$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi^i \partial^\mu \phi^i - \frac{1}{2} m^2 (\phi^i \phi^i) - \frac{\lambda}{4} (\phi^i \phi^i)^2$$

$$= \frac{1}{2}\partial_\mu \phi^1 \partial^\mu \phi^1 + \frac{1}{2}\partial_\mu \phi^2 \partial^\mu \phi^2 - \frac{1}{2} m^2 (\phi_1^2 + \phi_2^2) - \frac{\lambda}{4} (\phi_1^2 + \phi_2^2)^2$$

$$\phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}} \qquad \phi^+ = \frac{\phi_1 - i\phi_2}{\sqrt{2}}$$

$$\phi_1 = \frac{\phi + \phi^+}{\sqrt{2}} \qquad \phi_2 = -i\frac{\phi - \phi^+}{\sqrt{2}}$$

$$\mathcal{L} = \frac{1}{2}\left[\frac{1}{2} \partial_\mu (\phi + \phi^+) \partial^\mu (\phi + \phi^+) - \frac{1}{2} \partial_\mu (\phi - \phi^+) \partial^\mu (\phi - \phi^+) \right] -$$

$$- \frac{1}{2} m^2 \left[\frac{1}{2} (\phi + \phi^+)^2 - \frac{1}{2} (\phi - \phi^+)^2 \right] -$$

$$- \frac{\lambda}{4} \left[\frac{1}{4} (\phi + \phi^+)^4 + \frac{1}{4} (\phi - \phi^+)^4 - \frac{1}{2} (\phi + \phi^+)^2 (\phi - \phi^+)^2 \right]$$

$$\mathcal{L} = \partial_\mu \phi \, \partial^\mu \phi^+ - \frac{m^2}{4}\left[\phi^2 + \phi^{+2} + 2\phi\phi^+ - \phi^2 - \phi^{+2} + 2\phi\phi^+ \right] -$$

$$- \frac{\lambda}{4}\left[\frac{1}{2}(\phi + \phi^+)^2 - \frac{1}{2}(\phi - \phi^+)^2 \right]^2$$

$$= \partial_\mu \phi \, \partial^\mu \phi^+ - m^2 \phi\phi^+ - \frac{\lambda}{16}\left[\phi^2 + \phi^{+2} + 2\phi\phi^+ - \phi^2 - \phi^{+2} + 2\phi\phi^+ \right]^2$$

$$= \partial_\mu \phi^+ \partial^\mu \phi^+ - m^2 \phi\phi^+ - \lambda(\phi\phi^+)^2$$

$$\phi = \rho e^{i\theta} \qquad \phi^+ = \rho e^{-i\theta}$$

$$\partial_\mu \phi = \partial_\mu \left(\rho e^{i\theta} \right) = \left(\partial_\mu \rho + i\rho \partial_\mu \theta \right) e^{i\theta}$$

$$\partial_\mu \phi^+ \partial^\mu \phi = \left(\partial_\mu \rho + i\rho \partial_\mu \theta \right)\left(\partial_\mu \rho - i\rho \partial_\mu \theta \right)$$

$$= \partial_\mu \rho \, \partial^\mu \rho + \rho^2 \partial_\mu \theta \, \partial^\mu \theta$$

$$\phi\phi^+ = \rho^2 \qquad \Rightarrow (\phi\phi^+)^2 = \rho^4$$

$$\mathcal{L} = \partial_\mu \rho \partial^\mu \rho + \rho^2 \partial_\mu \theta \partial^\mu \theta - m^2 \rho^2 - \lambda \rho^4$$

$\Rightarrow \theta$ BOSONE DI GOLDSTONE