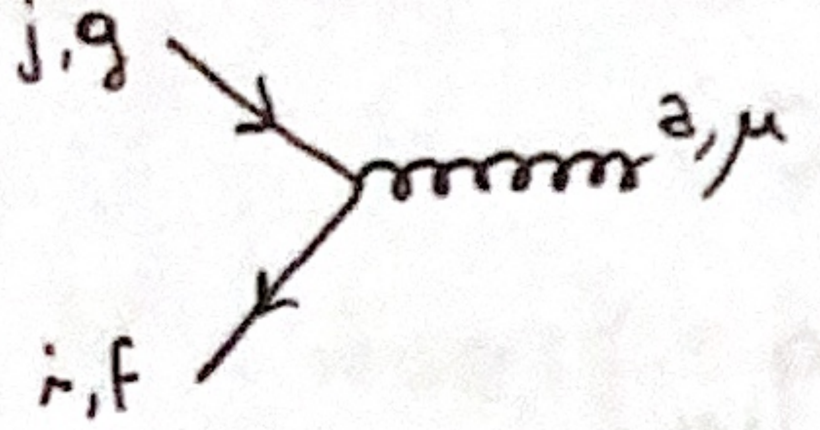
Some Calculation

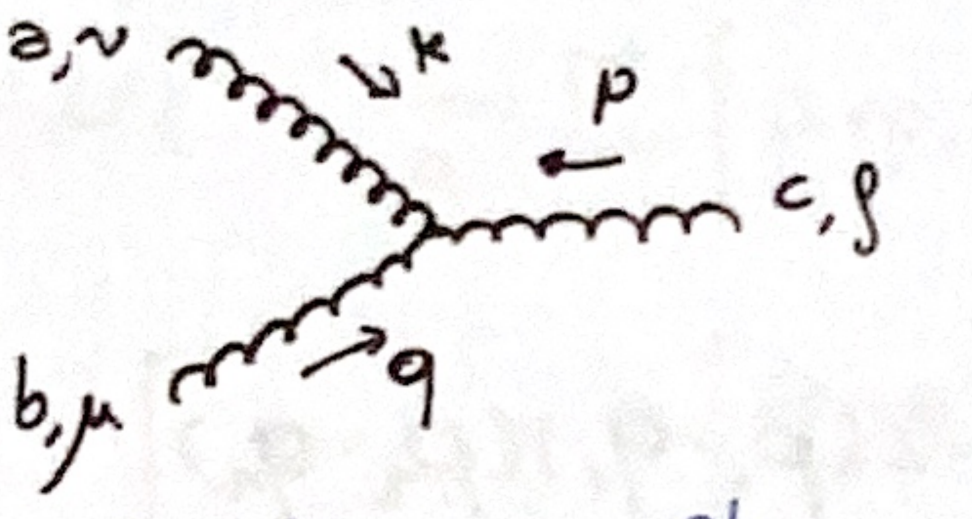
QCD

RULES:

$$\Delta^{ab}_{\mu\nu}(q) = \frac{-i\delta^{ab}}{q^2}\left(g_{\mu\nu} - (1-\xi)\frac{q_\mu q_\nu}{q^2}\right) \xrightarrow{(\xi=1)} \frac{-i\delta^{ab}}{q^2} g_{\mu\nu}$$



$$= i g_s \gamma^\mu t^a_{ij}\,\delta_{fg}$$

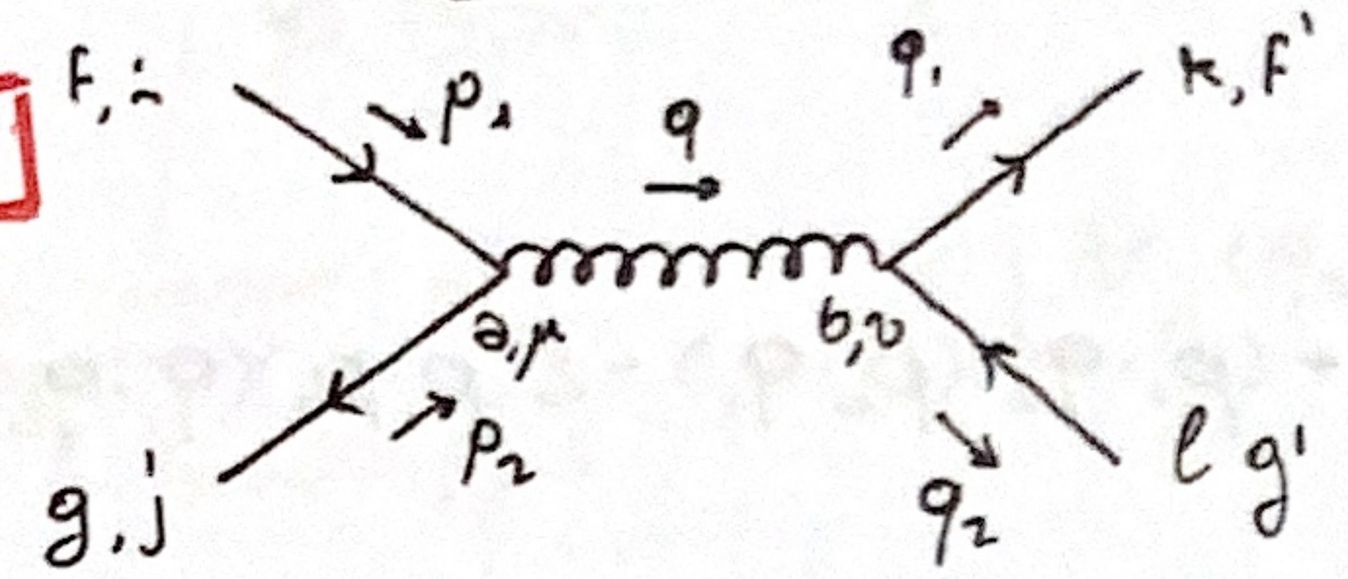


$$= g_s f^{abc}\left[(k-q)_\rho g_{\mu\nu} + (p-k)_\mu g_{\rho\nu} + (q-p)_\nu g_{\rho\mu}\right]$$

$t^a_{ij}t^b_{ji} = T_R\delta^{ab}$; $\delta^{ab}\delta^{ba} = \delta^{aa} = N^2-1$; $t^a_{ij}t^a_{ji} = C_F N$, $C_F = \frac{N^2-1}{2N}$

$(t^b_G)_{ac} = i f^{abc}$ $\quad Tr\{t^a_G t^b_G\} = C_A\delta^{ab}$

$\boxed{1}$



$$\mathcal{M} = \bar u(q_1)(i g_s t^b_{kl}\gamma^\nu)v(q_2)\frac{-i\delta_{ab}g_{\mu\nu}}{q^2}v(p_2)(i g_s t^a_{ij}\gamma^\mu)u(p_1)$$

$$|\mathcal{M}|^2 = \frac{1}{N_c^2}\frac{1}{4}\sum_{\substack{\text{SPIN}\\ \text{COLOR}}}\left[\bar u(q_1) i g_s t^a_{kl}\gamma^\mu v(q_2)\frac{-i}{q^2}\bar v(p_2) i g_s t^a_{ij}\gamma_\mu u(p_1)\right]\times$$

$$\times\left[\bar u(p_1)\gamma^\nu t^b_{ji}(-ig_s)v(p_2)\frac{i}{q^2}\bar v(q_2)\gamma_\nu t^b_{lk}(-ig_s)u(q_1)\right]$$

$$= \frac{1}{4N_c^2}\sum g_s^4\frac{1}{s^2}\left[\bar u(q_1)\gamma^\mu v(q_2)\bar v(p_2)\gamma_\mu u(p_1)\times\right.$$

$$\left.\times\,\bar u(p_1)\gamma^\nu v(p_2)\bar v(p_2)\gamma_\nu u(q_1)\right]t^a_{kl}t^a_{ij}t^b_{ji}t^b_{lk}$$

$$= \frac{g_s^4}{4N_c^2 s^2}\sum\left[\bar u_a(q_1)\gamma^\mu_{ab}v_b(q_2)\bar v_c(p_2)\gamma_{\mu\,cd}u_d(p_1)\times\right.$$

$$\left.\bar u_e(p_1)\gamma^\nu_{ef}v_f(p_2)\bar v_g(q_2)\gamma_{\nu\,gh}u_h(q_1)\right]t^a_{kl}\,T_R\delta^{ab}\,t^b_{lk}$$

$$= \frac{g_s^4}{4N_c^2 s^2}Tr\left\{\not p_{1\,de}\gamma^\nu_{ef}\not p_{2\,fc}\gamma_{\mu\,cd}\right\}Tr\left\{\not q_2\gamma_\nu\not q_1\gamma^\mu\right\}T_R C_F N_c$$

$$= \frac{g_s^4}{4N_c^2 s^2}T_R\cdot T_R\frac{N_c^2-1}{N_c}N_c\, Tr\left\{\not p_1\gamma^\nu\not p_2\gamma^\mu\right\}Tr\left\{\not q_2\gamma_\nu\not q_1\gamma_\mu\right\}$$

WE KNOW:

$$\mathrm{Tr}\{\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\} = 4\left(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}\right)$$

SO

$$\mathrm{Tr}\{\not p_1\gamma^\nu\not p_2\gamma^\mu\} = 4\left(p_1^\nu p_2^\mu - (p_1\cdot p_2)g^{\nu\mu} + p_1^\mu p_2^\nu\right)$$

$$\mathrm{Tr}\{\not q_2\gamma_\nu\not q_1\gamma_\mu\} = 4\left(q_{2\nu}q_{1\mu} - (q_1\cdot q_2)g_{\nu\mu} + q_{2\mu}q_{1\nu}\right)$$

$$\Rightarrow \mathrm{Tr}\{\not p_1\gamma^\nu\not p_2\gamma^\mu\}\mathrm{Tr}\{\not q_2\gamma_\nu\not q_1\gamma_\mu\} =$$

$$= 16\left[2(p_1\cdot q_2)(p_2\cdot q_1) - 2\varepsilon(q_1\cdot q_2)(p_1\cdot p_2) + 2(p_1\cdot q_1)(p_2\cdot q_2)\right]$$

$$= 32\left[(p_1\cdot q_1)(p_2\cdot q_2) + (p_1\cdot q_2)(p_2\cdot q_1) - \varepsilon(q_1\cdot q_2)(p_1\cdot p_2)\right]$$

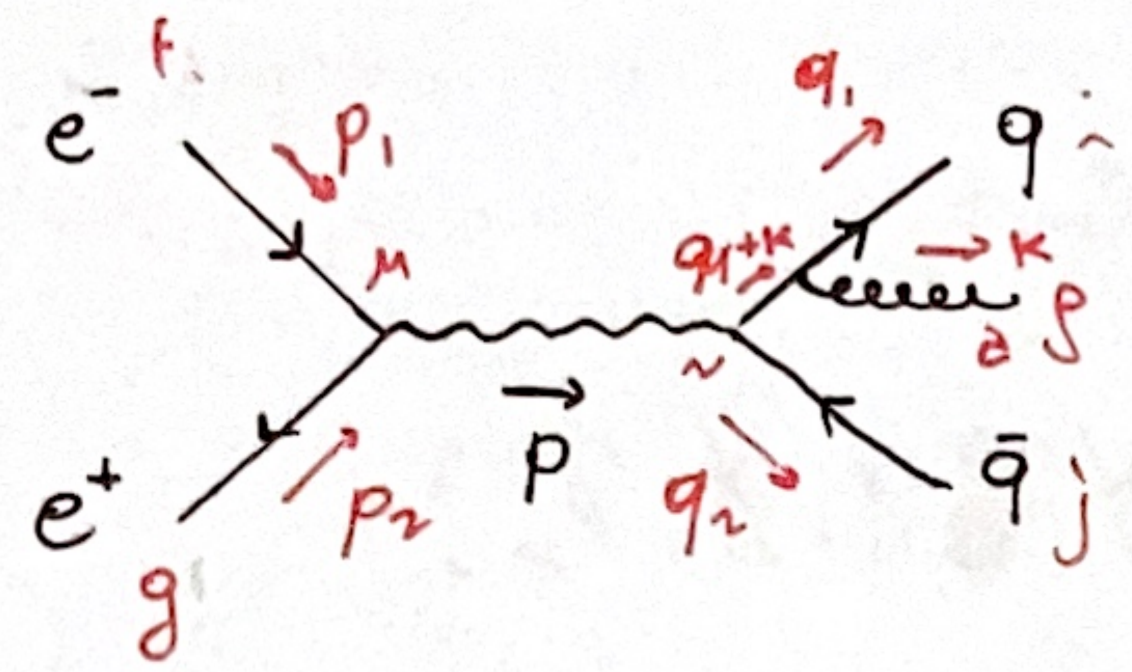
SO WE GET

$$|\mathcal{M}|^2 = \frac{g_s^4}{4N_c^2 s^2}T_R^2(N_c^2-1)\,32\left[(p_1\cdot q_1)(p_2\cdot q_2) + (p_1\cdot q_2)(p_2\cdot q_1) - \varepsilon(p_1\cdot p_2)(q_1\cdot q_2)\right]$$

$$\begin{cases} s = (p_1+p_2)^2 = (q_1+q_2)^2 \sim 2p_1\cdot p_2 = 2q_1\cdot q_2 \\ t = (p_1-q_1)^2 = (q_2-p_2)^2 \sim -2p_1\cdot q_1 = -2p_2\cdot q_2 \\ u = (p_1-q_2)^2 = (q_1-p_2)^2 \sim -2p_1\cdot q_2 = -2p_2\cdot q_1 \end{cases}$$

$$|\mathcal{M}|^2 = \frac{8g_s^4T_R^2}{N_c^2 s^2}(N_c^2-1)\left[\frac{t^2}{4} + \frac{u^2}{4} - \varepsilon\frac{s^2}{4}\right]$$

$$= \frac{2g_s^4T_R^2}{N_c^2 s^2}(N_c^2-1)\left[t^2 + u^2 - \varepsilon s^2\right]$$



$$\mathcal{M} = \bar u(p_1)(ie\gamma^\mu)v(p_2)\frac{-ig_{\mu\nu}}{p^2}\bar u_i(q_1)(ig_s\gamma^\rho t^a_{ij})\varepsilon_\rho(k)\frac{i(\not q_1+\not k)}{(q_1+k)^2}(ieQ_f\gamma^\nu)v_j(q_2)$$

$$|\mathcal{M}|^2 = \frac{1}{4}\sum\left[\bar u(p_1)e\gamma^\mu v(p_2)\frac{1}{p^2}\bar u_i(q_1)g_s\gamma^\rho t^a_{ij}\varepsilon_\rho(k)\frac{\not q_1+\not k}{(q_1+k)^2}eQ_f\gamma_\mu v_j(q_2)\right]\times$$

$$\times\left[\bar v_j(q_2)\gamma_\sigma Q_f e\frac{\not q_1+\not k}{(q_1+k)^2}\varepsilon^*_\tau(k)t^a_{ji}\gamma^\tau g_s u_i(q_1)\frac{1}{p^2}\bar v(p_2)e\gamma^\sigma u(p_1)\right]$$

$$=\frac{g_s^2e^4Q_f^2}{p^4(q_1+k)^4}\sum\left[\bar u(p_1)\gamma^\mu v(p_2)\bar u_i(q_1)\gamma^\rho t^a_{ij}(-g_{\rho\tau})(\not q_1+\not k)\gamma_\mu v_j(q_2)\right]\times$$

$$\times\left[\bar v_j(q_2)\gamma_\sigma(\not q_1+\not k)t^a_{ji}\gamma^\tau u_i(q_1)\bar v(p_2)\gamma^\sigma u(p_1)\right]$$

NEL GAUGE DI FEYNMAN

$\sum\varepsilon_\mu\varepsilon^*_\nu = -g_{\mu\nu}$

$$=\frac{g_s^2e^4Q_f^2}{s^2(q_1+k)^4}\sum\left[t^a_{ij}t^a_{ji}\right]\left[\bar u(p_1)\gamma^\mu v(p_2)\bar u_i(q_1)\gamma^\rho(\not q_1+\not k)\gamma_\mu v_j(q_2)\right]\times$$

$$\times\left[\bar v_j(q_2)\gamma_\sigma(\not q_1+\not k)\gamma_\rho u_i(q_1)\bar v(p_2)\gamma^\sigma u(p_1)\right]$$

$$=\frac{g_s^2e^4Q_f^2}{s^2(q_1+k)^4}T_R\delta^{aa}\,\mathrm{Tr}\left\{\not q_2\gamma_\sigma(\not q_1+\not k)\gamma_\rho\not q_1\gamma^\rho(\not q_1+\not k)\gamma_\mu\right\}\times$$

$$\times\mathrm{Tr}\left\{\not p_2\gamma^\sigma\not p_1\gamma^\mu\right\}$$

$$=\frac{g_s^2e^4Q_f^2}{s^2(q_1+k)^4}T_R(N^2-1)\Big\{\mathrm{Tr}\left[\not q_2\gamma_\sigma\not q_1\gamma_\rho\not q_1\gamma^\rho\not q_1\gamma_\mu\right]+$$

$$+\mathrm{Tr}\left[\not q_2\gamma_\sigma\not q_1\gamma_\rho\not q_1\gamma^\rho\not k\gamma_\mu\right]+\mathrm{Tr}\left[\not q_2\gamma_\rho\not k\gamma_\rho\not q_1\gamma^\rho\not q_1\gamma_\mu\right]+$$

$$+\mathrm{Tr}\left[\not q_2\gamma_\sigma\not k\gamma_\rho\not q_1\gamma^\rho\not k\gamma_\mu\right]\Big\}\times$$

$$\times 4\left\{p_2^\sigma p_1^\mu-(p_1p_2)g^{\sigma\mu}+p_2^\mu p_1^\sigma\right\}$$

PERÒ $\gamma^\mu\not p\gamma_\mu = -2\not p$

$$=(-2)\frac{g_s^2 e^4 Q_f^2}{s^2(q_1+k)^4} T_R(N^2-1)\Big\{ Tr[\not{q}_2 \gamma_\sigma \not{q}_1 \not{q}_1 \not{q}_1 \gamma_\mu] +$$

$$+ Tr[\not{q}_2 \gamma_\sigma \not{q}_1 \not{q}_1 \not{k} \gamma_\mu] + Tr[\not{q}_2 \gamma_\beta \not{k} \not{q}_1 \not{q}_1 \gamma_\mu] +$$

$$+ Tr[\not{q}_2 \gamma_\sigma \not{k} \not{q}_1 \not{k} \gamma_\mu] \Big\} \times 4\Big\{ p_2^\sigma p_1^\mu - (p_1 p_2) g^{\sigma\mu} + p_2^\mu p_1^\sigma \Big\}$$

PERÒ $\not{\partial}\not{\partial} = \partial^2$ MA SE SIAMO NEL LIMITE MASSLESS
$q_1^2 = 0$

$$= -8 \frac{g_s^2 e^4 Q_f^2}{s^2(q_1+k)^4} T_R(N^2-1)\, Tr\{\not{q}_2 \gamma_\sigma \not{k} \not{q}_1 \not{k} \gamma_\mu\}\{p_2^\sigma p_1^\mu - (p_1 p_2) g^{\sigma\mu} + p_2^\mu p_1^\sigma\}$$

VALE $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$ DUNQUE

$$\not{k}\not{q}_1\not{k} = k_\alpha q_{1\beta} k_\delta \gamma^\alpha \gamma^\beta \gamma^\delta$$

$$= k_\alpha q_{1\beta} k_\delta \gamma^\alpha [2g^{\beta\delta} - \gamma^\delta \gamma^\beta]$$

$$= 2\not{k}(k\cdot q_1) - \underbrace{\not{k}\not{k}}_{k^2=0}\not{q}_1$$

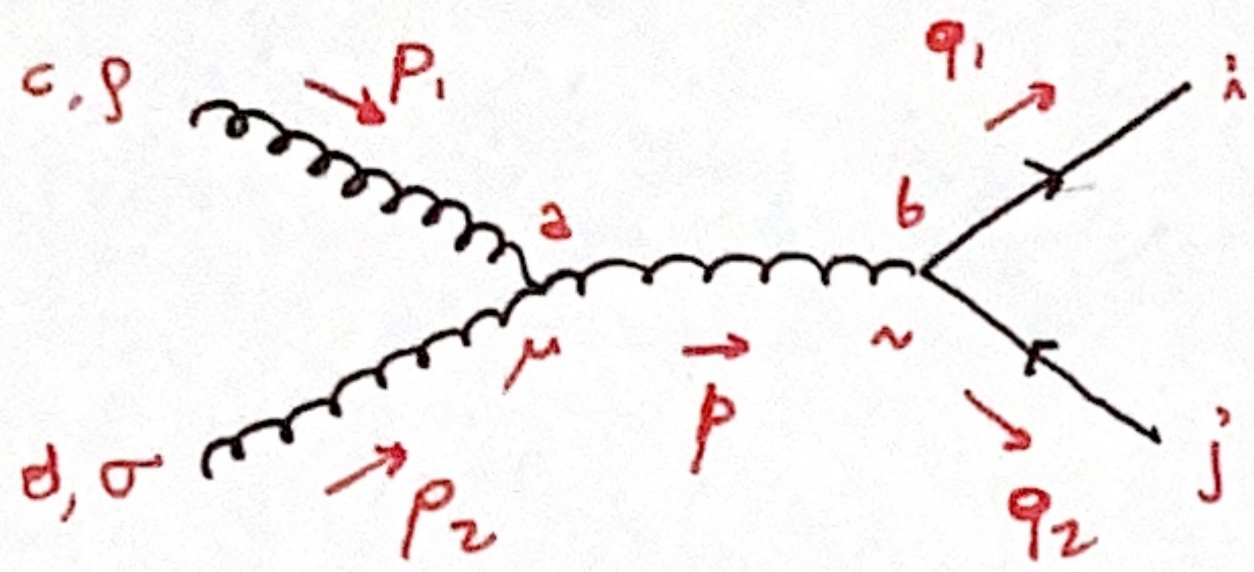
$$= -4\cdot 4 \frac{g_s^2 e^4 Q_f^2}{s^2(q_1+k)^4} T_R(N^2-1)\,(k\cdot q_1)\, Tr\{\not{q}_2 \gamma_\sigma \not{k} \gamma_\mu\}\{p_2^\sigma p_1^\mu - (p_1 p_2) g^{\sigma\mu} + p_2^\mu p_1^\sigma\}$$

$$= -4\cancel{4}\frac{g_s^2 e^4 Q_f^2 T_R(N^2-1)}{s^2[\cancel{2}(k\cdot q_1)]^{\cancel{4}}}(\cancel{k\cdot q_1})4[q_{2\sigma}k_\mu - (k\cdot q_2)g_{\sigma\mu} + q_{2\mu}k_\sigma] \times$$

$$\times [p_2^\sigma p_1^\mu - (p_1 p_2) g^{\sigma\mu} + p_2^\mu p_1^\sigma]$$

$$= -16 \frac{g_s^2 e^4 Q_f^2 T_R(N^2-1)}{s^2 (k\cdot q_1)}[2(p_2 q_2)(k p_1) + 2(p_1 q_2)(k p_2)]$$

FATTORE DI COLORE:



$$\mathcal{M} = \bar{u}(q_1)\left(i g_s t^b_{ij} \gamma^\nu\right) v(q_2) \frac{-i g_{\mu\nu} \delta^{ab}}{p^2} \varepsilon^*_\rho(p_1)\left(g_s f^{cda}[\dots]\right) \varepsilon^*_{d\sigma}(p_2)$$

IGNORANDO I FATTORI DI LORENTZ PROVENIENTI DAL VERTICE A 3 GLUONI, POSSIAMO VEDERE IL FATTORE DI COLORE

$$|\mathcal{M}|^2 \propto t^b_{ij} f^{cda} t^b_{ji} f^{adc}$$

$$= \left(t^b_{ij} t^b_{ji}\right)\left(f^{cda} f^{adc}\right)$$

$$= T_R \delta^{bb} (-i)^2 (t^d_b)_{ca} (t^d_b)_{ac}$$

$$= - T_R (N^2-1) C_A \delta^{dd}$$

$$= - T_R C_A N (N^2 - 1)$$