Relazioni utili

$\gamma_5^\dagger = \gamma_5$ $\qquad (\gamma^5)^2 = \mathbb{1}$ $\qquad \{\gamma_5, \gamma_\mu\} = 0$

$$\mathrm{Tr}\{\gamma_5 \gamma^\mu \gamma^\nu\} = 0$$

$$\mathrm{Tr}\{\gamma_5 (\#\text{DISPARI MATRICI } \gamma^{\text{DI}})\} = 0$$

$$\mathrm{Tr}\{\gamma_5 \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma\} = -4i\,\varepsilon^{\mu\nu\rho\sigma}$$

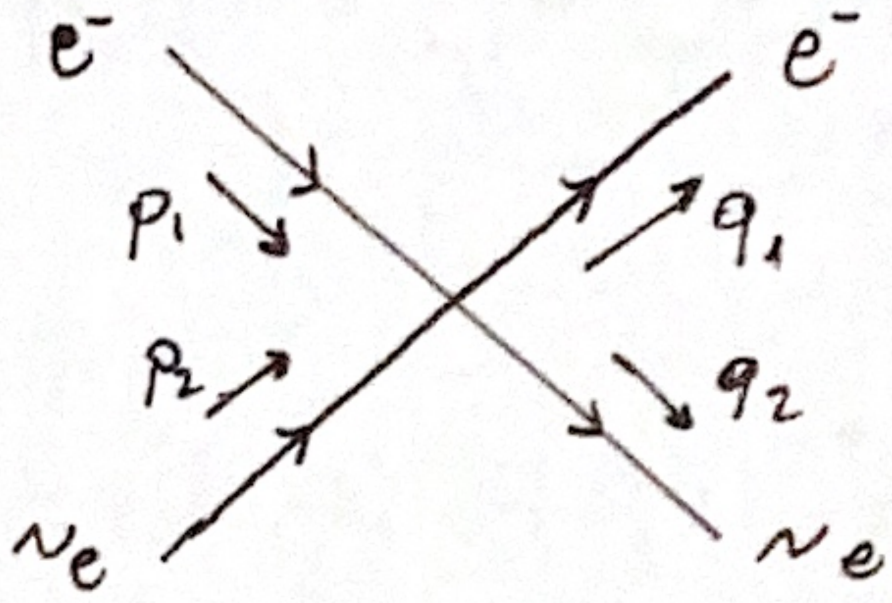
$$\mathrm{Tr}\{\gamma^\mu \gamma^\nu\} = 4g^{\mu\nu}$$

$$\mathrm{Tr}\{\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma\} = 4\left(g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\rho}\right)$$

VIOLAZIONE UNITARIETÀ TEORIA DI FERMI

CI SERVE CALCOLARE:

$$e^- + \nu_e \longrightarrow e^- + \nu_e$$



$$i\mathcal{M} = \frac{G_F}{\sqrt{2}} \bar{u}(q_1)\gamma^\mu(1-\gamma_5)u(p_1)\,\bar{u}(q_2)\gamma_\mu(1-\gamma_5)u(p_2)$$

DUNQUE:

$$|\mathcal{M}|^2 = \frac{1}{4}\frac{G_F^2}{2}\sum_{\text{pol.}}\left[\bar{u}(q_1)\gamma^\mu(1-\gamma_5)u(p_1)\bar{u}(q_2)\gamma_\mu(1-\gamma_5)u(p_2)\right]\times$$

$$\times\left[\bar{u}(p_2)\gamma^\nu(1-\gamma_5)u(q_2)\bar{u}(p_1)\gamma_\nu(1-\gamma_5)u(q_1)\right]$$

$$= \frac{G_F^2}{8}\mathrm{Tr}\left\{(\not p_1 + m_e)\gamma_\nu(1-\gamma_5)(\not q_1 + m_e)\gamma^\mu(1-\gamma_5)\right\}\times$$

$$\times\mathrm{Tr}\left\{\not p_2\gamma^\nu(1-\gamma_5)\not q_2\gamma_\mu(1-\gamma_5)\right\}$$

SPOSTO LE $(1-\gamma_5)$ VICINO TRA LORO USANDO $\{\gamma_5,\gamma_\mu\}=0$

$$= \frac{G_F^2}{8}\mathrm{Tr}\left\{(\not p_1 + m_e)\gamma_\nu \not q_1\gamma^\mu(1-\gamma_5)^2 +\right.$$

$$\left.+\cancel{(\not p_1 + m_e)\gamma_\nu m_e\gamma^\mu(1+\gamma_5)(1-\gamma_5)}\right\}\times$$

$$\times\mathrm{Tr}\left\{\not p_2\gamma^\nu\not q_2\gamma_\mu(1-\gamma_5)^2\right\}$$

= 0 PERCHÉ o Tr DI # DISPARI DI γ o PERCHÉ Tr DI # DISPARI DI γ E γ_5

$$= \frac{G_F^2}{8}\mathrm{Tr}\left\{2\not p_1\gamma_\nu\not q_1\gamma^\mu(1-\gamma_5) + \cancel{2m_e\gamma_\nu\not q_1\gamma^\mu(1-\gamma_5)}\right\}\times$$

$$\times 2\,\mathrm{Tr}\left\{\not p_2\gamma^\nu\not q_2\gamma_\mu - \not p_2\gamma^\nu\not q_2\gamma_\mu\gamma_5\right\}$$

$$= \frac{G_F^2}{8}4\,\mathrm{Tr}\left\{\not p_1\gamma^\nu\not q_1\gamma^\mu - \not p_1\gamma^\nu\not q_1\gamma^\mu\gamma_5\right\}\times$$

$$\times\mathrm{Tr}\left\{\not p_2\gamma_\nu\not q_2\gamma_\mu - \not p_2\gamma_\nu\not q_2\gamma_\mu\gamma_5\right\}$$

$$|\mathcal{M}|^2 = 16\frac{G_F^2}{2}\left[p_1^\nu q_1^\mu - (p_1q_2)g^{\mu\nu} + p_1^\mu q_1^\nu + i p_{1\alpha} q_{1\beta}\,\varepsilon^{\alpha\nu\beta\mu}\right]\times$$

$$\times\left[p_{2\nu} q_{2\mu} - (p_2q_2)g_{\mu\nu} + p_{2\mu} q_{2\nu} + i p_2^\rho q_2^\sigma\,\varepsilon_{\rho\nu\sigma\mu}\right]$$

$$= 8G_F^2\left[2(p_1p_2)(q_1q_2) + 2(p_1q_2)(p_2q_1) - p_{1\alpha}p_2^\rho q_{1\beta}q_2^\sigma\,\varepsilon^{\alpha\nu\beta\mu}\varepsilon_{\rho\nu\sigma\mu}\right]$$

$$\varepsilon^{\alpha\nu\beta\mu}\varepsilon_{\rho\nu\sigma\mu} = -2\left(\delta^\alpha{}_\rho\,\delta^\beta{}_\sigma - \delta^\alpha{}_\sigma\,\delta^\beta{}_\rho\right)$$

$$= 8G_F^2\left[2(p_1p_2)(q_1q_2) + 2(p_1q_2)(p_2q_1) + 2p_{1\alpha}p_2^\rho q_{1\beta}q_2^\sigma\left(\delta^\alpha_\rho\delta^\beta_\sigma - \delta^\alpha_\sigma\delta^\beta_\rho\right)\right]$$

$$= 16G_F^2\left[(p_1p_2)(q_1q_2) + \cancel{(p_1q_2)(p_2q_1)} + (p_1p_2)(q_1q_2) - \cancel{(p_1q_2)(p_2q_1)}\right]$$

$$= 32G_F^2\,(p_1p_2)(q_1q_2)$$

OSS NELLE NOTE VIENE RIPORTATO IL RISULTATO $|\mathcal{M}|^2 = 128G_F^2(p_1p_2)(q_1q_2)$ PERCHÈ NON È MEDIATO PER LO STATO INIZIALE (4).

VEDIAMO:

$s = (p_1+p_2)^2 \sim 2(p_1p_2)$ APPROSSIMANDO $p_1^2 = p_2^2 \sim 0$ (MASSLESS)

E VALENDO $p_1 + p_2 = q_1 + q_2$

COSÌ

$s = (q_1+q_2)^2 \sim 2(q_1q_2)$

DUNQUE:

$$|\mathcal{M}|^2 = 32G_F^2\left(\frac{s}{2}\right)^2$$

$$= 8G_F^2 s^2$$