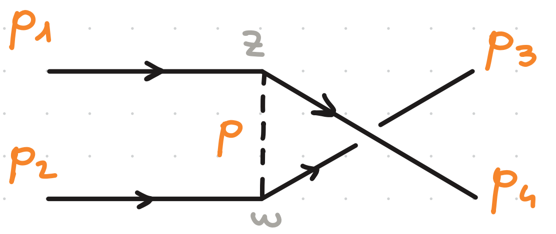
YUKAWA :

$$\boxed{e^- \quad e^- \longrightarrow e^- \quad e^-}$$

(FUNZIONA UGUALE PER $e^+ e^+ \longrightarrow e^+ e^+$)

I DIAGRAMMI SONO :





CANALE t: $p = p_1 - p_3$

CANALE u : $p = p_1 - p_4$

POSSIAMO VEDERE IL SEGNO DEI DUE DIAGRAMMI CON LE CONTRAZIONI :

$$t: \quad \langle 0 | T\left[\bar\psi(x_1)\bar\psi(x_2)\psi(x_3)\psi(x_4)\phi(z)\bar\psi(z)\psi(z)\phi(w)\bar\psi(w)\psi(w)\right] | 0 \rangle$$

$$\begin{cases} \psi(z)\bar\psi(x_1) & 5 \text{ SCAMBI} \\ \psi(w)\bar\psi(x_2) & 5 \text{ SCAMBI} \\ \psi(x_3)\bar\psi(z) & 1 \text{ SCAMBI} \\ \psi(x_4)\bar\psi(w) & 0 \text{ SCAMBI} \end{cases} \implies (-1)^{11} = -1$$

$$\mathcal{M}: \quad \langle 0 | T\left[\bar{\psi}(x_1)\bar{\psi}(x_2)\psi(x_3)\psi(x_4)\phi(z)\bar{\psi}(z)\psi(z)\phi(w)\bar{\psi}(w)\psi(w) \right] | 0 \rangle$$

$$\begin{cases} \psi(z)\bar{\psi}(x_1) & 5 \text{ SCAMBI} \\ \psi(w)\bar{\psi}(x_2) & 5 \text{ SCAMBI} \\ \psi(x_3)\bar{\psi}(w) & 2 \text{ SCAMBI} \\ \psi(x_4)\bar{\psi}(z) & 0 \text{ SCAMBI} \end{cases} \quad \Rightarrow \quad (-1)^{12} = +1$$

DUNQUE C'È UN SEGNO DI DIFFERENZA E DOVREMO CALCOLARE:

$$|\mathcal{M}|^2 = \frac{1}{4}\sum_s \left\{ |\mathcal{M}_t|^2 + |\mathcal{M}_\mu|^2 - 2\,\mathrm{Re}\,\mathcal{M}_t^* \mathcal{M}_\mu \right\}$$

IN CUI ABBIAMO:

$$\mathcal{M}_t = \frac{i(-ig)^2}{(p_1-p_3)^2 - m^2}\left(\bar{u}_\alpha(p_1)\,\delta_{\alpha\beta}\,u_\beta(p_3)\right)\left(\bar{u}_\gamma(p_2)\,\delta_{\gamma\lambda}\,u_\lambda(p_4)\right)$$

$$\mathcal{M}_\mu = \frac{i(-ig)^2}{(p_1-p_4)^2 - m^2}\left(\bar{u}_\alpha(p_1)\,\delta_{\alpha\beta}\,u_\beta(p_4)\right)\left(\bar{u}_\gamma(p_2)\,\delta_{\gamma\lambda}\,u_\lambda(p_3)\right)$$

CALCOLIAMO

$$\frac{1}{4}\sum_s |\mathcal{M}_t|^2 = \frac{(-ig)^4}{4(t-m^2)^2}\sum_s \left(\bar{u}_\alpha(p_1) u_\alpha(p_3)\right)\left(\bar{u}_\gamma(p_2) u_\gamma(p_4)\right) \times$$

$$\times \left(\bar{u}_\beta(p_3) u_\beta(p_1)\right)\left(\bar{u}_\delta(p_4) u_\delta(p_2)\right)$$

$$= \frac{(-ig)^4}{4(t-m^2)^2}(\not{p}_3 - m)_{\alpha\beta}(\not{p}_1 - m)_{\beta\alpha}(\not{p}_2 - m)_{\delta\gamma}(\not{p}_4 - m)_{\gamma\delta}$$

$$= \frac{(-ig)^4}{4t^2} Tr(\not{p}_3 \not{p}_1)\, Tr(\not{p}_2 \not{p}_4)$$

$$= \frac{(-ig)^4}{4t^2} 16 (p_3 p_1)(p_2 p_4)$$

$$= + \frac{g^4}{4t^2} 16 \frac{t^2}{4}$$

$$= + g^4$$

$$\frac{1}{4}\sum |\mathcal{M}_\mu|^2 = \frac{(-ig)^4}{4(\mu-m^2)^2}\sum_s \left(\bar{u}_\alpha(p_1)u_\alpha(p_4)\right)\left(\bar{u}_\gamma(p_2)u_\gamma(p_3)\right) \times$$

$$\times \left(\bar{u}_\beta(p_4)u_\beta(p_1)\right)\left(\bar{u}_\delta(p_3)u_\delta(p_2)\right)$$

$$= \frac{(-ig)^4}{4(\mu-m^2)^2}(\not{p}_4 - m)_{\alpha\beta}(\not{p}_1 - m)_{\beta\alpha}(\not{p}_2 - m)_{\delta\gamma}(\not{p}_3 - m)_{\gamma\delta}$$

$$= \frac{(-ig)^4}{4\mu^2}\,\mathrm{Tr}(\not{p}_4\not{p}_1)\,\mathrm{Tr}(\not{p}_2\not{p}_3)$$

$$= \frac{(-ig)^4}{\mu^2}\,16(p_4p_1)(p_2p_3)$$

$$= +\frac{g^4}{4\mu^2}\,16\,\frac{\mu^2}{4}$$

$$= +g^4$$

ORA

$$\frac{1}{4}\sum_s \mathcal{M}_t^* \mathcal{M}_u = \frac{1}{4}(-ig)^4 \sum_s \left(\bar{u}_\alpha(p_3) u_\alpha(p_1)\right)\left(\bar{u}_\beta(p_4) u_\beta(p_2)\right) \times$$

$$\times \frac{+1}{[t-m^2][u-m^2]} \times \left(\bar{u}_\gamma(p_1) u_\gamma(p_4)\right)\left(\bar{u}_\delta(p_2) u_\delta(p_3)\right)$$

$$= +\frac{g^4}{4tu}(\not p_1 - m)_{\alpha\gamma}(\not p_4 - m)_{\gamma\beta}(\not p_2 - m)_{\beta\delta}(\not p_3 - m)_{\delta\alpha}$$

$$\simeq +\frac{g^4}{4tu} p_{1\mu} p_{4\nu} p_{2\rho} p_{3\sigma} \, \mathrm{Tr}\left\{\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma\right\}$$

$$= +\frac{g^4}{4tu} p_{1\mu} p_{4\nu} p_{2\rho} p_{3\sigma} \, 4\left(\eta^{\mu\nu}\eta^{\rho\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \eta^{\mu\rho}\eta^{\nu\sigma}\right)$$

$$= +\frac{g^4}{tu}\left\{(p_1 p_4)(p_2 p_3) + (p_1 p_3)(p_2 p_4) - (p_1 p_2)(p_3 p_4)\right\}$$

$$= +\frac{g^4}{tu}\left\{\frac{u^2}{4} + \frac{t^2}{4} - \frac{s^2}{4}\right\}$$

$$= +\frac{g^4}{4tu}\left(u^2 + t^2 - (t+u)^2\right)$$

$$= + \frac{g^4}{4tu}(-2tu)$$

$$= -\frac{g^4}{2}$$

DUNQUE

$$|\mathcal{M}|^2 = g^4 \left\{ 1 + 1 + 2 \cdot \frac{1}{2} \right\} = 3g^4$$

RISULTATO SREDNICKI:

$$|\mathcal{M}|^2 \sim 3g^4$$ ✓

(CAMBIA L'ESPRESSIONE RISPETTO $e^+e^- \to e^+e^-$ SE NON GUARDIAMO IL LIMITE AD ALTE ENERGIE)

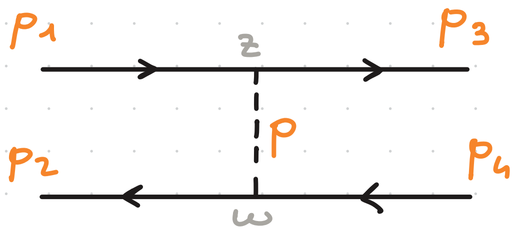
LA SEZIONE D'URTO:

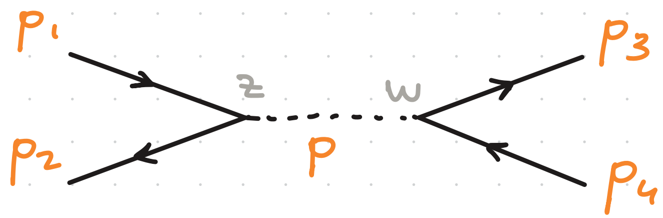
$$\left.\frac{d\sigma}{d\Omega}\right|_{cm} = \frac{|\mathcal{M}|^2}{64\pi^2(p_1+p_2)^2} = \frac{3g^4}{64\pi^2 s}$$

Yukawa :

$$\boxed{e^+ \quad e^- \longrightarrow e^+ \quad e^-}$$

I diagrammi sono :





canale t : $p = p_1 - p_3$ canale s : $p = p_1 + p_2$

Possiamo vedere il segno dei due diagrammi con le contrazioni :

$$t: \quad \langle 0 | T\left[\bar{\psi}(x_1)\psi(x_2)\psi(x_3)\bar{\psi}(x_4)\phi(z)\bar{\psi}(z)\psi(z)\phi(w)\bar{\psi}(w)\psi(w)\right] | 0 \rangle$$

$$\begin{cases} \psi(z)\bar{\psi}(x_1) & 5 \text{ scambi} \\ \psi(x_2)\bar{\psi}(w) & 3 \text{ scambi} \\ \psi(x_3)\bar{\psi}(z) & 1 \text{ scambi} \\ \psi(w)\bar{\psi}(x_4) & 0 \text{ scambi} \end{cases} \quad \Rightarrow \quad (-1)^9 = -1$$

s. $\langle 0 | T\left[\bar{\psi}(x_1)\psi(x_2)\psi(x_3)\bar{\psi}(x_4)\phi(z)\bar{\psi}(z)\psi(z)\phi(\omega)\bar{\psi}(\omega)\psi(\omega)\right] | 0 \rangle$

$$\begin{cases} \psi(z)\bar{\psi}(x_1) & 5 \text{ SCAMBI} \\ \psi(x_2)\bar{\psi}(z) & 2 \text{ SCAMBI} \\ \psi(x_3)\bar{\psi}(\omega) & 1 \text{ SCAMBI} \\ \psi(\omega)\bar{\psi}(x_4) & 0 \text{ SCAMBI} \end{cases} \Rightarrow (-1)^8 = +1$$

CALCOLIAMO QUINDI

$$|\mathcal{M}|^2 = \frac{1}{4}\sum_s \left\{ |\mathcal{M}_s|^2 + |\mathcal{M}_t|^2 - 2\,\mathrm{Re}\,\mathcal{M}_s^*\mathcal{M}_t \right\}$$

CON

$$\mathcal{M}_t = \frac{(-ig)^2\, i}{(p_1-p_3)^2 - m^2}\Big(\bar{u}_\alpha(p_3)u_\alpha(p_1)\Big)\Big(\bar{v}_\beta(p_2)v_\beta(p_4)\Big)$$

$$\mathcal{M}_s = \frac{(-ig)^2\, i}{(p_1+p_2)^2 - m^2}\Big(\bar{v}_\gamma(p_2)u_\gamma(p_1)\Big)\Big(\bar{u}_\delta(p_3)v_\delta(p_4)\Big)$$

CALCOLIAMO

$$\frac{1}{4}\sum_s |\mathcal{M}_t|^2 = \frac{g^4}{4(t-m^2)^2}\sum_s \left(\bar{u}_\alpha(p_1)\, u_\alpha(p_3)\right)\left(\bar{v}_\beta(p_4)\, v_\beta(p_2)\right) \times$$

$$\times \left(\bar{u}_\gamma(p_3)\, u_\gamma(p_1)\right)\left(\bar{v}_\delta(p_2)\, v_\delta(p_4)\right)$$

$$= \frac{g^4}{4(t-m^2)^2}\left(\not{p}_3 - m\right)_{\alpha\gamma}\left(\not{p}_1 - m\right)_{\gamma\alpha} \times$$

$$\times\left(\not{p}_2 + m\right)_{\beta\delta}\left(\not{p}_4 + m\right)_{\delta\beta}$$

$$\sim \frac{g^4}{4t^2}\,\mathrm{Tr}\left\{\not{p}_3\not{p}_1\right\}\mathrm{Tr}\left\{\not{p}_2\not{p}_4\right\}$$

$$= \frac{g^4}{4t^2}\,16(p_1 p_3)(p_2 p_4)$$

$$= \frac{4g^4}{t^2}\cdot\frac{t^2}{4}$$

$$= g^4$$

$$\frac{1}{4}\sum_s |\mathcal{M}_s|^2 = \frac{g^4}{4(s-m^2)^2} \sum_s \left(\bar{u}_\alpha(p_1) v_\alpha(p_2)\right)\left(\bar{v}_\beta(p_4) u_\beta(p_3)\right) \times$$

$$\times \left(\bar{v}_\gamma(p_2) u_\gamma(p_1)\right)\left(\bar{u}_\delta(p_3) v_\delta(p_4)\right)$$

$$= \frac{g^4}{4(s-m^2)^2} (\not{p}_2 + m)_{\alpha\gamma} (\not{p}_1 - m)_{\gamma\alpha} \times$$

$$(\not{p}_3 - m)_{\beta\delta} (\not{p}_4 + m)_{\delta\beta}$$

$$\sim \frac{g^4}{4s^2} \mathrm{Tr}\{\not{p}_2 \not{p}_1\} \mathrm{Tr}\{\not{p}_3 \not{p}_4\}$$

$$= \frac{g^4}{4s^2} 16 (p_1 p_2)(p_3 p_4)$$

$$= \frac{4g^4}{s^2} \frac{s^2}{4}$$

$$= g^4$$

$$\frac{1}{4} \sum \mathcal{M}_s^+ \mathcal{M}_t = \frac{(-i)^i i g^4}{4[s-m^2][t-m^2]} \sum_s \left(\bar{u}_\gamma(p_1) v_\gamma(p_2)\right)\left(\bar{v}_\delta(p_4) u_\delta(p_3)\right) \times$$

$$\times \left(\bar{u}_\alpha(p_3) u_\alpha(p_1)\right)\left(\bar{v}_\beta(p_2) v_\beta(p_4)\right)$$

$$= \frac{+g^4}{4[s-m^2][t-m^2]} (\not{p}_2 + m)_{\gamma\beta} (\not{p}_4 + m)_{\beta\delta} (\not{p}_3 - m)_{\delta\alpha} (\not{p}_1 - m)_{\alpha\gamma}$$

$$\sim \frac{+g^4}{4st} \mathrm{Tr}\left\{ \not{p}_2 \not{p}_4 \not{p}_3 \not{p}_1 \right\}$$

$$= \frac{+g^4}{4st} p_{2\mu} p_{4\nu} p_{3\rho} p_{1\sigma} \, 4\left(\eta^{\mu\nu}\eta^{\rho\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \eta^{\mu\rho}\eta^{\nu\sigma}\right)$$

$$= \frac{+g^4}{st}\left\{ (p_2 p_4)(p_1 p_3) + (p_1 p_2)(p_3 p_4) - (p_2 p_3)(p_1 p_4) \right\}$$

$$= \frac{+g^4}{st}\left(\frac{t^2}{4} + \frac{s^2}{4} - \frac{u^2}{4} \right)$$

$$= \frac{+g^4}{st4}\left(t^2 + s^2 - (s+t)^2 \right)$$

$$= -\frac{g^4}{2}$$

DUNQUE

$$|\mathcal{M}|^2 = g^4 + g^4 + 2 \cdot \frac{g^4}{2} = 3g^4$$

RISULTATO SREDNICKI:

$$|\mathcal{M}|^2 = g^4 \left\{ \frac{(s-4m^2)^2}{(M^2-s)^2} + \frac{st - 4m^2 u}{(M^2-s)(M^2-t)} + \frac{(t-4m^2)^2}{(M^2-t)^2} \right\} \sim g^4 \left\{ 1 + 1 + 1 \right\} \checkmark$$

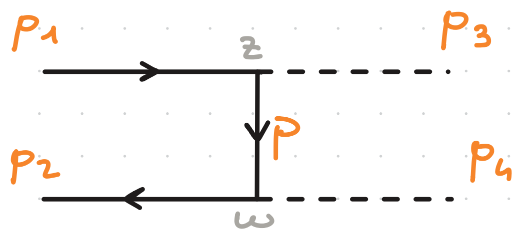
LA SEZIONE D'URTO È:

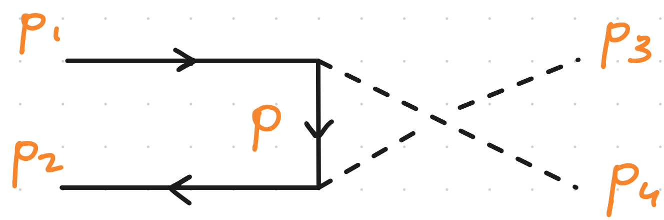
$$\left.\frac{d\sigma}{d\Omega}\right|_{cm} = \frac{|\mathcal{M}|^2}{64\pi^2 s} = \frac{3g^4}{64\pi^2 s}$$

YUKAWA :

$$\boxed{e^+ \quad e^- \longrightarrow 2\phi}$$

I DIAGRAMMI SONO :





CANALE t : $p = p_1 - p_3$

CANALE u : $p = p_1 - p_4$

POSSIAMO VEDERE IL SEGNO DEI DUE DIAGRAMMI CON LE CONTRAZIONI :

$$t: \quad \langle 0 | T\left[\bar\psi(x_1)\psi(x_2)\phi(x_3)\phi(x_4)\phi(z)\bar\psi(z)\psi(z)\phi(w)\bar\psi(w)\psi(w) \right] | 0 \rangle$$

$$\begin{cases} \psi(z)\bar\psi(x_1) & 3 \text{ SCAMBI} \\ \psi(x_2)\bar\psi(w) & 1 \text{ SCAMBI} \\ \psi(w)\bar\psi(z) & 0 \text{ SCAMBI} \end{cases} \quad \Longrightarrow \quad (-1)^4 = +1$$

u : $\langle 0 | T\left[\bar\psi(x_1)\psi(x_2)\phi(x_3)\phi(x_4)\phi(z)\bar\psi(z)\psi(z)\phi(w)\bar\psi(w)\psi(w)\right]|0\rangle$

$$\begin{cases} \psi(z)\bar\psi(x_1) & 3 \text{ SCAMBI} \\ \psi(x_2)\bar\psi(w) & 1 \text{ SCAMBI} \\ \psi(w)\bar\psi(z) & 0 \text{ SCAMBI} \end{cases} \Rightarrow (-1)^4 = +1$$

DUNQUE NON C'È SEGNO RELATIVO E DOBBIAMO CALCOLARE:

$$|\mathcal{M}|^2 = \frac{1}{4}\sum_s \left\{ |\mathcal{M}_t|^2 + |\mathcal{M}_u|^2 + 2\,\mathrm{Re}\,\mathcal{M}_t^\dagger \mathcal{M}_u \right\}$$

CON

$$\mathcal{M}_t = (-ig)^2\, i\left(\bar v_\alpha(p_2)\left(\not{p}_t - m + i\varepsilon\right)^{-1}_{\alpha\beta} u_\beta(p_1)\right)$$

$$\mathcal{M}_u = (-ig)^2\, i\left(\bar v_\alpha(p_2)\left(\not{p}_u - m + i\varepsilon\right)^{-1}_{\alpha\beta} u_\beta(p_1)\right)$$

SAPENDO $\quad i(\not{p} - m)^{-1}_{\alpha\beta} = \dfrac{i(\not{p}+m)_{\alpha\beta}}{p^2 - m^2 + i\varepsilon}$

DUNQUE

$$\frac{1}{4}\sum_s |\mathcal{M}_t|^2 = \frac{(-ig)^4}{4(p_t^2-m^2)^2}\sum_s \left(\bar{v}_\alpha(p_2)(\not{p}_t+m)_{\alpha\beta}\, u_\beta(p_1)\right) \times$$

$$\times \left(\bar{u}_\gamma(p_1)(\not{p}_t+m)_{\gamma\delta}\, v_\delta(p_2)\right)$$

$$\sim \frac{g^4}{4t^2}\,\mathrm{Tr}\left\{\not{p}_2\not{p}_t\not{p}_1\not{p}_t\right\}$$

$$\sim \frac{g^4}{4t^2}\,4\Big\{2\left[p_2(p_1-p_3)\right]\left[p_1(p_1-p_3)\right] +$$

$$-(p_1-p_3)(p_1-p_3)(p_2p_1)\Big\}$$

$$= \frac{g^4}{t^2}\left\{2\left[p_2(p_1-p_3)\right]\left[p_1^2-p_1p_3\right]-t(p_1p_2)\right\}$$

$$\sim \frac{g^4}{t^2}\left\{-2(p_1p_3)\left[p_1p_2-p_2p_3\right]-\frac{st}{2}\right\}$$

$p_1^2 \sim 0$

$$= \frac{g^4}{t^2}\left\{+2\frac{t}{2}\left[\frac{s}{2}+\frac{u}{2}\right]-\frac{st}{2}\right\}$$

$$= \frac{g^4}{t^2}\left\{\frac{t}{2}(s+u) - \frac{st}{2}\right\}$$

$$= \frac{g^4}{t^2}\frac{1}{2}\left(-t^2-st\right)$$

$$= \frac{-g^4}{2t}(t+s)$$

$$= +\frac{g^4}{2}\frac{u}{t}$$

IL CONTO DI $|\mathcal{M}_u|^2$ È IDENTICO MA CON IL CAMBIO $p_t \rightarrow p_u$

DUNQUE

$$\frac{1}{4}\sum_s |\mathcal{M}_u|^2 = \frac{(-ig)^4}{4(u-m^2+i\varepsilon)^2}\sum_s \Big(\bar v_\alpha(p_2)\,(\not p_u - m)_{\alpha\beta}\, u_\beta(p_1)\Big)\times$$

$$\times\Big(\bar u_\gamma(p_1)\,(\not p_u - m)_{\gamma\delta}\, v_\delta(p_2)\Big)$$

$$\sim \frac{g^4}{4u^2}\,\mathrm{Tr}\{\not p_1 \not p_u \not p_2 \not p_u\}$$

$$= \frac{g^4}{4u^2}4\Big[(p_1 p_u)(p_2 p_u) + (p_1 p_u)(p_2 p_u) - p_u^2(p_1 p_2)\Big]$$

$$= \frac{g^4}{u^2}\Big[2\, p_1(p_1-p_4)\cdot p_2(p_1-p_4) - u\frac{s}{2}\Big]$$

$$= \frac{g^4}{u^2}\Big[-2(p_1p_4)\big[p_1p_2 - p_2p_4\big] - \frac{us}{2}\Big]$$

$$= \frac{g^4}{u^2}\Big[+u\Big[\frac{s}{2}+\frac{t}{2}\Big] - \frac{us}{2}\Big]$$

$$= \frac{g^4}{2u}(-u-s) = \frac{g^4}{2}\frac{t}{u}$$

ORA

$$\frac{1}{4}\sum_s \mathcal{M}_t^+ \mathcal{M}_u = \frac{+(-ig)^4}{4[t-m^2][u-m^2]} \sum_s \left(\bar{u}_\alpha(p_1) (\not{p}_t + m)_{\alpha\beta} v_\beta(p_2) \right) \times$$

$$\times \left(\bar{v}_\gamma(p_2) (\not{p}_u + m)_{\gamma\delta} u_\delta(p_1) \right)$$

$$\sim \frac{+g^4}{4tu} (\not{p}_2)_{\beta\gamma}\, p_{u\mu} \gamma^\mu_{\gamma\delta}\, (\not{p}_1)_{\delta\alpha}\, p_{t\nu} \gamma^\nu_{\alpha\beta}$$

$$= \frac{+g^4}{4tu} p_{u\mu} p_{t\nu} p_{2\rho} p_{1\sigma} \mathrm{Tr}\{\gamma^\rho \gamma^\mu \gamma^\sigma \gamma^\nu\}$$

$$= +\frac{g^4}{tu} p_{u\mu} p_{t\nu} p_{2\rho} p_{1\sigma} \left(\eta^{\rho\mu}\eta^{\sigma\nu} + \eta^{\rho\nu}\eta^{\mu\sigma} - \eta^{\rho\sigma}\eta^{\mu\nu} \right)$$

$$= +\frac{g^4}{tu} \left\{ (p_u p_2)(p_t p_1) + (p_t p_2)(p_u p_1) - (p_1 p_2)(p_t p_u) \right\}$$

$$= \frac{g^4}{tu} \Big\{ [p_2(p_1 - p_4)][p_1(p_1 - p_3)] + [p_2(p_1 - p_3)][p_1(p_1 - p_4)] -$$

$$- \frac{s}{2}(p_1 - p_3)(p_1 - p_4) \Big\}$$

$$= \frac{g^4}{tu}\Big\{ -(p_1p_3)\left[p_1p_2 - p_2p_4\right] - (p_1p_4)\left[p_1p_2 - p_2p_3\right] - $$
$$- \frac{s}{2}\left[-p_1p_4 - p_1p_3 + p_3p_4\right]\Big\}$$

$$= \frac{g^4}{tu}\left\{ \frac{t}{2}\left[\frac{s}{2}+\frac{t}{2}\right] + \frac{u}{2}\left[\frac{s}{2}+\frac{u}{2}\right] - \frac{s}{2}\underbrace{\left[\frac{u}{2}+\frac{t}{2}+\frac{s}{2}\right]}_{\sim 0}\right\}$$

$$= \frac{+g^4}{4tu}\left\{ t(-u) + u(-t) \right\}$$

$$= -\frac{g^4}{2}$$

VEDIAMO QUINDI

$$|\mathcal{M}|^2 = g^4\left(\frac{1}{2}\frac{t}{u} + \frac{1}{2}\frac{u}{t} - 1\right)$$
$$= \frac{g^4}{2}\left(\frac{t}{u}+\frac{u}{t}\right) - g^4$$

RISULTATO SREDNICKI :

$$|\mathcal{M}|^2 = \frac{g^4}{2}\left(\frac{t}{u}+\frac{u}{t}\right) - g^4 \quad \checkmark$$

QUINDI LA SEZIONE D'URTO DIFFERENZIALE

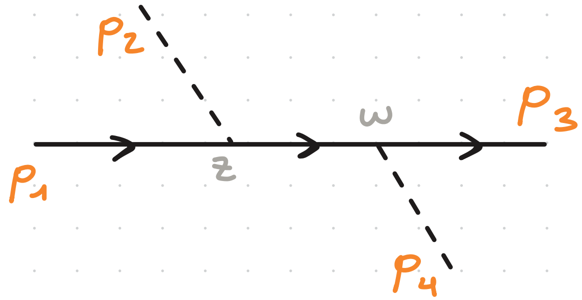
$$\left.\frac{d\sigma}{d\Omega}\right|_{cm} = \frac{g^4}{64\pi S}\left\{\frac{1}{2}\left(\frac{t}{u}+\frac{u}{t}\right)-1\right\}$$

YUKAWA :

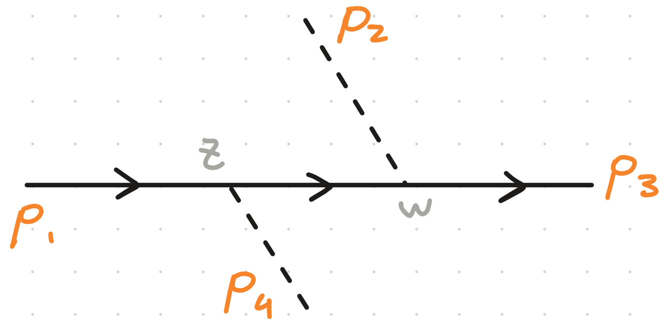
$$\boxed{e^- \quad \phi \longrightarrow e^- \quad \phi}$$

(FUNZIONA UGUALE PER $e^+ \phi \to e^+ \phi$)

I DIAGRAMMI SONO :



CANALE S: $p = p_1 + p_2$



CANALE u: $p = p_1 - p_4$

CONTROLLIAMO IL SEGNO DEI DIAGRAMMI :

$$s: \quad \langle 0| T\left[\bar\psi(x_1)\phi(x_2)\psi(x_3)\phi(x_4)\,\phi(z)\bar\psi(z)\psi(z)\phi(w)\bar\psi(w)\psi(w)\right]|0\rangle$$

$$\begin{cases} \psi(z)\bar\psi(x_1) & 3 \text{ SCAMBI} \\ \psi(x_4)\bar\psi(w) & 1 \text{ SCAMBI} \\ \psi(w)\bar\psi(z) & 0 \text{ SCAMBI} \end{cases} \quad \Rightarrow (-1)^4 = +1$$

$$s: \quad \langle 0 | T\left[\bar\psi(x_1)\phi(x_2)\psi(x_3)\phi(x_4)\,\phi(z)\bar\psi(z)\psi(z)\phi(w)\bar\psi(w)\psi(w)\right]|0\rangle$$

$$\begin{cases} \psi(z)\bar\psi(x_1) & 3 \text{ SCAMBI} \\ \psi(x_3)\bar\psi(w) & 1 \text{ SCAMBI} \\ \psi(w)\bar\psi(z) & 0 \text{ SCAMBI} \end{cases} \quad \Rightarrow \quad (-1)^4 = +1$$

DOBBIAMO DUNQUE CALCOLARE :

$$|\mathcal{M}|^2 = \frac{1}{2}\sum \left\{ |\mathcal{M}_s|^2 + |\mathcal{M}_u|^2 + 2\,\mathrm{Re}\,\mathcal{M}_s^\dagger \mathcal{M}_u \right\}$$

CON

$$\mathcal{M}_s = \frac{(-ig)^2\, i}{p_s^2 - m^2 + i\varepsilon}\left(\bar u(p_3)\,(\not p_s + m)\,u(p_1)\right)$$

$$\mathcal{M}_u = \frac{(-ig)^2\, i}{p_u^2 - m^2 + i\varepsilon}\left(\bar u(p_3)\,(\not p_u + m)\,u(p_1)\right)$$

POSSIAMO QUINDI CALCOLARE

$$\frac{1}{2}\sum_s |\mathcal{M}_s|^2 = \frac{(-ig)^4}{2(s-m^2)^2} \sum_s \left(\bar{u}(p_1)(\not{p}_s + m) u(p_3) \right) \times$$

$$\times \left(\bar{u}(p_3)(\not{p}_s + m) u(p_1) \right)$$

$$\sim \frac{g^4}{2s^2} \mathrm{Tr}\left\{ \not{p}_3 \not{p}_s \not{p}_1 \not{p}_s \right\}$$

$$= \frac{g^4}{2s^2} 4 \left\{ (p_3 p_s)(p_1 p_s) + (p_3 p_s)(p_1 p_s) - (p_1 p_3) p_s^2 \right\}$$

$p_s = p_1 + p_2$
$= p_3 + p_4$

$$\sim 2 \frac{g^4}{s^2} \left\{ 2 [p_3(p_3 + p_4)][p_1(p_1 + p_2)] + \frac{ts}{2} \right\}$$

$$= 2 \frac{g^4}{s^2} \left\{ 2 (p_3 p_4)(p_1 p_2) + \frac{ts}{2} \right\}$$

$$= 2 \frac{g^4}{s^2} \left\{ \frac{s^2}{2} + \frac{ts}{2} \right\}$$

$$= \frac{g^4}{s} (s + t)$$

$$= -g^4 \frac{u}{s}$$

$$\frac{1}{2}\sum_s |\mathcal{M}_u|^2 = \frac{(-ig)^4}{2(p_u^2 - m^2 + i\varepsilon)^2}\sum_s \left(\bar{u}(p_3)(\not{p}_u + m)u(p_1)\right)\left(\bar{u}(p_1)(\not{p}_u + m)u(p_3)\right)$$

$$\sim \frac{g^4}{2u^2}\,\mathrm{Tr}\{\not{p}_1 \not{p}_u \not{p}_3 \not{p}_u\}$$

$$= \frac{g^4}{2u^2}\,4\{(p_1 p_u)(p_3 p_u) + (p_1 p_u)(p_3 p_u) - (p_1 p_3)p_u^2\}$$

$$= 2\frac{g^4}{u^2}\left\{2[p_1(p_1 - p_4)][p_3(p_3 - p_2)] + \frac{t}{2}u\right\}$$

$$= 2\frac{g^4}{u^2}\left\{+2(p_1 p_4)(p_2 p_3) + \frac{tu}{2}\right\}$$

$$= 2\frac{g^4}{u^2}\left\{+\frac{u^2}{2} + \frac{tu}{2}\right\}$$

$$= -g^4\frac{s}{u}$$

$$\frac{1}{2}\sum \mathcal{M}_s^\dagger \mathcal{M}_u = \frac{(-i)i(-ig)^4}{2[p_u^2-m^2][p_s^2-m^2]} \sum_s \left(\bar{u}(p_1)(\not{p}_s+m)u(p_3)\right) \times$$

$$\times \left(\bar{u}(p_3)(\not{p}_u+m)u(p_1)\right)$$

$$\sim \frac{+g^4}{2us} \mathrm{Tr}\{\not{p}_3 \not{p}_u \not{p}_1 \not{p}_s\}$$

$$= +\frac{g^4}{2us} 4\{(p_3 p_u)(p_1 p_s) + (p_3 p_s)(p_1 p_u) - (p_1 p_3)(p_u p_s)\}$$

$$= +\frac{2g^4}{us}\{-(p_2 p_3)(p_1 p_2) + [p_3(p_3+p_4)][p_1(p_1-p_4)] -$$

$$-(p_1 p_3)[(p_1-p_4)(p_1+p_2)]\}$$

$$= +\frac{2g^4}{us}\left\{\frac{su}{4} - (p_3 p_4)(p_1 p_4) - (p_1 p_3)[(p_1 p_2) - (p_4 p_1) - (p_2 p_4)]\right\}$$

$$= +\frac{2g^4}{us}\left\{\frac{su}{4} + \frac{su}{4} + \frac{t}{2}\left[\frac{s}{2} + \frac{u}{2} + \frac{t}{2}\right]\right\}$$

$$= +g^4$$

DUNQUE

$$|\mathcal{M}|^2 = -g^4\left(\frac{u}{s}+\frac{s}{u}\right)+2g^4$$

RISULTATO SREDNICKI :

$$|\mathcal{M}|^2 = g^4\left\{\frac{1}{(m^2-s)^2}\left[-su+m^2(9s+u)+7m^4-8m^2M^2+M^4\right]+\right.$$

$$+\frac{2}{(m^2-s)(M^2-u)}\left[su+3m^2(s+u)+9m^4-8m^2M^2-M^4\right]+$$

$$\left.+\frac{1}{(m^2-u)^2}\left[-su+m^2(9u+s)+7m^4-8m^2M^2+M^4\right]\right\}$$

$$\sim g^4\left\{\frac{1}{s^2}(-su)+\frac{2}{su}(su)+\frac{1}{u^2}(-su)\right\}$$

$$= -g^4\left(\frac{u}{s}+\frac{s}{u}\right)+2g^4$$

E LA SEZIONE D'URTO DIFFERENZIALE

$$\left.\frac{d\sigma}{d\Omega}\right|_{cm} = \frac{g^4}{64\pi s}\left\{-\left(\frac{u}{s}+\frac{s}{u}\right)+2\right\}$$

QED

RIFERIMENTI : [SREDNICKI] : CAP. 58 (P. 351 - 356) (REGOLE)

CAP. 59 (P. 357 - 361) (RISULTATI) → NEGLI ESERCIZI

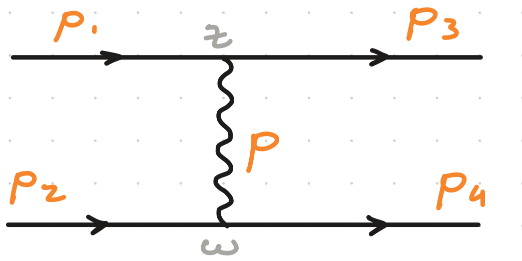
CAP. 60 (P. 362 - 370) (SEGNI RELATIVI DIAGRAMMI E RISULTATI)

MØLLER SCATTERING

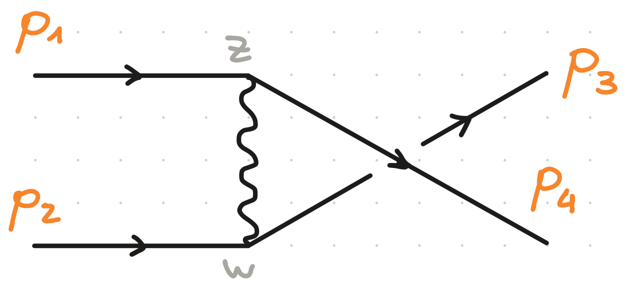
$$\boxed{e^- \; e^- \longrightarrow e^- \; e^-}$$

(FUNZIONA UGUALE PER $e^+ e^+ \to e^+ e^+$)

I DIAGRAMMI SONO:



CANALE t: $p = p_1 - p_3$



CANALE u: $p = p_1 - p_4$

CONTROLLIAMO IL SEGNO DEI DIAGRAMMI:

$$t. \quad \langle 0 | T \left[\bar\psi(x_1) \bar\psi(x_2) \psi(x_3) \psi(x_4) A_\mu(z) \bar\psi(z) \psi(z) A_\mu(\omega) \bar\psi(\omega) \psi(\omega) \right] | 0 \rangle$$

$$\begin{cases} \psi(z)\bar\psi(x_1) & 5 \text{ SCAMBI} \\ \psi(\omega)\bar\psi(x_2) & 5 \text{ SCAMBI} \\ \psi(x_3)\bar\psi(z) & 1 \text{ SCAMBI} \\ \psi(x_4)\bar\psi(\omega) & 0 \text{ SCAMBI} \end{cases} \quad \Rightarrow \quad (-1)^{11} = -1$$

$$\mathcal{M}. \quad \langle 0| T\left[\bar\psi(x_1)\bar\psi(x_2)\psi(x_3)\psi(x_4)A_\mu(z)\bar\psi(z)\psi(z)A_\mu(w)\bar\psi(w)\psi(w)\right]|0\rangle$$

$$\begin{cases} \psi(z)\bar\psi(x_1) & 5 \ \text{SCAMBI} \\ \psi(w)\bar\psi(x_2) & 5 \ \text{SCAMBI} \\ \psi(x_4)\bar\psi(z) & 0 \ \text{SCAMBI} \\ \psi(x_3)\bar\psi(w) & 0 \ \text{SCAMBI} \end{cases} \Rightarrow (-1)^{10} = +1$$

DOBBIAMO CALCOLARE

$$|\mathcal{M}|^2 = \frac{1}{4}\sum\left\{|\mathcal{M}_t|^2 + |\mathcal{M}_u|^2 - 2\mathrm{Re}\,\mathcal{M}_t^\dagger \mathcal{M}_u\right\}$$

CON

$$\mathcal{M}_t = \frac{i(-ie)^2}{(p_1-p_3)^2+i\varepsilon}\left(\bar u(p_3)\gamma^\mu u(p_1)\right)\left(\bar u(p_4)\gamma_\mu u(p_2)\right)$$

$$\mathcal{M}_u = \frac{i(-ie)^2}{(p_1-p_4)^2+i\varepsilon}\left(\bar u(p_4)\gamma^\mu u(p_1)\right)\left(\bar u(p_3)\gamma_\mu u(p_2)\right)$$

COMINCIAMO CON:

$$\frac{1}{4}\sum |\mathcal{M}_t|^2 = \frac{(-ie)^4}{4t^2} \sum \left(\bar{u}_\alpha(p_3)(\gamma^\mu)_{\alpha\beta}\, u_\beta(p_1)\right)\left(\bar{u}_\delta(p_4)(\gamma_\mu)_{\delta\gamma}\, u_\gamma(p_2)\right) \times$$

$$\times \left(\bar{u}_\rho(p_1)(\gamma^\nu)_{\rho\sigma}\, u_\sigma(p_3)\right)\left(\bar{u}_\varepsilon(p_2)(\gamma_\nu)_{\varepsilon\tau}\, u_\tau(p_4)\right)$$

$$\sim \frac{e^4}{4t^2} (\not{p}_1)_{\beta\rho} (\gamma^\nu)_{\rho\sigma} (\not{p}_3)_{\sigma\alpha} (\gamma^\mu)_{\alpha\beta} \cdot (\not{p}_2)_{\gamma\varepsilon} (\gamma_\nu)_{\varepsilon\tau} (\not{p}_4)_{\tau\delta} (\gamma_\mu)_{\delta\gamma}$$

$$= \frac{e^4}{4t^2} (p_1)_\rho (p_2)^\sigma (p_3)_\delta (p_4)^\tau \, \mathrm{Tr}\left\{\gamma^\rho \gamma^\nu \gamma^\delta \gamma^\mu\right\} \mathrm{Tr}\left\{\gamma_\sigma \gamma_\nu \gamma_\tau \gamma_\mu\right\}$$

$$= \frac{4e^4}{t^2} (p_1)_\rho (p_2)^\sigma (p_3)_\delta (p_4)^\tau \left[\eta^{\rho\nu}\eta^{\delta\mu} - \eta^{\rho\delta}\eta^{\nu\mu} + \eta^{\rho\mu}\eta^{\nu\delta}\right] \times$$

$$\times \left[\eta_{\sigma\nu}\eta_{\tau\mu} - \eta_{\sigma\tau}\eta_{\nu\mu} + \eta_{\sigma\mu}\eta_{\nu\tau}\right]$$

$$= \frac{4e^4}{t^2} (p_1)_\rho (p_2)^\sigma (p_3)_\delta (p_4)^\tau \left[2\eta^\rho{}_\sigma \eta^\delta{}_\tau + 2\eta^\delta{}_\sigma \eta^\rho{}_\tau\right]$$

$$= \frac{8e^4}{t^2}\left\{(p_1p_2)(p_3p_4) + (p_1p_4)(p_2p_3)\right\}$$

$$= \frac{2e^4}{t^2}\left\{s^2 + u^2\right\}$$

ANALOGAMENTE

$$\frac{1}{4}\sum |\mathcal{M}_u|^2 = \frac{e^4}{4u^2}\sum \left(\bar{u}_\alpha(p_4)(\gamma^\mu)_{\alpha\beta}\, u_\beta(p_1)\right)\left(\bar{u}_\rho(p_3)(\gamma_\mu)_{\rho\sigma}\, u_\sigma(p_2)\right) \times$$

$$\times \left(\bar{u}_\varepsilon(p_1)(\gamma^\nu)_{\varepsilon\tau}\, u_\tau(p_4)\right)\left(\bar{u}_\lambda(p_2)(\gamma_\nu)_{\lambda\delta}\, u_\delta(p_3)\right)$$

$$\sim \frac{e^4}{4u^2}(\not{p}_1)_{\beta\varepsilon}(\gamma^\nu)_{\varepsilon\tau}(\not{p}_4)_{\tau\alpha}(\gamma^\mu)_{\alpha\beta}\cdot(\not{p}_2)_{\sigma\lambda}(\gamma_\nu)_{\lambda\delta}(\not{p}_3)_{\delta\rho}(\gamma_\mu)_{\rho\sigma}$$

$$= \frac{e^4}{4u^2}(p_1)_\rho(p_4)_\sigma(p_2)^\varepsilon(p_3)^\tau \,\mathrm{Tr}\left\{\gamma^\rho\gamma^\nu\gamma^\sigma\gamma^\mu\right\}\mathrm{Tr}\left\{\gamma_\varepsilon\gamma_\nu\gamma_\tau\gamma_\mu\right\}$$

$$= \frac{4e^4}{u^2}(p_1)_\rho(p_4)_\sigma(p_2)^\varepsilon(p_3)^\tau\left[\eta^{\rho\nu}\eta^{\sigma\mu} - \eta^{\rho\sigma}\eta^{\nu\mu} + \eta^{\rho\mu}\eta^{\nu\sigma}\right]\left[\eta_{\varepsilon\nu}\eta_{\tau\mu} - \eta_{\varepsilon\tau}\eta_{\nu\mu} + \eta_{\varepsilon\mu}\eta_{\nu\tau}\right]$$

$$= \frac{4e^4}{u^2} (p_1)_\rho (p_4)_\sigma (p_2)^\varepsilon (p_3)^\tau \left[2\eta^\rho{}_\varepsilon \eta^\sigma{}_\tau + 2\eta^\rho{}_\tau \eta^\sigma{}_\varepsilon \right]$$

$$= \frac{8e^4}{u^2} \left[(p_1 p_2)(p_3 p_4) + (p_1 p_3)(p_2 p_4) \right]$$

$$= \frac{2e^4}{u^2} \left(s^2 + t^2 \right)$$

E IL TERMINE MISTO

$$\frac{1}{4} \sum \mathcal{M}_t^\dagger \mathcal{M}_u = \frac{e^4}{4ut} \sum \left(\bar{u}_\alpha(p_1) (\gamma^\mu)_{\alpha\beta} u_\beta(p_3) \right) \left(\bar{u}_\delta(p_2) (\gamma_\mu)_{\delta\sigma} u_\sigma(p_4) \right) \times$$

$$\times \left(\bar{u}_\varepsilon(p_4) (\gamma^\nu)_{\varepsilon\tau} u_\tau(p_1) \right) \left(\bar{u}_\rho(p_3) (\gamma_\nu)_{\rho\gamma} u_\gamma(p_2) \right)$$

$$\sim \frac{e^4}{4ut} (\not{p}_3)_{\beta\rho} (\gamma_\nu)_{\rho\gamma} (\not{p}_2)_{\gamma\delta} (\gamma_\mu)_{\delta\sigma} (\not{p}_4)_{\sigma\varepsilon} (\gamma^\nu)_{\varepsilon\tau} (\not{p}_1)_{\tau\alpha} (\gamma^\mu)_{\alpha\beta}$$

$$= \frac{e^4}{4ut} (p_3)_\rho (p_2)_\sigma (p_4)_\varepsilon (p_1)_\tau \, \mathrm{Tr} \left\{ \gamma^\rho \gamma_\nu \gamma^\sigma \gamma_\mu \gamma^\varepsilon \gamma^\nu \gamma^\tau \gamma^\mu \right\}$$

$$= \frac{e^4}{4ut} (p_1)_\tau (p_2)_\sigma (p_3)_\rho (p_4)_\varepsilon \left(-32 \eta^{\sigma\tau} \eta^{\rho\varepsilon}\right)$$

$$= -\frac{8e^4}{ut} (p_1 p_2)(p_3 p_4)$$

$$= -\frac{2e^4}{ut} s^2$$

DUNQUE ABBIAMO

$$|\mathcal{M}|^2 = 2e^4 \left\{ \frac{s^2+u^2}{t^2} + \frac{s^2+t^2}{u^2} + \frac{2s^2}{ut} \right\}$$

$$= 2e^4 \left\{ \frac{s^2u^2 + u^4 + s^2t^2 + t^4 + 2s^2ut}{(ut)^2} \right\}$$

$$= 2e^4 \frac{s^2(u^2+t^2+2ut) + u^4 + t^4}{(ut)^2}$$

$$= 2e^4 \frac{u^4 + t^4 + s^2(u+t)^2}{u^2t^2}$$

$$= 2e^4 \frac{u^4 + t^4 + s^4}{u^2t^2}$$

RISULTATO SREDNICKI

$$|\mathcal{M}|^2 = 2e^4 \frac{s^4 + t^4 + u^4}{t^2 u^2}$$

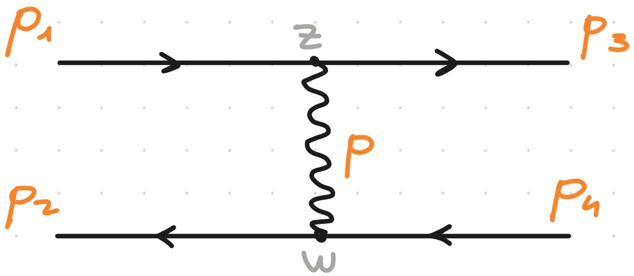
LA SEZIONE D'URTO È

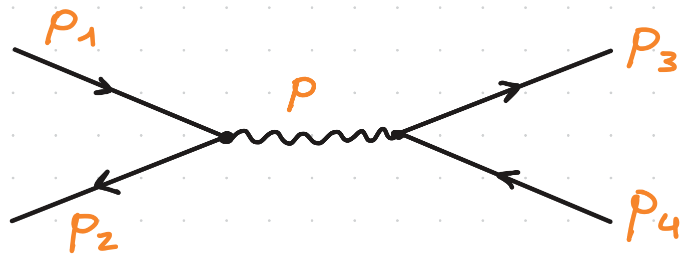
$$\left.\frac{d\sigma}{d\Omega}\right|_{CM} = \frac{1}{64\pi s} 2e^4 \frac{s^4 + t^4 + u^4}{t^2 u^2}$$

BHABHA SCATTERING

$$\boxed{e^+ \quad e^- \longrightarrow e^+ \quad e^-}$$

I DIAGRAMMI SONO:





CANALE t: $p = p_1 - p_3$

CANALE s: $p = p_1 + p_2$

CONTROLLIAMO IL SEGNO DEI DIAGRAMMI:

$$t: \quad \langle 0 | T\left[\bar\psi(x_1)\psi(x_2)\psi(x_3)\bar\psi(x_4)A_\mu(z)\bar\psi(z)\psi(z)A_\nu(w)\bar\psi(w)\psi(w)\right]|0\rangle$$

$$\begin{cases} \psi(z)\bar\psi(x_1) & 5 \text{ SCAMBI} \\ \psi(x_2)\bar\psi(w) & 3 \text{ SCAMBI} \\ \psi(x_3)\bar\psi(z) & 1 \text{ SCAMBI} \\ \psi(w)\bar\psi(x_4) & 0 \text{ SCAMBI} \end{cases} \implies (-1)^9 = -1$$

s: $\quad \langle 0 | T\left[\bar\psi(x_1)\psi(x_2)\psi(x_3)\bar\psi(x_4) A_\mu(z)\bar\psi(z)\psi(z) A_\nu(w)\bar\psi(w)\psi(w)\right] |0\rangle$

$$\begin{cases} \psi(z)\bar\psi(x_1) & 5 \text{ SCAMBI} \\ \psi(x_2)\bar\psi(z) & 2 \text{ SCAMBI} \\ \psi(x_3)\bar\psi(w) & 1 \text{ SCAMBI} \\ \psi(w)\bar\psi(x_4) & 0 \text{ SCAMBI} \end{cases} \Rightarrow (-1)^8 = +1$$

DUNQUE C'È UN SEGNO RELATIVO E DOBBIAMO CALCOLARE:

$$|\mathcal{M}|^2 = \frac{1}{4}\sum_s \left\{ |\mathcal{M}_s|^2 + |\mathcal{M}_t|^2 - 2\,\mathrm{Re}\,\mathcal{M}_s^\dagger \mathcal{M}_t \right\}$$

CON

$$\mathcal{M}_s = \frac{-i(-ie)^2}{(p_1+p_2)^2+i\varepsilon}\left(\bar v_\alpha(p_2)\gamma^\mu_{\alpha\beta}u_\beta(p_1)\right)\eta_{\mu\nu}\left(\bar u_\delta(p_3)\gamma^\nu_{\delta\sigma}v_\sigma(p_4)\right)$$

$$\mathcal{M}_t = \frac{-i(-ie)^2}{(p_1-p_3)^2+i\varepsilon}\left(\bar u_\alpha(p_3)\gamma^\mu_{\alpha\beta}u_\beta(p_1)\right)\eta_{\mu\nu}\left(\bar v_\delta(p_2)\gamma^\nu_{\delta\sigma}v_\sigma(p_4)\right)$$

COMINCIAMO A CALCOLARE:

$$\frac{1}{4}\sum|\mathcal{M}_s|^2 = \frac{e^4}{4s^2}\sum\left(\underline{\bar{v}_\alpha(p_2)}\,\underline{\gamma^\mu_{\alpha\beta}}\,\underline{u_\beta(p_1)}\right)\left(\underline{\bar{u}_\delta(p_3)}\,\underline{(\gamma_\mu)}_{\delta\sigma}\,\underline{v_\sigma(p_4)}\right)\times$$

$$\times\left(\underline{\bar{u}_\lambda(p_1)}\,\underline{\gamma^\nu_{\lambda\tau}}\,\underline{v_\tau(p_2)}\right)\left(\underline{\bar{v}_\varepsilon(p_4)}\,\underline{(\gamma_\nu)}_{\varepsilon\rho}\,\underline{u_\rho(p_3)}\right)$$

$$\sim \frac{e^4}{4s^2}(\not{p}_2)_{\tau\alpha}\gamma^\mu_{\alpha\beta}\,(\not{p}_1)_{\beta\lambda}\gamma^\nu_{\lambda\tau}\,(\not{p}_4)_{\sigma\varepsilon}(\gamma_\nu)_{\varepsilon\rho}\,(\not{p}_3)_{\rho\delta}(\gamma_\mu)_{\delta\sigma}$$

$$= \frac{e^4}{4s^2}\,p_{2\rho}\,p_{1\sigma}\,\mathrm{Tr}\left(\gamma^\rho\gamma^\mu\gamma^\sigma\gamma^\nu\right)p_4^\varepsilon\,p_3^\tau\,\mathrm{Tr}\left(\gamma_\varepsilon\gamma_\nu\gamma_\tau\gamma_\mu\right)$$

$$= \frac{4e^4}{s^2}\,p_{1\sigma}p_{2\rho}p_3^\tau p_4^\varepsilon\left[\eta^{\rho\mu}\eta^{\sigma\nu} - \eta^{\rho\sigma}\eta^{\mu\nu} + \eta^{\rho\nu}\eta^{\mu\sigma}\right]\times$$

$$\times\left[\eta_{\varepsilon\nu}\eta_{\tau\mu} - \eta_{\varepsilon\tau}\eta_{\nu\mu} + \eta_{\varepsilon\mu}\eta_{\nu\tau}\right]$$

$$= \frac{4e^4}{s^2}\,p_{1\sigma}p_{2\rho}p_3^\tau p_4^\varepsilon\Big[2\eta^\rho{}_\tau\eta^\sigma{}_\varepsilon - 4\eta_{\varepsilon\tau}\eta^{\rho\sigma} + 2\eta^\rho{}_\varepsilon\eta^\sigma{}_\tau +$$

$$+\eta^{\mu\nu}\eta_{\mu\nu}\eta_{\varepsilon\tau}\eta^{\rho\sigma}\Big]$$

4

$$= \frac{4e^4}{s^2}\, 2\left[(p_2 p_3)(p_1 p_4) + (p_2 p_4)(p_1 p_3) \right]$$

$$= \frac{8e^4}{s^2}\left[\frac{u^2}{4} + \frac{t^2}{4} \right]$$

$$= 2e^4\, \frac{u^2+t^2}{s^2}$$

PROCEDIAMO.

$$\frac{1}{4}\sum |\mathcal{M}_t|^2 = \frac{e^4}{4t^2}\sum \left(\bar{u}_\alpha(p_3)\,\gamma^\mu_{\alpha\beta}\,u_\beta(p_1)\right)\left(\bar{v}_\delta(p_2)(\gamma_\mu)_{\delta\gamma}\,v_\gamma(p_4)\right) \times$$

$$\times \left(\bar{u}_\rho(p_1)\,\gamma^\nu_{\rho\sigma}\,u_\sigma(p_3)\right)\left(\bar{v}_\varepsilon(p_4)(\gamma_\nu)_{\varepsilon\tau}\,v_\tau(p_2)\right)$$

$$\sim \frac{e^4}{4t^2}(\not{p}_1)_{\beta\rho}\,\gamma^\nu_{\rho\sigma}\,(\not{p}_3)_{\sigma\alpha}\,\gamma^\mu_{\alpha\beta} \cdot (\not{p}_4)_{\gamma\varepsilon}(\gamma_\nu)_{\varepsilon\tau}\,(\not{p}_2)_{\tau\delta}\,(\gamma_\mu)_{\delta\gamma}$$

$$= \frac{e^4}{4t^2} p_{1\rho} p_{3\sigma} \operatorname{Tr}\{\gamma^\rho \gamma^\nu \gamma^\sigma \gamma^\mu\} p_4^{\ \tau} p_2^{\ \varepsilon} \operatorname{Tr}\{\gamma_\tau \gamma_\nu \gamma_\varepsilon \gamma_\mu\}$$

$$= \frac{4e^4}{t^2} p_{1\rho} p_2^{\ \varepsilon} p_{3\sigma} p_4^{\ \tau} \left[\eta^{\rho\nu}\eta^{\sigma\mu} - \eta^{\rho\sigma}\eta^{\nu\mu} + \eta^{\rho\mu}\eta^{\nu\sigma}\right] \times$$

$$\times \left[\eta_{\tau\nu}\eta_{\varepsilon\mu} - \eta_{\tau\varepsilon}\eta_{\nu\mu} + \eta_{\tau\mu}\eta_{\nu\varepsilon}\right]$$

$$= \frac{4e^4}{t^2} p_{1\rho} p_2^{\ \varepsilon} p_{3\sigma} p_4^{\ \tau} \left[2\eta^\rho{}_\tau \eta^\sigma{}_\varepsilon - 4\eta^{\rho\sigma}\eta_{\varepsilon\tau} + 2\eta^\rho{}_\varepsilon \eta^\sigma{}_\tau + \eta^{\mu\nu}\eta_{\mu\nu}\eta^{\rho\sigma}\eta_{\varepsilon\tau}\right]$$

$$= \frac{8e^4}{t^4}\left[(p_1 p_4)(p_2 p_3) + (p_1 p_2)(p_3 p_4)\right]$$

$$= \frac{8e^4}{t^4}\left[\frac{u^2}{4} + \frac{s^2}{4}\right]$$

$$= 2e^4 \frac{s^2 + u^2}{t^2}$$

L'ULTIMO TERMINE :

$$\frac{1}{4}\sum \mathcal{M}_s^+ \mathcal{M}_t = \frac{+e^4}{4st}\sum\left(\bar{u}_\alpha(p_1)\gamma^\mu_{\alpha\beta} v_\beta(p_2)\right)\left(\bar{v}_\sigma(p_4)(\gamma_\mu)_{\sigma\rho} u_\rho(p_3)\right)\times$$

$$\times\left(\bar{u}_\delta(p_3)\gamma^\nu_{\delta\lambda} u_\lambda(p_1)\right)\left(\bar{v}_\varepsilon(p_2)(\gamma_\nu)_{\varepsilon\tau} v_\tau(p_4)\right)$$

$$\sim \frac{+e^4}{4st}(\not{p}_2)_{\beta\varepsilon}(\gamma_\nu)_{\varepsilon\tau}(\not{p}_4)_{\tau\sigma}(\gamma_\mu)_{\sigma\rho}(\not{p}_3)_{\rho\delta}(\gamma^\nu)_{\delta\lambda}(\not{p}_1)_{\lambda\alpha}(\gamma^\mu)_{\alpha\beta}$$

$$= \frac{+e^4}{4st} p_2{}^\rho p_4{}^\sigma p_3{}^\varepsilon p_1{}^\tau \underbrace{\mathrm{Tr}\left\{\gamma_\rho\gamma^\nu\gamma_\sigma\gamma^\mu\gamma_\varepsilon\gamma_\nu\gamma_\tau\gamma_\mu\right\}}_{=-32\,\eta_{\sigma\tau}\eta_{\varepsilon\rho}}$$

$$= \frac{+e^4}{4st}(-32)(p_1 p_4)(p_2 p_3)$$

$$= -2e^4\frac{\mu^2}{st}$$

DUNQUE ABBIAMO

$$|\mathcal{M}|^2 = 2e^4 \left\{ \frac{u^2+t^2}{s^2} + \frac{s^2+u^2}{t^2} + \frac{u^2}{st} \right\}$$

$$= 2e^4 \left\{ \frac{u^2t^2+t^4+s^4+s^2u^2+u^2st}{s^2t^2} \right\}$$

$$= 2e^4 \left\{ \frac{s^4+t^4+u^2(s^2+t^2+st)}{s^2t^2} \right\}$$

$$= 2e^4 \left\{ \frac{s^4+t^4+u^2(s+t)^2}{s^2t^2} \right\}$$

$$= 2e^4 \left\{ \frac{s^4+t^4+u^4}{s^2t^2} \right\}$$

RISULTATO SREDNICKI:

$$|\mathcal{M}|^2 \sim 2e^4 \left(\frac{u^4+s^4+t^4}{s^2t^2} \right)$$

LA SEZIONE D'URTO DIFFERENZIALE

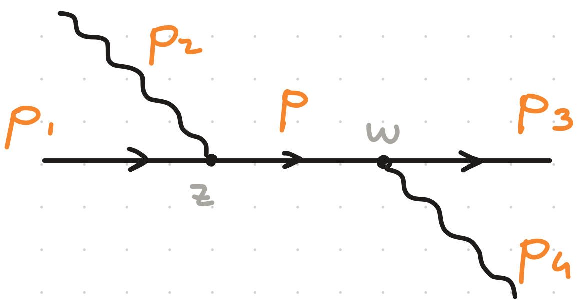
$$\left.\frac{d\sigma}{d\Omega}\right|_{cm} = \frac{2e^4}{64\pi s} \left\{ \frac{u^4+s^4+t^4}{s^2t^2} \right\}$$

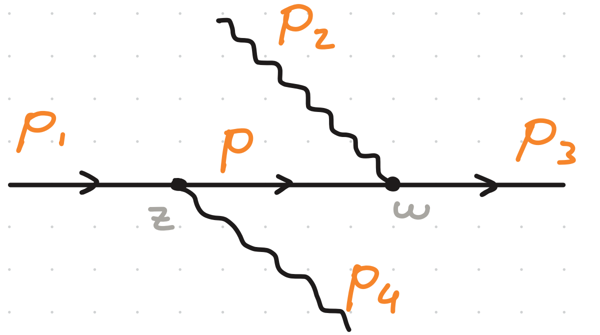
COMPTON SCATTERING

$$\boxed{e^- \quad \gamma \longrightarrow e^- \quad \gamma}$$

(FUNZIONA UGUALE PER $e^+\gamma \to e^+\gamma$)

I DIAGRAMMI SONO :





CANALE s : $p = p_1 + p_2$

CANALE u : $p = p_1 - p_4$

CONTROLLIAMO IL SEGNO DEI DIAGRAMMI :

$$s: \quad \langle 0| T\left[\bar\psi(x_1) A_\mu(x_2) \psi(x_3) A_\nu(x_4) A_\rho(z) \bar\psi(z) \psi(z) A_\sigma(w) \bar\psi(w) \psi(w) \right] |0\rangle$$

$$\begin{cases} \psi(z)\bar\psi(x_1) & 3 \text{ SCAMBI} \\ \psi(x_3)\bar\psi(w) & 1 \text{ SCAMBI} \\ \psi(w)\bar\psi(z) & 0 \text{ SCAMBI} \end{cases} \qquad \Rightarrow (-1)^4 = +1$$

$$\mathcal{M}: \quad \langle 0| T\left[\bar{\psi}(x_1) A_\mu(x_2) \psi(x_3) A_\nu(x_4) A_\rho(z) \bar{\psi}(z) \psi(z) A_\sigma(w) \bar{\psi}(w) \psi(w)\right] |0\rangle$$

$$\begin{cases} \psi(z)\bar{\psi}(x_1) & 3 \text{ SCAMBI} \\ \psi(x_3)\bar{\psi}(w) & 1 \text{ SCAMBI} \\ \psi(w)\bar{\psi}(z) & 0 \text{ SCAMBI} \end{cases} \quad \Rightarrow \quad (-1)^4 = +1$$

DOBBIAMO QUINDI CALCOLARE

$$|\mathcal{M}|^2 = \frac{1}{4} \sum_{s,\lambda} \left\{ |\mathcal{M}_s|^2 + |\mathcal{M}_u|^2 + 2 \operatorname{Re} \mathcal{M}_s^\dagger \mathcal{M}_u \right\}$$

CON

$$\mathcal{M}_s = \frac{i(-ie)^2}{(p_1+p_2)^2 - m^2} \left(\bar{u}(p_3) \not{\varepsilon}(p_4) (\not{p}_s + m) \not{\varepsilon}^*(p_2) u(p_1) \right)$$

$$\mathcal{M}_u = \frac{i(-ie)^2}{(p_1-p_4)^2 - m^2} \left(\bar{u}(p_3) \not{\varepsilon}^*(p_2) (\not{p}_u + m) \not{\varepsilon}(p_4) u(p_1) \right)$$

POSSIAMO DUNQUE CALCOLARE:

$$\frac{1}{4}\sum_{s,\lambda}|\mathcal{M}_s|^2 = \frac{e^4}{4s^2}\sum_{s,\lambda}\Big\{ \varepsilon_\mu^{(\lambda_4)}(p_4)\,\varepsilon_\nu^{(\lambda_2)*}(p_2)\,(p_s)_\rho \times$$

$$\times\left[\bar{u}_\alpha(p_3)(\gamma^\mu)_{\alpha\beta}(\gamma^\rho)_{\beta\delta}(\gamma^\nu)_{\delta\gamma}\,u_\gamma(p_1)\right]\times$$

$$\times\,\varepsilon_\sigma^{(\lambda_4)*}(p_4)\,\varepsilon_\tau^{(\lambda_2)}(p_2)\,(p_s)_\varepsilon \times$$

$$\times\left[\bar{u}_g(p_1)(\gamma^\tau)_{gd}(\gamma^\varepsilon)_{db}(\gamma^\sigma)_{ba}\,u_a(p_3)\right]\Big\}$$

$$= \frac{e^4}{4s^2}\sum_{\lambda_4}\varepsilon_\mu^{(\lambda_4)}(p_4)\,\varepsilon_\sigma^{(\lambda_4)*}(p_4)\sum_{\lambda_2}\varepsilon_\tau^{(\lambda_2)}(p_2)\,\varepsilon_\nu^{(\lambda_2)*}(p_2)\times$$

$$\times(p_s)_\rho\,(p_s)_\varepsilon\,(\not{p}_3+m)_{a\alpha}(\gamma^\mu)_{\alpha\beta}(\gamma^\rho)_{\beta\delta}(\gamma^\nu)_{\delta\gamma}\times$$

$$\times(\not{p}_1+m)_{\gamma g}(\gamma^\tau)_{gd}(\gamma^\varepsilon)_{db}(\gamma^\sigma)_{ba}$$

$$\sim \frac{e^4}{4s^2}(-\eta_{\mu\sigma})(-\eta_{\tau\nu})(p_s)_\rho (p_s)_\varepsilon \times$$

$$\times (p_3)_\lambda (p_1)_\delta \, \mathrm{Tr}\left\{\gamma^\lambda \gamma^\mu \gamma^\rho \gamma^\nu \gamma^\delta \gamma^\tau \gamma^\varepsilon \gamma^\sigma\right\}$$

$$= \frac{e^4}{4s^2}(p_s)_\rho (p_s)_\varepsilon (p_3)_\lambda (p_1)_\delta \, \mathrm{Tr}\left\{\gamma^\lambda \gamma^\mu \gamma^\rho \gamma^\nu \gamma^\delta \gamma_\nu \gamma^\varepsilon \gamma_\mu\right\}$$

VERSO $\quad \gamma^\nu \gamma^\delta \gamma_\nu = -\gamma^\delta \gamma^\nu \gamma_\nu + 2\eta^{\delta\nu}\gamma_\nu$

$$= -4\gamma^\delta + 2\gamma^\delta$$

$$= -2\gamma^\delta$$

COSÌ NELLA TRACCIA ABBIAMO

$$-2\gamma^\lambda \gamma^\mu \gamma^\rho \gamma^\delta \gamma^\varepsilon \gamma_\mu = -2\gamma^\mu \gamma^\rho \gamma^\delta \gamma^\varepsilon \gamma_\mu \gamma^\lambda$$

CICLICITÀ DELLA TRACCIA

$$= -2\gamma^\rho \gamma^\delta \gamma^\varepsilon \gamma_\mu \gamma^\lambda \gamma^\mu$$

$$= + 4 \gamma^{\rho} \gamma^{\delta} \gamma^{\varepsilon} \gamma^{\lambda}$$

COSÌ ABBIAMO

$$\frac{1}{2} \sum_{s,\lambda} |\mathcal{M}_s|^2 = \frac{e^4}{4s^2} (p_s)_\rho (p_s)_\varepsilon (p_3)_\lambda (p_1)_\delta \, 4 \, \mathrm{Tr}\{\gamma^\rho \gamma^\delta \gamma^\varepsilon \gamma^\lambda\}$$

$$= \frac{e^4}{s^2} (p_s)_\rho (p_s)_\varepsilon (p_3)_\lambda (p_1)_\delta \, 4 \left[\eta^{\rho\delta}\eta^{\varepsilon\lambda} - \eta^{\rho\varepsilon}\eta^{\delta\lambda} + \eta^{\rho\lambda}\eta^{\delta\varepsilon}\right]$$

$$= \frac{4e^4}{s^2} \left[(p_1 p_s)(p_3 p_s) - (p_s p_s)(p_1 p_3) + (p_1 p_s)(p_3 p_s)\right]$$

CON $p_s = p_1 + p_2 = p_3 + p_4$ ABBIAMO

$$= \frac{4e^4}{s^2} \left\{ 2 \left[p_1 (p_1 + p_2)\right]\left[p_3 (p_3 + p_4)\right] + \frac{t}{2} s \right\}$$

$$= \frac{4e^4}{s^2} \left\{ 2 (p_1 p_2)(p_3 p_4) + \frac{ts}{2} \right\}$$

$$= \frac{4e^4}{s^2}\left\{ \frac{s^2}{2} + \frac{ts}{2} \right\}$$

$$= \frac{2e^4}{s}(s+t)$$

$$= -2e^4 \frac{u}{s}$$

ANALOGAMENTE CALCOLIAMO:

$$\frac{1}{4}\sum_{s\lambda} |\mathcal{M}_u|^2 = \frac{e^4}{4u^2} \sum_{s,\lambda} \varepsilon_\mu^{(\lambda_4)}(p_4)\, \varepsilon_\nu^{(\lambda_2)*}(p_2)\, (p_\mu)_\rho \times$$

$$\times \left(\bar{\mu}_\alpha(p_3) (\gamma^\mu)_{\alpha\beta} (\gamma^\rho)_{\beta\gamma} (\gamma^\nu)_{\gamma\delta}\, \mu_\delta(p_1) \right) \times$$

$$\times\, \varepsilon_\varepsilon^{(\lambda_2)}(p_2)\, \varepsilon_\tau^{(\lambda_4)*}(p_4)\, (p_\mu)_\sigma \times$$

$$\times \left(\bar{\mu}_a(p_1) (\gamma^\varepsilon)_{ab} (\gamma^\sigma)_{bc} (\gamma^\tau)_{cd}\, \mu_d(p_3) \right)$$

$$= \frac{e^4}{4m^2} \sum_{\lambda} \varepsilon_{\mu}^{(\lambda_2)}(p_2)\, \varepsilon_{\tau}^{(\lambda_2)*}(p_2) \cdot \varepsilon_{\varepsilon}^{(\lambda_4)}(p_4)\, \varepsilon_{\nu}^{(\lambda_4)*}(p_4) \times$$

$$\times (p_\mu)_\rho (p_\mu)_\sigma (\not{p}_1)_{\delta a} (\gamma^\varepsilon)_{ab} (\gamma^\sigma)_{bc} (\gamma^\tau)_{cd} (\not{p}_3)_{d\alpha} (\gamma^\mu)_{\alpha\beta} (\gamma^\rho)_{\beta\gamma} (\gamma^\nu)_{\gamma\delta}$$

$$= \frac{e^4}{4m^2} (-\eta_{\mu\tau})(-\eta_{\varepsilon\nu}) (p_\mu)_\rho (p_\mu)_\sigma (p_1)_\lambda (p_3)_\delta \times$$

$$\times \mathrm{Tr}\{ \gamma^\lambda \gamma^\varepsilon \gamma^\sigma \gamma^\tau \gamma^\delta \gamma^\mu \gamma^\rho \gamma^\nu \}$$

$$= \frac{e^4}{4m^2} (p_\mu)_\rho (p_\mu)_\sigma (p_1)_\lambda (p_3)_\delta \, \mathrm{Tr}\{ \gamma^\lambda \gamma^\varepsilon \gamma^\sigma \gamma^\tau \gamma^\delta \gamma_\tau \gamma^\rho \gamma_\varepsilon \}$$

SAPENDO $\gamma^\tau \gamma^\delta \gamma_\tau = -2\gamma^\delta$ ALLORA ABBIAMO

$$-2\, \mathrm{Tr}\{ \gamma^\lambda \gamma^\varepsilon \gamma^\sigma \gamma^\delta \gamma^\rho \gamma_\varepsilon \} =$$

$$= -2\, \mathrm{Tr}\{ \gamma^\sigma \gamma^\delta \gamma^\rho \gamma_\varepsilon \gamma^\lambda \gamma^\varepsilon \}$$

$$= +4\,\mathrm{Tr}\{\gamma^\sigma\gamma^\delta\gamma^\rho\gamma^\lambda\}$$

$$= 16\left[\eta^{\sigma\delta}\eta^{\rho\lambda} - \eta^{\sigma\rho}\eta^{\delta\lambda} + \eta^{\sigma\lambda}\eta^{\delta\rho}\right]$$

DUNQUE

$$\frac{1}{4}\sum|\mathcal{M}_\mu|^2 = \frac{4e^4}{\mu^2}(p_\mu)_\rho(p_\mu)_\sigma(p_1)_\lambda(p_3)_\delta \times$$

$$\times\left[\eta^{\sigma\delta}\eta^{\rho\lambda} - \eta^{\sigma\rho}\eta^{\delta\lambda} + \eta^{\sigma\lambda}\eta^{\delta\rho}\right]$$

$$= \frac{4e^4}{\mu^2}\left[(p_\mu p_1)(p_\mu p_3) - p_\mu^2(p_1 p_3) + (p_\mu p_1)(p_\mu p_3)\right]$$

ABBIAMO $\quad p_\mu = p_1 - p_4 = p_3 - p_2$

$$= \frac{4e^4}{\mu^2}\left[2(-p_1p_4)(-p_2p_3) + \mu\frac{t}{2}\right]$$

$$=$$

$$= \frac{4e^4}{u^2}\left[\frac{u^2}{2} + \frac{ut}{2}\right]$$

$$= 2e^4 \, \frac{u+t}{u}$$

$$= -2e^4 \frac{s}{u}$$

IL TERMINE DI INTERFERENZA:

$$\frac{1}{4}\sum_{s,\lambda} \mathcal{M}_s^+ \mathcal{M}_u = \frac{e^4}{4su} \sum_{\lambda,s} \varepsilon_\mu^{(\lambda_4)*}(p_4)\, \varepsilon_\nu^{(\lambda_2)}(p_2)\, (p_s)_\tau \times$$

$$\times \left(\bar{u}_\alpha(p_1) (\gamma^\nu)_{\alpha\beta} (\gamma^\tau)_{\beta\gamma} (\gamma^\mu)_{\gamma\delta}\, u_\delta(p_3)\right) \times$$

$$\times\, \varepsilon_\rho^{(\lambda_4)}(p_4)\, \varepsilon_\sigma^{(\lambda_2)*}(p_2)\, (p_u)_\varepsilon \times$$

$$\times \left(\bar{u}_a(p_3) (\gamma^\sigma)_{ab} (\gamma^\varepsilon)_{bc} (\gamma^\rho)_{cd}\, u_d(p_1)\right)$$

$$\sim \frac{e^4}{4s\mu} \sum_{\lambda_2} \varepsilon_\nu^{(\lambda_2)} \varepsilon_\sigma^{(\lambda_2)*} \sum_{\lambda_4} \varepsilon_\rho^{(\lambda_4)} \varepsilon_\mu^{(\lambda_4)*} \times (p_5)_\tau (p_\mu)_\varepsilon$$

$$\times (\not{p}_3)_{\delta a} (\gamma^\sigma)_{ab} (\gamma^\varepsilon)_{bc} (\gamma^\rho)_{cd} (\not{p}_1)_{d\alpha} (\gamma^\nu)_{\alpha\beta} (\gamma^\tau)_{\beta\gamma} (\gamma^\mu)_{\gamma\delta}$$

$$= \frac{e^4}{4s\mu} (-\eta_{\nu\sigma})(-\eta_{\rho\mu}) \ldots$$

$$= \frac{e^4}{4s\mu} (p_5)_\tau (p_\mu)_\varepsilon \operatorname{Tr}\{ \not{p}_3 \gamma^\sigma \gamma^\varepsilon \gamma^\rho \not{p}_1 \gamma_\sigma \gamma^\tau \gamma_\rho \}$$

$$= \frac{e^4}{4s\mu} (p_5)_\tau (p_\mu)_\varepsilon (p_3)_\mu (p_1)_\nu \operatorname{Tr}\{ \gamma^\mu \gamma_\sigma \gamma^\varepsilon \gamma_\rho \gamma^\nu \gamma^\sigma \gamma^\tau \gamma^\rho \}$$

$$= \frac{e^4}{4s\mu} (p_5)_\tau (p_\mu)_\varepsilon (p_3)_\mu (p_1)_\nu \; (-32) \eta^{\varepsilon\tau} \eta^{\nu\mu}$$

$$= \frac{e^4}{4s\mu} (-32) (p_5 p_\mu)(p_1 p_3)$$

$$= -32 \frac{e^4}{4su} \left[(p_1 + p_2)(p_1 - p_4) \right] \left(-\frac{t}{2} \right)$$

$$= -32 \frac{e^4}{4su} \left(-\frac{t}{2} \right) \left[-p_1 p_4 + p_1 p_2 - p_2 p_4 \right]$$

$$= \frac{8e^4}{su} \frac{t}{2} \left[\frac{u}{2} + \frac{s}{2} + \frac{t}{2} \right]$$

$$= 0$$

QUINDI L'AMPIEZZA È :

$$|\mathcal{M}|^2 = -2e^4 \left(\frac{s}{u} + \frac{u}{s} \right)$$

RISULTATO SREDNICKI : (METRICA DIVERSA $\longrightarrow$ CAMBIA IL SEGNO DI $\sum \varepsilon_\mu \varepsilon_\nu^* = \eta_{\mu\nu}$)

$$|\mathcal{M}|^2 = 2e^4 \left(\left| \frac{s}{u} \right| + \left| \frac{u}{s} \right| \right) \quad \checkmark$$

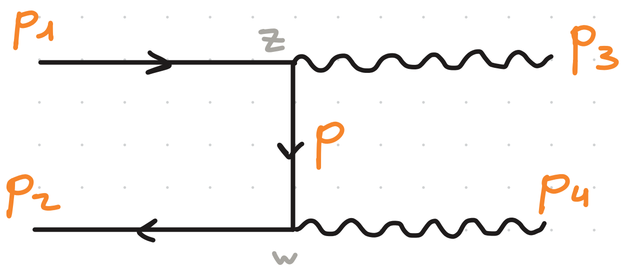
LA SEZIONE D'URTO DIFFERENZIALE :

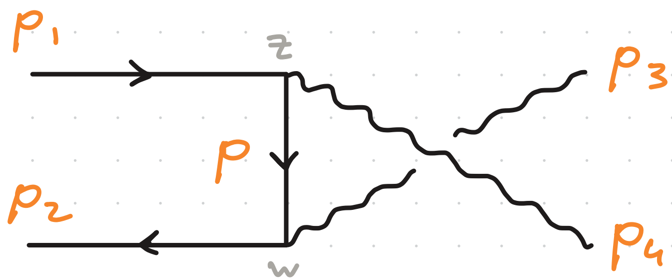
$$\left. \frac{d\sigma}{d\Omega} \right|_{cm} = \frac{1}{64 \pi s} (-2e^4) \left(\frac{s}{u} + \frac{u}{s} \right)$$

(NO NAME) SCATTERING

$$\boxed{e^+ \quad e^- \longrightarrow 2\gamma}$$

I DIAGRAMMI SONO:





CANALE t: $p = p_1 - p_3$

CANALE u: $p = p_1 - p_4$

CONTROLLIAMO IL SEGNO DEI DIAGRAMMI:

$$t: \quad \langle 0 | T\left[\bar\psi(x_1)\psi(x_2)A_\mu(x_3)A_\nu(x_4)A_\rho(z)\bar\psi(z)\psi(z)A_\sigma(w)\bar\psi(w)\psi(w)\right]|0\rangle$$

$$u: \quad \langle 0 | T\left[\bar\psi(x_1)\psi(x_2)A_\mu(x_3)A_\nu(x_4)A_\rho(z)\bar\psi(z)\psi(z)A_\sigma(w)\bar\psi(w)\psi(w)\right]|0\rangle$$

L'UNICA DIFFERENZA È LA CONTRAZIONE TRA GLI A_μ, CHE NON DANNO SEGNO, PER CUI I DIAGRAMMI HANNO STESSO SEGNO.

DOBBIAMO CALCOLARE

$$|\mathcal{M}|^2 = \frac{1}{4}\sum_{s,\lambda}\left\{ |\mathcal{M}_t|^2 + |\mathcal{M}_u|^2 + 2\,\mathrm{Re}\,\mathcal{M}_t^\dagger \mathcal{M}_u \right\}$$

IN CUI

$$\mathcal{M}_t = \frac{i(-ie)^2}{(p_1-p_3)^2-m}\left(\bar{u}(p_2)\,\not{\varepsilon}(p_4)\,(\not{p}_t+m)\,\not{\varepsilon}(p_3)\,u(p_1)\right)$$

$$\mathcal{M}_u = \frac{i(-ie)^2}{(p_1-p_4)^2-m^2}\left(\bar{u}(p_2)\,\not{\varepsilon}(p_3)\,(\not{p}_u+m)\,\not{\varepsilon}(p_4)\,u(p_1)\right)$$

CALCOLO I VARI PEZZI

$$\frac{1}{4}\sum_{s,\lambda}|\mathcal{M}_t|^2 = \frac{e^4}{4t^2}\sum_{s,\lambda}\varepsilon_\mu^{(\lambda_4)}\varepsilon_\nu^{(\lambda_3)}\left(\bar{u}_\alpha(p_2)(\gamma^\mu)_{\alpha\beta}(\not{p}_t)_{\beta\gamma}(\gamma^\nu)_{\gamma\delta}u_\delta(p_1)\right)\times$$

$$\times\,\varepsilon_\rho^{(\lambda_4)*}\varepsilon_\sigma^{(\lambda_3)*}\left(\bar{u}_\varepsilon(p_1)(\gamma^\sigma)_{\varepsilon\tau}(\not{p}_t)_{\tau\kappa}(\gamma^\rho)_{\kappa\omega}u_\omega(p_2)\right)$$

$$\sim \frac{e^4}{4t^2}(-\eta_{\mu\rho})(-\eta_{\nu\sigma})(\not{p}_1)_{\delta\varepsilon}(\gamma^\sigma)_{\varepsilon\tau}(\not{p}_t)_{\tau\kappa}(\gamma^\rho)_{\kappa\omega}(\not{p}_2)_{\omega\alpha}(\gamma^\mu)_{\alpha\beta}(p_t)_{\beta\gamma}(\gamma^\nu)_{\gamma\delta}$$

$$= \frac{e^4}{4t^2}\,\mathrm{Tr}\left\{\not{p}_1\gamma^\sigma\not{p}_t\gamma^\rho\not{p}_2\gamma_\rho\not{p}_t\gamma_\sigma\right\}$$

$$\underbrace{\qquad\qquad}_{-2\not{p}_2}$$

$$= \frac{e^4}{4t^2}\,\mathrm{Tr}\left\{\not{p}_t\,\not{p}_2\,\not{p}_t\,\underbrace{\gamma_\sigma\,\not{p}_1\,\gamma^\sigma}_{-2\not{p}_1}\right\}(-2)$$

$$= \frac{e^4}{t^2}\,\mathrm{Tr}\left\{\not{p}_t\,\not{p}_2\,\not{p}_t\,\not{p}_1\right\}$$

$$= \frac{e^4}{t^2}\,4\left[(p_2p_t)(p_1p_t) - (p_tp_t)(p_1p_2) + (p_1p_t)(p_2p_t)\right]$$

$$= \frac{4e^4}{t^2}\left[2(p_2p_4)(-p_1p_3) - t\,\frac{s}{2}\right]$$

$$= \frac{4e^4}{t^2}\left[-\frac{t^2}{2} - \frac{t\,s}{2}\right]$$

$$= -2e^4\,\frac{t+s}{t}$$

$$= 2e^4\,\frac{u}{t}$$

$$\frac{1}{4}\sum_{s,\lambda}|\mathcal{M}_\mu|^2 = \frac{e^4}{4\mu^2}\sum_{s,\lambda}\varepsilon_\mu^{(\lambda_4)*}\,\varepsilon_\nu^{(\lambda_3)*}\,\varepsilon_\rho^{(\lambda_4)}\,\varepsilon_\sigma^{(\lambda_3)}\;\times$$

$$\times\left(\bar{u}_\alpha(p_2)(\gamma^\sigma)_{\alpha\beta}(\not{p}_\mu)_{\beta\gamma}(\gamma^\rho)_{\gamma\delta}\,u_\delta(p_1)\right)\left(\bar{u}_\varepsilon(p_1)(\gamma^\mu)_{\varepsilon\tau}(\not{p}_\mu)_{\tau\kappa}(\gamma^\nu)_{\kappa\omega}\,u_\omega(p_2)\right)$$

$$= \frac{e^4}{4\mu^2}(-\eta_{\rho\mu})(-\eta_{\sigma\nu})\,(\not{p}_1)_{\delta\varepsilon}(\gamma^\mu)_{\varepsilon\tau}(\not{p}_\mu)_{\tau\kappa}(\gamma^\nu)_{\kappa\omega}(\not{p}_2)_{\omega\alpha}(\gamma^\sigma)_{\alpha\beta}(\not{p}_\mu)_{\beta\gamma}(\gamma^\rho)_{\gamma\delta}$$

$$= \frac{e^4}{4\mu^2}\,\mathrm{Tr}\left\{\not{p}_1\,\gamma^\mu\,\not{p}_\mu\,\underbrace{\gamma^\nu\,\not{p}_2\,\gamma_\nu}_{-2\not{p}_2}\,\not{p}_\mu\,\gamma_\mu\right\}$$

$$= \frac{e^4}{4\mu^2}(-2)\,\mathrm{Tr}\left\{\not{p}_\mu\,\not{p}_2\,\not{p}_\mu\,\underbrace{\gamma_\mu\,\not{p}_1\,\gamma^\mu}_{-2\not{p}_1}\right\}$$

$$= \frac{e^4}{\mu^2}\,\mathrm{Tr}\left\{\not{p}_\mu\,\not{p}_2\,\not{p}_\mu\,\not{p}_1\right\}$$

$$= \frac{4e^4}{\mu^2}\left[\,2(p_1p_\mu)(p_2p_\mu) - p_\mu^2(p_1p_2)\right]$$

$$= \frac{4e^4}{\mu^2}\left[2(-p_1p_4)(p_2p_3) - \mu\,\frac{s}{2}\right]$$

$$= \frac{4e^4}{u^2}\left[-\frac{u^2}{2} - \frac{us}{2}\right]$$

$$= 2e^4\left[\frac{-u-s}{u}\right]$$

$$= 2e^4\,\frac{t}{u}$$

IL TERMINE DI INTERFERENZA

$$\frac{1}{4}\sum_{s,\lambda} \mathcal{M}_t^{+}\mathcal{M}_u = \frac{e^4}{4ut}\sum_{\lambda_3,\lambda_4} \varepsilon_\mu^{(\lambda_3)*}\,\varepsilon_\nu^{(\lambda_4)*}\,\varepsilon_\rho^{(\lambda_3)}\,\varepsilon_\sigma^{(\lambda_4)}\,\times$$

$$\times\left(\bar{u}_\alpha(p_1)(\gamma^\nu)_{\alpha\beta}(\not{p}_t)_{\beta\gamma}(\gamma^\mu)_{\gamma\delta}\,u_\delta(p_2)\right)\times$$

$$\times\left(\bar{u}_\tau(p_2)(\gamma^\sigma)_{\tau\varepsilon}(\not{p}_u)_{\varepsilon\kappa}(\gamma^\rho)_{\kappa\omega}\,u_\omega(p_1)\right)$$

$$\sim \frac{e^4}{4ut} (-\eta_{\rho\mu})(-\eta_{\sigma\nu}) (\not{p}_2)_{\delta\tau} (\gamma^\sigma)_{\tau\varepsilon} (\not{p}_u)_{\varepsilon\kappa} (\gamma^\rho)_{\kappa\omega} (\not{p}_1)_{\omega\alpha} \times$$

$$\times (\gamma^\nu)_{\alpha\beta} (\not{p}_t)_{\beta\gamma} (\gamma^\mu)_{\gamma\delta}$$

$$= \frac{e^4}{4ut} \mathrm{Tr}\left\{ \not{p}_2 \gamma^\sigma \not{p}_u \gamma^\rho \not{p}_1 \gamma_\sigma \not{p}_t \gamma_\rho \right\}$$

$$= \frac{e^4}{4ut} (p_2)_\mu (p_u)_\nu (p_1)_\varepsilon (p_t)_\tau \mathrm{Tr}\left\{ \gamma^\mu \gamma_\sigma \gamma^\nu \gamma^\rho \gamma^\varepsilon \gamma^\sigma \gamma^\tau \gamma_\rho \right\}$$

$$= \frac{e^4}{4ut} (p_2)_\mu (p_u)_\nu (p_1)_\varepsilon (p_t)_\tau (-32) \eta^{\nu\tau} \eta^{\mu\varepsilon}$$

$$= -\frac{8e^4}{ut} \left[(p_1 p_2)(p_u p_t) \right]$$

$$= -\frac{8e^4}{ut} \frac{s}{2} \left[\underbrace{(p_1 - p_4)(p_1 - p_3)}_{-p_1p_3 - p_1p_4 + p_3p_4 \,=\, \frac{t}{2} + \frac{u}{2} + \frac{s}{2} \,=\, 0} \right]$$

$= 0$

DUNQUE L'AMPIEZZA È:

$$|\mathcal{M}|^2 = 2e^4\left(\frac{t}{u} + \frac{u}{t}\right)$$

RISULTATO SREDNICKI:

$$|\mathcal{M}|^2 = 2e^4\left(\left|\frac{t}{u}\right| + \left|\frac{u}{t}\right|\right)$$

LA SEZIONE D'URTO DIFFERENZIALE:

$$\left.\frac{d\sigma}{d\Omega}\right|_{cm} = \frac{1}{64\pi\, s}\, 2e^4\left(\frac{t}{u} + \frac{u}{t}\right)$$