

UTOPAGGIO FISICA III

5

TUMORAGGI : 40, 41, 12

ESERCIZI : 73-87-84

—
D ES 85 e 87
SONO NEI FILE
DELLA PROVA DESANTE

86: MANCA LA PARTE FINALE

88: SODAGNATO QUALCHE SEGNO (a V")
VA PROCEDIMENTO CORRETTO

$$\boxed{1} \text{ VALGONO } \begin{cases} E_0 = m_0 c^2 \\ E_{cin} = m_0 (\gamma - 1) c^2 \\ E_{TOT} = \gamma m_0 c^2 \end{cases}$$

ABBIAMO $E_{cin} = q E_0$, $|\vec{v}| = ?$
 $m_0 e = 9.1 \cdot 10^{-31} \text{ Kg}$, $|p| = ?$

IMPONIAMO $m_0 (\gamma - 1) c^2 = q m_0 c^2$
 $\Rightarrow \gamma = 10$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \Rightarrow \beta = \frac{1}{\sqrt{1-\frac{1}{\gamma^2}}} = \frac{10}{\sqrt{100-1}} \approx 0.995$$

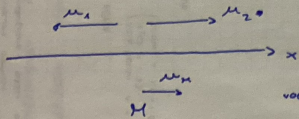
TROVO $v = \beta c$

ORA, SAPPIAMO $p = m_0 \gamma \beta c = 2.7 \cdot 10^{-21} \frac{\text{m Kg}}{\text{s}}$

$$\boxed{2} \text{ VALGONO } p^\mu = (m \gamma c, \gamma v_x m, \gamma v_y m, \gamma v_z m)$$

IN $\underline{K}$ $m_1 = m$ $m_2 = \frac{3}{4} m$

$u_1 = -\frac{3}{5} c$ $u_2 = \frac{4}{5} c$ u_H



VOGLIO: $\frac{H}{m} = ?$
 $u_H = ?$

HO $P^\mu = (H \gamma(u_H) c, H \gamma(u_H) u_H, 0, 0)$

$$P_1^\mu = (m_1 \gamma(u_1) c, m_1 \gamma(u_1) u_1, 0, 0)$$

$$P_2^\mu = (m_2 \gamma(u_2) c, m_2 \gamma(u_2) u_2, 0, 0)$$

SO CHE 4 MOMENTO SI CONSERVA
 TROVO

$$\gamma(u_1) = \frac{1}{\sqrt{1-\frac{9}{25}}} = \frac{5}{4} > 1$$

UGUALE PER u_2

$$\gamma(u_2) = \frac{1}{\sqrt{1-\frac{16}{25}}} = \frac{5}{3} > 1$$

così

$$P_1^\mu = (m \frac{5}{4} c, -\frac{3}{4} m c, 0, 0)$$

$$P_2^\mu = (\frac{5}{4} m c, m c, 0, 0)$$

IMPONGO LA CONSERVAZIONE

$$P^\mu = P_1^\mu + P_2^\mu$$

$$H \gamma(u_H) c = m_1 \gamma(u_1) c + m_2 \gamma(u_2) c$$

$$= \frac{5}{2} m c \quad (1)$$

$$H \gamma(u_H) u_H = \frac{1}{4} m c \quad (2)$$

DA (1) $\frac{H}{m} = \frac{5}{2 \gamma}$

DA (2) $\frac{H}{m} = \frac{c}{4 \gamma u_H}$

$$\Rightarrow \frac{5}{2 \gamma} = \frac{c}{4 \gamma u_H} \Rightarrow u_H = \frac{c}{10}$$

POSSIAMO RICAVARE

$$\gamma = \frac{1}{\sqrt{1 - \frac{1}{100}}} = \frac{10}{3\sqrt{11}}$$

E POI

$$\Rightarrow \frac{M}{m} = \frac{2}{2} \frac{3\sqrt{11}}{10} = \frac{3\sqrt{11}}{4}$$

(*) $B'^{\mu} = (0, 0, 1, 0)$ DI TIPO SPAZIO

↳ LO SCEGLIAMO

E $A'^{\mu} B'_{\mu} = 0$ È

SODDISFATTO PER ESEMPIO

SE $A'^{\mu} = (0, 1, 0, 0)$ CHE NON È DI TIPO TEMPO.

3 $A'^{\mu} B'_{\mu} = 0 \quad B'^{\mu} = (B^0, B^1, B^2, B^3)$

$A'^{\mu} B'_{\mu} = 0$

CONDIZIONE

$|B^1| > |B^3|$

↳ B'^{μ} TIPO TEMPO

a) CON $A'^{\mu} = (A^0, A^1, A^2, A^3)$ DOBBIAMO DIMOSTRARE CHE È TIPO SPAZIO, OVVERO $|A^0| < |A^1|$

SCRIVIAMO

$A'^{\mu} B'_{\mu} = -A^0 B_0 + \vec{A} \cdot \vec{B} = 0$

$\Rightarrow A_0 B_0 = \vec{A} \cdot \vec{B}$

SE PRENDO I MODULI VALE ANCORA L'UGUAGLIANZA

$|A_0 B_0| = |\vec{A} \cdot \vec{B}| \leq |\vec{A}| \cdot |\vec{B}|$ (SCHWARTZ)

$|A_0| |B_0|$

$\Rightarrow |A_0| |B_0| \leq |\vec{A}| |\vec{B}|$

$\frac{|A_0|}{|\vec{A}|} \leq \frac{|\vec{B}|}{|B_0|} < 1$

DA $|B^1| > |B^3|$

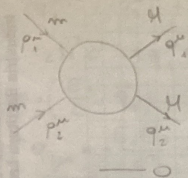
$\Rightarrow |A_0| < |\vec{A}|$

b) CON $A'^{\mu} B'_{\mu} = 0$ CON B'_{μ} TIPO SPAZIO DIMOSTRA CHE A'^{μ} NON NECESSARIAMENTE È TIPO TEMPO

PER LORENZ SAPPIAMO L'INVARIANZA

$A'^{\mu} B'_{\mu} = A'_{\mu} B^{\mu} = 0$

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VALE

$$p_1^\mu + q_1^\mu = p_2^\mu + q_2^\mu$$

a) DA SUGGERIMENTO CALCOLO

$$\begin{aligned} (p_1^\mu + q_1^\mu)(p_{2\mu} + q_{2\mu}) &= \\ &= (p_1^\mu p_{2\mu}) + p_1^\mu q_{2\mu} + q_1^\mu p_{2\mu} + (q_1^\mu q_{2\mu}) \\ &= -m^2 c^2 + p_1^\mu q_{2\mu} + q_1^\mu p_{2\mu} \quad \text{per } p_1^\mu - q_2^\mu = -M^2 c^2 \\ &= -m^2 c^2 - M^2 c^2 + 2p_1^\mu q_{2\mu} \end{aligned}$$

$$\begin{aligned} (p_2^\mu + q_2^\mu)(p_{2\mu} + q_{2\mu}) &= -m^2 c^2 - M^2 c^2 + 2p_2^\mu q_{2\mu} \\ \Rightarrow -m^2 c^2 - M^2 c^2 + 2p_1^\mu q_{2\mu} &= -m^2 c^2 - M^2 c^2 + 2p_2^\mu q_{2\mu} \\ \Rightarrow p_1^\mu q_{2\mu} &= p_2^\mu q_{2\mu} \end{aligned}$$

ORA $p_1^\mu + q_1^\mu = p_2^\mu + q_2^\mu \Rightarrow p_1^\mu - q_2^\mu = p_2^\mu - q_1^\mu$

FACCIO LO STESSO PROCEDIMENTO

$$\begin{aligned} (p_1^\mu - q_2^\mu)(p_{2\mu} - q_{2\mu}) &= p_1^\mu p_{2\mu} - q_2^\mu p_{2\mu} - p_1^\mu q_{2\mu} + q_2^\mu q_{2\mu} \\ &= (p_1^\mu)^2 + (q_2^\mu)^2 - 2p_1^\mu q_{2\mu} = -m^2 c^2 - M^2 c^2 - 2p_1^\mu q_{2\mu} \end{aligned}$$

ISEM

$$(p_1^\mu - q_2^\mu)(p_{2\mu} - q_{2\mu}) = -m^2 c^2 - M^2 c^2 - 2p_1^\mu q_{2\mu}$$

ALLORA

$$\Rightarrow p_1^\mu q_{2\mu} = p_2^\mu q_{2\mu}$$

b) $q_1^\mu q_{2\mu} - p_1^\mu p_{2\mu}$

CALCOLO LA NORMA QUADRA

$$p_1^\mu q_{2\mu} = q_2^\mu + p_2^\mu \Rightarrow (p_1^\mu - p_2^\mu) = (q_2^\mu - q_1^\mu)$$

$$\begin{aligned} (p_1^\mu - p_2^\mu)(p_{2\mu} - p_{2\mu}) &= (q_2^\mu - q_1^\mu)(q_{2\mu} - q_{1\mu}) \\ (p_1^\mu)^2 + (p_2^\mu)^2 - 2p_1^\mu p_{2\mu} &= (q_1^\mu)^2 + (q_2^\mu)^2 - 2q_1^\mu q_{2\mu} \\ -2m^2 c^2 - 2p_1^\mu p_{2\mu} &= -2M^2 c^2 - 2q_1^\mu q_{2\mu} \\ \Rightarrow q_1^\mu q_{2\mu} - p_1^\mu p_{2\mu} &= -c^2(M^2 - m^2) \end{aligned}$$

c) $A^\mu = q_1^\mu + q_2^\mu \quad B^\mu = p_1^\mu - p_2^\mu$

$$A^\mu B_{2\mu} = (q_1^\mu + q_2^\mu)(p_{2\mu} - p_{2\mu})$$

$$q_1^\mu p_{2\mu} - q_1^\mu p_{2\mu} + q_2^\mu p_{2\mu} - q_2^\mu p_{2\mu}$$

UO LE RELAZIONI DIMOSTRARE PRIMA

$$p_1^\mu q_{2\mu} - p_2^\mu q_{2\mu} + p_1^\mu q_{2\mu} - p_2^\mu q_{2\mu} = 0$$

∂_1

∂_2

∂_2

∂_1

$$A''_{\mu} = (q_1'')^2 + (q_2'')^2 + 2q_1'' q_2''$$

$$= -2M^2 c^2 + 2q_1'' q_2''$$

so $q_1'' = (\frac{E_1}{c}, \vec{q}_1)$ $q_2'' = (\frac{E_2}{c}, \vec{q}_2)$

$$q_1'' q_2'' = -\frac{E_1 E_2}{c^2} + \vec{q}_1 \cdot \vec{q}_2$$

se $E = \sqrt{m^2 c^4 + |\vec{q}|^2 c^2} \Rightarrow E > |\vec{q}| c > 0$

MA $\vec{q}_1 \cdot \vec{q}_2 \leq |\vec{q}_1| |\vec{q}_2| < \frac{E_1 E_2}{c^2}$

$$\Rightarrow q_1'' q_2'' = -\frac{E_1 E_2}{c^2} + \vec{q}_1 \cdot \vec{q}_2$$

$$\leq -\frac{E_1 E_2}{c^2} + |\vec{q}_1| |\vec{q}_2| < 0$$

se A'' TIPO TEMPO $A''_{\mu} A''^{\mu} = (A'')^2 < 0$

$$\Rightarrow -|\vec{A}|^2 + |A^0|^2 < 0 \Rightarrow |A^0|^2 > |\vec{A}|^2$$

CALCOLO $A''_{\mu} B''^{\mu} = 0$

$$-A_0 B_0 + \vec{A} \cdot \vec{B} = 0 \Rightarrow A_0 B_0 = \vec{A} \cdot \vec{B}$$

$$\Rightarrow |\vec{A}| |\vec{B}| = \vec{A} \cdot \vec{B} \leq |\vec{A}| |\vec{B}|$$

$$\Rightarrow \frac{|A_0|}{|\vec{A}|} \leq \frac{|\vec{B}|}{|B_0|} > 1 \Rightarrow |\vec{B}| > |B_0|$$

$$\Rightarrow B''_{\mu} B''^{\mu} > 0 \Rightarrow B'' \text{ TIPO SPAZIO}$$

74 D'' TIPO TEMPO IN K

K' CON V RISPETTO K $\beta = \frac{v}{c}$

D' STESSO SECONDO DI $D^0 = \gamma(v)(D^0 - \beta \cdot \vec{D})$

— 0 —

se $D''_{\mu} D''^{\mu} < 0 \Rightarrow$ TIPO TEMPO

$$\Rightarrow D''_{\mu} D''^{\mu} = -D_0'^2 + |\vec{D}'|^2 \Rightarrow |\vec{D}'|^2 < D_0'^2$$

$$\Rightarrow |\vec{D}'| < |D_0'|$$

IN PIÙ SO CHE $\beta < 1$

$$\Rightarrow D^0 = \gamma(v)(D^0 - \beta \cdot \vec{D}) < \gamma(D^0 - \vec{D})$$

se $D^0 > 0 \Rightarrow D^0' > 0$

se $D^0 < 0 \Rightarrow D^0' < 0$

OPPURE ESSENDO D'' DI TIPO TEMPO

LO SELEGO $D'' = (D'', \vec{0})$

CON $\vec{D} = \vec{0}$

$$\text{E } D^0' = \gamma(D^0 - \beta \cdot \vec{0}) = \gamma D^0$$

$$\boxed{75} \quad E = 3 \cdot 10^9 \text{ J} \quad q = 1 \cdot 10^{17} \text{ kg}_{18}$$

$$E = \sqrt{m_0^2 c^4 + q^2 c^2} \Rightarrow E^2 = m_0^2 c^4 + q^2 c^2$$

$$\Rightarrow m_0 = \sqrt{\frac{E^2}{c^4} - \frac{q^2}{c^2}}$$

$$\text{uso } c = 399792458 \cdot 10^8 \text{ m/s}$$

$$\Rightarrow m_0 = 1,24 \cdot 10^{27} \text{ kg}$$

$$T = E - E_0 = E - m_0 c^2 = 2,88 \cdot 10^9 \text{ J}$$

$$T = m(\gamma - 1)c^2 \Rightarrow \gamma = \frac{T}{mc^2} + 1$$

$$= 26,84$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow v = c \left(1 - \frac{1}{\gamma^2}\right)^{1/2}$$

$$= 2,99 \cdot 10^8 \text{ m/s}$$

F6 $A, m_0 \quad \vec{u}_A = \left(\frac{3}{5}c, 0, 0\right) \quad \text{in } K$

$B, \frac{7}{4}m_0 \quad \text{PERNA in } K$

$A + B = C$

a) $p_A^\mu = m_0 u_A^\mu = m_0 (\gamma c, \gamma \vec{u})$
 $= m_0 (\gamma c, \frac{3}{5} \gamma c, 0, 0)$

$\gamma(u) = \frac{1}{\sqrt{1 - \frac{9}{25}}} = \frac{5}{4} = \frac{5}{4}$

$\Rightarrow p_A^\mu = m_0 \left(\frac{5}{4}c, \frac{3}{4}c, 0, 0\right)$

b) URTO INELASTICO

$p_B^\mu = \frac{7}{4}m_0(c, \vec{u}_B) = \left(\frac{7}{4}m_0c, \vec{0}\right)$

SO CHE SI CONSERVA IL QUADRIIMPULSO

$p_A^\mu + p_B^\mu = p_C^\mu$

$m_0 \gamma(u)c + m_0 \gamma(u_0)c = m_c \gamma(u_c)c$
 $\frac{12}{4}m_0 = m_c \gamma(u_c)$

$m_0 \gamma(u) \vec{u} + \vec{0} = m_c \gamma(u_c) \vec{u}_c$

$\frac{3}{4}m_0c = m_c \gamma(u_c) u_c^1$

$\left\{ \begin{array}{l} \frac{12}{4}m_0 = m_c \gamma \\ \frac{3}{4}m_0c = m_c \gamma u_c^1 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} m_c = \frac{12m_0}{4\gamma} \\ \frac{3}{4}m_0c = \frac{12m_0}{4\gamma} \gamma u_c^1 \end{array} \right.$

$\left\{ \begin{array}{l} m_c = \frac{12m_0}{4\gamma} = 3m_0 \frac{1}{\gamma} \\ \frac{3}{12}c = u_c^1 \rightarrow \vec{u}_c = \left(\frac{1}{4}c, 0, 0\right) \end{array} \right.$

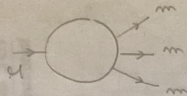
$\gamma(u_c) = \frac{1}{\sqrt{1 - \frac{1}{16}}} = \frac{4}{\sqrt{15}}$

$\Rightarrow m_c = 3m_0 \frac{\sqrt{15}}{4} = \frac{3\sqrt{15}}{4}m_0$

QUINDI

$p_C^\mu = (m_c \gamma(u_c)c, \vec{u}_c m_c \gamma)$
 $= \left(\frac{3\sqrt{15}}{4} \frac{4m_0}{\sqrt{15}}c, \frac{c}{4} \frac{3\sqrt{15}}{4} \frac{4}{\sqrt{15}}, 0, 0\right)$
 $= \left(3m_0c, \frac{3}{4}m_0c, 0, 0\right)$

FF



M in K A RIPOSO

$$m_0 = \frac{M}{5}$$

QUADRI-PULSO INIZIALE È:

$$P^M = M(\gamma(v) c, \gamma(v) \vec{v}) = (Mc, 0, 0, 0)$$

SO CHE SI CONSERVA

$$P^M = Q_1^M + Q_2^M + Q_3^M$$

$$\begin{cases} Mc = \gamma(v_1)mc + \gamma(v_2)mc + \gamma(v_3)mc \\ \vec{0} = \gamma(v_1)\vec{v}_1 + \gamma(v_2)\vec{v}_2 + \gamma(v_3)\vec{v}_3 \end{cases}$$

$$a) |\vec{v}_1| = |\vec{v}_2| = |\vec{v}_3| = v$$

$$\begin{cases} Mc = 3\gamma(v)mc \\ 0 = 3\gamma(v)\vec{v} \end{cases} \quad \gamma(v) = \frac{Mc}{3mc} = \frac{M}{3m} = \frac{5}{3}$$

$$\gamma^2(v) = \frac{1}{1 - \frac{v^2}{c^2}} \Rightarrow \frac{1}{\gamma^2} = \frac{c^2 - v^2}{c^2} \Rightarrow \gamma^2 = c^2 \left(1 - \frac{1}{\gamma^2}\right)$$

$$\Rightarrow v = c \left(1 - \frac{1}{\gamma^2}\right)^{1/2} = \left(1 - \frac{9}{25}\right)^{1/2} c = \sqrt{\frac{16}{25}} c = \frac{4}{5} c$$

b) SE UNA È FERMA $\rightarrow v$ ALTRE 2

$$\begin{cases} Mc = (2\gamma(v) + 1)mc \\ 0 = \gamma(v)\vec{v}_1 + \gamma(v_2)\vec{v}_2 \end{cases} \Rightarrow \begin{cases} Mc = \frac{2}{5}\gamma(v)Mc + \frac{1}{5}Mc \\ \gamma(v)\vec{v}_1 = -\gamma(v_2)\vec{v}_2 \end{cases}$$

LE ASSUMO UGUALI

$$\Rightarrow 1 - \frac{1}{5} = \frac{2}{5} \gamma(v)$$

$$\frac{4}{5} = \frac{2}{5} \gamma(v) \Rightarrow \gamma(v) = 2$$

$$\Rightarrow v = c \left(1 - \frac{1}{\gamma^2}\right)^{1/2} \Rightarrow v = \left(1 - \frac{1}{4}\right)^{1/2} c$$

$$\Rightarrow v = \frac{\sqrt{3}}{2} c$$

78 $C \rightarrow A, I$

GH SODIO $\lambda_1 = 589 \text{ nm}$, $I = 0$ $V_{\text{min},1} = 0,361 \text{ V}$

MERCURIO $\lambda_2 = 283,7 \text{ nm}$ $V_{\text{min},2} = 3,146 \text{ V}$

NOTA $e = 1,6 \cdot 10^{-19} \text{ C}$, $m_e = 9,109 \cdot 10^{-31} \text{ kg}$

TROVA h, L_e DI POTASSIO, $I_0 \text{ MAX} \Rightarrow \nu_0 \text{ MIN}$
PER EFFETTO NEL POTASSIO

T_{max} , ν_{max} DAL POTASSIO

$T_{\text{max}} = e\Delta V = h\nu - L_e$

IL LAVORO DI ESTRAZIONE SE EN. FOTONI
= EN. PER TIRARE FUORI GLI E

POSSO SCRIVERE

$$\begin{cases} eV_{\text{min},1} = h\nu_1 - L_e \\ eV_{\text{min},2} = h\nu_2 - L_e \end{cases}$$

NE FACCIAMO LA DIFFERENZA

$$eV_{\text{min},1} - eV_{\text{min},2} = h\nu_1 - h\nu_2$$

$$\Rightarrow h = \frac{e(V_{\text{min},1} - V_{\text{min},2})}{\nu_1 - \nu_2} = \frac{e(V_{\text{min},1} - V_{\text{min},2})}{c(\frac{1}{\lambda_1} - \frac{1}{\lambda_2})} = 6,62 \cdot 10^{-34} \text{ J s}$$

ORA PRENDO

$$eV_{\text{min},1} = h\nu_1 - L_e \Rightarrow L_e = h\nu_1 - eV_{\text{min},1} = 2,73 \cdot 10^{-19} \text{ J} \\ = 1,75 \text{ eV}$$

SCRIVO $e\Delta V = h\nu_0 - L_e$

$$\Rightarrow \nu_0 = \frac{1}{h}(e\Delta V + L_e) = \frac{L_e}{h} = 4,21 \cdot 10^{14} \text{ s}^{-1}$$

$\Delta V = 0$
PER LA CONDIZIONE DI
MINIMO

$$\lambda_0 = \frac{c}{\nu_0} = 7,12 \cdot 10^{-7} \text{ m} = 712 \text{ nm}$$

SO $T_{\text{max}} = e\Delta V = h\nu - L_e$

SODIO $T_{\text{max}} = h\nu_1 - L_e = \frac{hc}{\lambda_1} - L_e$

$$\Rightarrow T_{\text{max}} = 5,84 \cdot 10^{-20} \text{ kg} \frac{\text{m}^2}{\text{s}^2}$$

MERCURIO $T_{\text{max}} = \frac{hc}{\lambda_2} - L_e = 5,046 \cdot 10^{-19} \text{ kg} \frac{\text{m}^2}{\text{s}^2}$

SODIO $\nu_{\text{max}} = \sqrt{\frac{2T_{\text{max}}}{m_e}} = 3,58 \cdot 10^5 \text{ m/s}$

MERCURIO $\nu_{\text{max}} = 1,05 \cdot 10^6 \text{ m/s}$

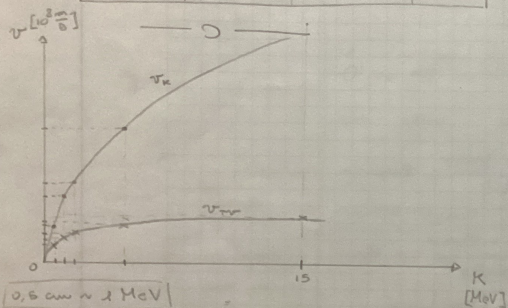
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$$v_{tr} = \frac{e}{\Delta t_{tr}}$$

$$v_k = \sqrt{\frac{2K}{m}}$$

VALORE CINETICA $K = e \Delta V$

$K [MeV]$	$v_k [10^8 \frac{m}{s}]$	$v_{tr} [10^8 \frac{m}{s}]$	β_{tr}
0,5	4,19	2,60	0,87
1,0	5,94	2,73	0,91
1,5	7,28	2,85	0,96
4,5	12,6	2,96	0,99
15,0	23,0	~3,0	~1,0



$$0,5 \text{ cm} \sim 1 \text{ MeV}$$

$$1 \text{ cm} \sim 3 \cdot 10^8 \frac{m}{s}$$

$$\text{su } T = m_e(\gamma - 1)c^2 \Rightarrow \frac{T}{m_e c^2 + 1} = \gamma$$

$$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{T + m_e c^2}{m_e c^2} \Rightarrow 1 - \frac{v^2}{c^2} = \left(\frac{m_e c^2}{T + m_e c^2} \right)^2$$

$$\Rightarrow \frac{v^2}{c^2} = 1 - \left(\frac{m_e c^2}{T + m_e c^2} \right)^2$$

$$\Rightarrow v = \sqrt{1 - \left(\frac{m_e c^2}{T + m_e c^2} \right)^2} c$$

$$K = (8,011 \cdot 10^{-14}; 1,6022 \cdot 10^{-13}; 2,41 \cdot 10^{-13}; 7,2 \cdot 10^{-13}; 2,4 \cdot 10^{-12}) J$$

$$v = (0,863; 0,941; 0,967; 0,995; 0,999) \frac{m}{s}$$

$$= (2,939; 2,813; 2,901; 2,985; 2,997) \cdot 10^8 \frac{m}{s}$$

$$\frac{v}{v_{tr}} = (0,935; 0,967; 0,99; 0,99; 0,999)$$

$$\sim (93,5; 96,7; 99; 99; 99,9) \%$$

QUIVIA L'ESPERIMENTO E L'APPROSSIMAZIONE CON LA MISURA DEL TEMPO DI VOLO È MOLTO BUONA.

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TROVA α_1 e α_2

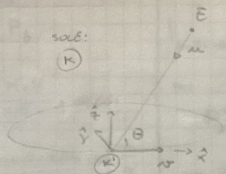
CONSIDERO

VALE

$$\vec{u}: \begin{cases} u_x = -c \cos \theta \\ u_y = 0 \\ u_z = -c \sin \theta \end{cases}$$

SOLE:

(K)



(K') SR DELLA TERRA

$$u'_x = \frac{u_x - v}{1 - \frac{v u_x}{c^2}}$$

$$u'_y = 0 \quad u'_z = \frac{u_z}{(1 - \frac{v u_x}{c^2})^{1/2}}$$

È L'ANGOLO DI OSSERVAZIONE DALLA TERRA SARÀ θ' , PER DEF:

$$u'_x = -c \cos \theta'$$

$$\cos \theta' \text{ VEDU} \quad \cos \theta' = \frac{u'_x}{c} = -\frac{-c \cos \theta - v}{1 + \frac{v c \cos \theta}{c^2}} \cdot \frac{1}{c}$$

$$\Rightarrow \cos \theta' = \frac{c \cos \theta + v}{1 + \beta \cos \theta} \cdot \frac{1}{c} \quad \text{MA SO } \beta \ll 1$$

SVILUPPO TAYLOR

$$\cos \theta' = \frac{c \cos \theta + \beta}{1 + \beta \cos \theta} \sim (\cos \theta + \beta)(1 - \beta \cos \theta + o(\beta^2))$$

$$= (\cos \theta + \beta - \beta \cos^2 \theta - \beta^2 \cos \theta + o(\beta^2))$$

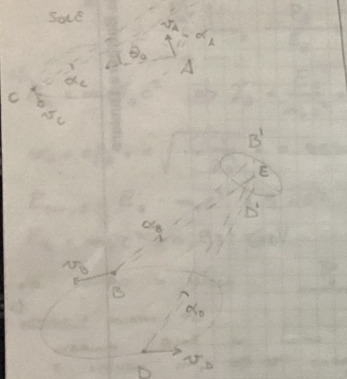
$$= (\cos \theta + \beta(1 - \cos^2 \theta) + \dots) = (\cos \theta + \beta \sin^2 \theta)$$

SO $\cos \theta' = \cos(\theta \pm \alpha) = \cos \theta \cos \alpha \mp \sin \theta \sin \alpha \sim \cos \theta \pm \alpha \sin \theta$
 COSTRUITO, DUE $\cos \theta'$

$$\cos \theta + \beta \sin^2 \theta = \cos \theta \pm \alpha \sin \theta$$

$$\Rightarrow \alpha = \mp \beta \sin \theta$$

$$\left\{ \begin{array}{l} \text{AC: } \theta = \pi/2 \quad \theta' = \theta - \alpha, \Rightarrow \alpha = \beta \\ \text{BC: } \theta = 0 \quad \theta' = \theta \pm \alpha \Rightarrow \alpha = \beta \sin \theta \end{array} \right.$$



SE TERRA IN AC $\rightarrow A'C'$

È TRA $\vec{v}_{AC}$ E LUCE STELLA

MA DALLA TERRA APPARE $\frac{\pi}{2} - \alpha_1$

$$\alpha_1 = \alpha_A = \alpha_C$$

SE TERRA IN B, D $\rightarrow B', D'$

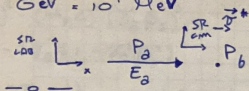
ANGOLO TRA $\vec{v}_{BD}$ E LUCE È θ_0

MA DALLA TERRA È $\theta' = \theta_0 \pm \alpha_2$

$$\alpha_2 = \alpha_B = \alpha_D$$

28) $m_p = 938 \frac{\text{MeV}}{c^2} = 0,938 \frac{\text{GeV}}{c^2}$

$E_a = 10 \text{ GeV} = 10^4 \text{ MeV}$

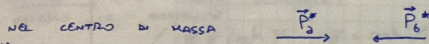


a) $E_a = m_p \gamma_a c^2 \Rightarrow \gamma_a = \frac{E_a}{m_p c^2} = \frac{10^4 \text{ MeV}}{938 \frac{\text{MeV}}{c^2} c^2} = 10,661$

$v_a = c \beta_a = c \sqrt{1 - \frac{1}{\gamma_a^2}} = 0,99956 c = 2,985 \cdot 10^8 \frac{\text{m}}{\text{s}}$

$E_{\text{cin},a} = E_a - m_p c^2 = 9,062 \text{ GeV}$

$E_b = m_p c^2 = 0,938 \text{ GeV}$



b) METODO 1 USIAMO TL

AVRAMO Q.TÀ DI MOTO UGUALE
E AVENDO $m_p \Rightarrow v$ SARANNO UGUALI
 $\Rightarrow$ STESSA ENERGIA

$\vec{p} = m_p \gamma \beta c$

$|\vec{p}_a^*| = |\vec{p}_b^*| = p^* \Rightarrow E_a^* = E_b^* = E^*$

SCRIVO IL QUADRIKUPLO

$P_a^\mu = (\frac{E_a}{c}, p_a, 0, 0)$

$P_b^\mu = (m_p c, \vec{0})$

con $\beta_{\text{cm}} = \frac{v^*}{c}$

SCRIVO LE TL

$$P_a^{\mu*} = \begin{pmatrix} \gamma_{\text{cm}} & -\beta_{\text{cm}} \gamma_{\text{cm}} & 0 & 0 \\ -\gamma_{\text{cm}} \beta_{\text{cm}} & \gamma_{\text{cm}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} E_a/c \\ p_a \\ 0 \\ 0 \end{pmatrix}$$

ANALOGA PER $P_b^{\mu*}$

POI PER TROVARE IL CM IMPONIAMO

$P_a^{\mu*} + P_b^{\mu*} = 0 \Rightarrow \beta_{\text{cm}} = \frac{\gamma_a \beta_a}{\gamma_a + 1} = 0,91$

$\Rightarrow \gamma_{\text{cm}} = \sqrt{\frac{\gamma_a + 1}{2}} = 2,41$

ESI POSSONO TROVARE LE NUOVE ENERGIE

$E^* = 2,26 \text{ GeV}$

METODO 2 UNOSCO P_a^μ E P_b^μ

E AVRO $P_a^{\mu*} = (\frac{E^*}{c}, \vec{p}^*)$

$P_b^{\mu*} = (\frac{E^*}{c}, -\vec{p}^*)$

POSSO ~~SCRIVERE~~ ~~SCRIVERE~~ ~~SCRIVERE~~ CALCOLARE
LA Q.TÀ DI MOTO TOTALE

$P_{\text{TOT}}^\mu = (\frac{E_a}{c} + m_p c, p_a, 0, 0)$

$P_{\text{TOT}}^{\mu*} = (\frac{2E^*}{c}, \vec{0})$

CALCOLO $P_{\text{TOT}}^\mu P_{\text{TOT},\mu} = P_{\text{TOT}}^{\mu*} P_{\text{TOT},\mu}^*$

NORMA QUADRA DEL QUADRIKUPLO È UN INVARIANTE

$-(\frac{E_a}{c} + m_p c)^2 + p_a^2 = -\frac{4E^{*2}}{c^2}$

$$-(m_p \gamma_a c + m_p c)^2 + (m_p \gamma_a \beta_a c)^2 = -\frac{4E^2}{c^2}$$

$$(m_p c)^2 (\gamma_a + 1)^2 - (m_p c)^2 (\gamma_a \beta_a)^2 =$$

$$= (m_p c)^2 (\gamma_a^2 + 2\gamma_a + 1 - \gamma_a^2 \beta_a^2)$$

$$= (m_p c)^2 (1 + 2\gamma_a + 1)$$

POICHÉ

$$\gamma_a^2 - \gamma_a^2 \beta_a^2 = \gamma_a^2 (1 - \beta_a^2)$$

$$\frac{1}{1 - \beta_a^2} (1 - \beta_a^2) = 1$$

E COSÌ TROVO

$$E^* = m_p c^2 \sqrt{\frac{\gamma_a + 1}{2}}$$

METODO 3 (NON TANTO GENERALE)
USIAMO LA RAPIDITÀ

$$\beta = \tanh Y, \quad \gamma = \cosh Y$$

$$\sinh Y = \beta \gamma$$

È ADDITIVA

SE HO v_1 QUINDI β_1 QUINDI Y_1
HA ANCHE $v_2 \rightarrow \beta_2 \rightarrow Y_2$

~~VARIA~~ $v'_1 = \frac{v_1 - v_2}{1 - \frac{v_1 v_2}{c^2}}$ NO ADDITIVA

E $Y'_1 = Y_1 + Y_2$ ADDITIVA

ORA NEL CASO IN CUI ABBIAMO STESSA
MASSA VARDA CHE LA:

$$\gamma^* = \frac{1}{2} \gamma_a \quad \text{E QUINDI} \quad \beta_a = \tanh Y_a$$

$$\gamma^* = \cosh Y^* = \cosh \frac{Y_a}{2}$$

$$\Rightarrow \gamma^* = \sqrt{\frac{1 + \cosh Y_a}{2}} = \sqrt{\frac{1 + \gamma_a}{2}}$$

c) $\pi \approx$ PIONE

$$m_\pi = 140 \frac{\text{MeV}}{c^2}$$

$$p + p \rightarrow p + p + N\pi$$

TRAVIAMO N ED IMPIANO LA
CONSERVAZIONE DELL'ENERGIA (CI METTIAMO
NEL SR DEL LAM)

$$E_{\text{TOT}}^* = E_a^* + E_b^* = \underbrace{2E^*}_{\text{INIZIALE}} + E_{a,\text{FIN}}^* + \sum_{i=1}^N E_{\pi,i}^*$$

$$\geq m_p c^2 + m_p c^2 + N m_\pi c^2$$

(CI METTIAMO NEL CASO IN CUI HO
LA MINIMA ENERGIA POSSIBILE
(QUELLA DI RIPOSO))

$$\Rightarrow N \leq \frac{E_{\text{TOT}}^* - 2m_p c^2}{m_\pi c^2} = \frac{4,52 \text{ GeV} - 2 \cdot 0,938 \frac{\text{GeV}}{c^2} c^2}{0,140 \frac{\text{GeV}}{c^2} c^2}$$

$$= 18,9$$

$$1) \vec{u}_A = \left(\frac{3}{5}c, 0, 0\right) \quad \vec{u}_B = \left(\frac{4}{5}c, 0, 0\right)$$

$$l_0 = 520 \text{ mm}$$

$$|\vec{u}_A| = \frac{3}{5}c$$

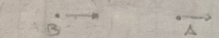
$$|\vec{u}_B| = \frac{4}{5}c$$

$$\gamma_A = \frac{1}{\sqrt{1 - \frac{9}{25}}} = \frac{5}{4}$$

$$\gamma_B = \frac{5}{3}$$

VALGOWO IN GENERALE

$$\lambda = \frac{v}{\nu}; \omega = 2\pi\nu$$

5)  B DIETRO A
ED A EMETTE LUCE.
CHE VEDE B?

MI METTO NEL SIZ DI B

$$W = \frac{u_{Ax} - u_{Bx}}{1 - \frac{u_{Ax} u_{Bx}}{c^2}} = \frac{\left(\frac{3}{5} - \frac{4}{5}\right)c}{1 - \frac{12}{25}} = -\frac{5}{13}c$$

$$\gamma(W) = \frac{1}{\sqrt{1 - \frac{25}{169}}} = \frac{13}{12}$$

EFFETTO DOPPLER

$$\omega' = \gamma(W) \omega \left(1 - \frac{W}{c}\right) = \frac{13}{12} 2\pi\nu \left(1 + \frac{5/13 c}{c}\right)$$

$$= \frac{13}{12} 2\pi\nu \left(\frac{13}{13}\right) = \frac{13}{6} \pi c \nu = 3\pi\nu c$$

$$\kappa' = \gamma \kappa \left(1 - \frac{W \cdot c}{c^2}\right) = \frac{13}{12} \kappa \left(1 + \frac{5}{12}\right) = \frac{3}{2} \kappa$$

$$\lambda = \frac{2\pi}{\kappa} \rightarrow \lambda' = \frac{2\pi}{\kappa'} = \frac{4\pi}{3\kappa} = \frac{4\pi}{3 \cdot 2\pi/l_0} = \frac{2}{3} l_0 = 347 \text{ mm}$$

$$b) \begin{array}{c} \bullet \xrightarrow{\quad} \\ A \end{array} \quad \begin{array}{c} \bullet \xrightarrow{\quad} \\ B \end{array}$$

B OGGI
ED EMETTE
OGGI VESCE

$$W = \frac{(u_{Bx} - u_{Ax})}{1 - \frac{u_{Bx} u_{Ax}}{c^2}} = \frac{\left(\frac{4}{5} - \frac{3}{5}\right)c}{1 - \frac{12}{25}} = \frac{1}{5} \frac{25}{13} c = \frac{5}{13} c$$

$$\gamma(W) = \frac{13}{12}$$

$$\kappa' = \gamma \kappa \left(1 - \frac{Wc}{c^2}\right) = \frac{13}{12} \kappa \left(1 - \frac{5}{13}\right)$$

$$= \frac{13}{12} \kappa \frac{8}{13} = \frac{2}{3} \kappa$$

$$\lambda = \frac{2\pi}{\kappa} \rightarrow \lambda' = \frac{2\pi}{\kappa'} = \frac{2\pi}{\frac{2}{3}\kappa} = \frac{2\pi}{2\kappa} 3 = \frac{2\pi \cdot 3}{2 \cdot 2\pi/l_0} = \frac{3}{2} l_0$$

$$= 780 \text{ mm}$$

81

$$\mu_A = \frac{\mu_B - W}{1 - W \frac{\mu_B}{c^2}} \rightarrow \mu_A \left(1 - W \frac{\mu_B}{c^2}\right) = \mu_B - W$$

$$W \left(\frac{\mu_B}{c^2} - 1 \right) = \mu_A - \mu_B$$

$$W = \frac{\mu_A - \mu_B}{\frac{\mu_B}{c^2} - 1} = -\frac{1}{5}c - \frac{1}{-13}25 = \boxed{\frac{5}{13}c}$$

$$\boxed{\gamma(W) = \frac{13}{12}}$$

$$\lambda' = \gamma \lambda \left(1 - \frac{W}{c}\right) \rightarrow \frac{1}{\lambda'} = \gamma \frac{1}{\lambda_0} \underbrace{\left(1 - \frac{W}{c}\right)}_{8/13}$$

$$\boxed{\frac{1}{\lambda'} = \frac{12}{13} \frac{1}{\lambda_0} \frac{13}{8} = \frac{3}{2} \frac{1}{\lambda_0}}$$

$$\frac{1}{\lambda'} = \gamma \frac{1}{\lambda_0} \underbrace{\left(1 + \frac{W}{c}\right)}_{18/13} \rightarrow \frac{1}{\lambda'} = \frac{12}{13} \frac{1}{\lambda_0} \frac{13}{8} = \frac{6}{2} \frac{1}{\lambda_0} = \boxed{\frac{2}{3} \frac{1}{\lambda_0}}$$

IBL spa

Industria Battitura Lamiere

82

$$v = \frac{c}{2}$$

3 ANNI (SECONDO LUI)

L'ANDATA

OGNI ANNO INVIÀ UN SEGNALE ALL'ANNIVERS.

$$\Delta t = \delta \Delta \tau$$

$$\Delta t = \frac{2}{\sqrt{3}} \Delta \tau$$

$$\delta = \frac{1}{\sqrt{1 - \frac{1}{4}}} = \frac{2}{\sqrt{3}}$$

$$3 \text{ ANNI} = 34608000 \text{ s} = \Delta \tau_A$$

$$\Delta t_{\text{TERRA}} = \frac{2}{\sqrt{3}} \Delta \tau_A = 3,46 \text{ ANNI}$$

A TERRA, AVVIENE L'INVERSIONE

$$\text{IL VIAGGIO COMPLETO DURA } 2 \Delta t_A = 6,928 \text{ ANNI}$$

ORA VOGLIO L'INTERVALLO TRA 2 SEGNALE ALL'ANDATA.

LO MANDA AD x_A, t_A E POI AD

$$x_B = x_A + v \cdot t = x_A + \frac{c}{2} (1 \text{ ANNO}) \quad t_B = t_A + (1 \text{ ANNO})$$

MA ANZILTRÒ SEGNALE

$$1 \text{ SEGNALE ARRIVANO AD } t_A = \frac{x_A}{c} \quad t_B = \frac{x_B}{c} + 1 \text{ ANNO}$$

$$\Rightarrow t_B = \frac{x_A + \frac{c}{2} (1 \text{ ANNO})}{c} + 1 \text{ ANNO} = \frac{x_A}{c} + \frac{1 \text{ ANNO}}{2} + 1 \text{ ANNO}$$

$$E \text{ LA MISURA A } t'_A = \gamma \frac{x_A}{c} \quad t'_B = \gamma \frac{x_A}{c} + \gamma \frac{3 \text{ ANNO}}{2}$$

$$\Delta t'_{AB} = \gamma \frac{3 \text{ ANNI}}{2} = \frac{2}{\sqrt{3}} \frac{3 \text{ ANNI}}{2} = \frac{3}{\sqrt{3}} \text{ ANNI} = 1,73 \text{ ANNI}$$

CALCOLO L'INTERVALLO TRA
CHE CHETTE AL RITORNO

$$x_A, t_A$$

$$x_B = x_A - v t_B = x_A - \frac{c}{2} (1 \text{ ANNO})$$

$$t_B = t_A + 1 \text{ ANNO} = \frac{x_B}{c} + 1 \text{ ANNO} = \frac{x_A}{c} + 1 \text{ ANNO} - \frac{1 \text{ ANNO}}{2}$$

LA TERRA LI RICEVE

$$t'_A = \gamma t_A = \gamma \frac{x_A}{c}$$

$$t'_B = \gamma t_B = \gamma \frac{x_B}{c} + \gamma 1 \text{ ANNO} = \gamma \frac{x_A}{c} + \gamma \frac{1 \text{ ANNO}}{2}$$

$$\Delta t'_{AB} = \gamma \frac{1 \text{ ANNO}}{2} = \frac{1 \text{ ANNO}}{\sqrt{3}} = 0,58 \text{ ANNI}$$

83

A: m

B: 2m PERNA

TOTALMENTE ANELASTICA → GENERA C: H

— 0 —
1 2

$$\left\{ \begin{aligned} \sigma_A + 2 &= \sigma_A \frac{v_A}{v_C} \Rightarrow v_C = \frac{\sigma_A v_A}{\sigma_A + 2} \\ \sigma_C &= \frac{m}{M} \sigma_A \frac{v_A}{v_C} \end{aligned} \right.$$

$$\frac{5}{3} C \frac{1}{11} = \frac{5}{3} C \frac{3}{11} = \frac{4}{11} C$$

a)

$$P_A^M = (m\sigma_A C, m\sigma_A v_A, 0, 0)$$

$$P_B^M = (2mc, 0, 0, 0)$$

CORRETTO

$$\text{AVIZO } P_C^M = (M\sigma_C C, M\sigma_C v_C, 0, 0)$$

$$\text{VALC } P_A^M + P_B^M = P_C^M$$

USO MASS-SHELL

$$P_C^M P_{CM} = (P_A^M + P_B^M)(P_{AM} + P_{BM})$$

$$| -H^2 C^2 = -m^2 C^2 - (2m)^2 C^2 + 2 P_A^M P_{AM}$$

$$-H^2 C^2 = -5m^2 C^2 - 2(m\sigma_A C)(2mc)$$

$$\Rightarrow 4\sigma_A = \frac{H^2}{m^2} - 5 \quad \text{se } M \geq 5m$$

$$\Rightarrow 4\sigma_A \geq 20 \Rightarrow \sigma_A \geq 5 \rightarrow \beta_A \geq \frac{2\sqrt{5}}{5}$$

$$\Rightarrow v \geq \frac{2\sqrt{5}}{5} C$$

$$\sigma_C = \frac{m}{M} \sigma_A \frac{v_A}{v_C}$$

83

A: m

B: 2m PERMA

TOTALMENTE ANELASTICA → GENERA C: H

— 0 —

di A → $\vec{v}_A$

5 m

2VO COMPONENTE 0 DEL QUADRIIMPULSO

$$m\vec{v}_A + E_{\text{imp}} = E_{\text{ARM}} + E_{\text{CFIN}} + E_{\text{CFIN}}$$

$$m\vec{v}_A^2 + 2m\vec{v}_A^2 = E_{\text{CFIN}} \geq 5m\vec{v}_A^2 \text{ EN A RIPOSO}$$

$$\vec{v} \geq \frac{3m\vec{v}_A^2}{m\vec{v}_A^2} \Rightarrow \vec{v} \geq 3$$

$$\frac{v^2 - v_A^2}{c^2} \leq \frac{1}{3} \Rightarrow v \geq c \left(1 - \frac{1}{9}\right)^{1/2}$$

$$\Rightarrow v \geq \sqrt{\frac{8}{9}} c \Rightarrow v \geq \frac{2\sqrt{2}}{3} c$$

$$b) v_A = \frac{4}{5} c$$

CONSERVO IL QUADRIIMPULSO

$$m\vec{v}_A^2 + 2m\vec{v}_A^2 = M\vec{v}_C^2$$

$$m\vec{v}_A v_A = M\vec{v}_C v_C$$

$$\text{CALCOLO } \vec{v}_A = \frac{1}{\sqrt{1 - \frac{16}{25}}} = \frac{5}{3}$$

$$\left\{ \vec{v}_A^2 + 2\vec{v}_A^2 = \frac{M}{m} \frac{m}{M} \vec{v}_A \frac{\vec{v}_A}{\vec{v}_C} \right.$$

$$\vec{v}_C = \frac{m}{M} \vec{v}_A \frac{\vec{v}_A}{\vec{v}_C}$$

$$\left\{ \vec{v}_A + 2 = \vec{v}_A \frac{\vec{v}_A}{\vec{v}_C} \Rightarrow \vec{v}_C = \frac{\vec{v}_A \vec{v}_A}{\vec{v}_A + 2} \right.$$

$$\vec{v}_C = \frac{m}{M} \vec{v}_A \frac{\vec{v}_A}{\vec{v}_C}$$

$$\vec{v}_C = \frac{5}{3} \frac{4}{5} c \frac{1}{\frac{5}{3} + 2} = \frac{8}{3} \frac{4}{9} c \frac{3}{11} = \frac{4}{11} c$$

$$\text{COS. } \vec{v}_C = \frac{1}{\sqrt{1 - \frac{16}{121}}} = \frac{11}{\sqrt{105}}$$

QUINDI:

$$\Rightarrow \frac{M}{m} = \frac{1}{\vec{v}_C} \vec{v}_A \frac{\vec{v}_A}{\vec{v}_C} = \frac{\sqrt{105}}{11} \frac{4}{3} \frac{4}{5} \frac{11}{4} = \frac{\sqrt{105}}{3}$$

$$\Rightarrow \frac{M}{m} = \sqrt{\frac{36 \cdot 3}{8}} = \sqrt{\frac{36}{3}}$$

34 A, B : m

(K) $\vec{v}_A = (\frac{3}{4}c, 0, 0)$ $\vec{v}_B = (\frac{c}{3}, \frac{c}{3}, 0)$

a) $v_{Bx}' = \frac{v_{Bx} - v_A}{1 - \frac{v_A v_{Bx}}{c^2}} = \frac{\frac{1}{3} - \frac{3}{4}}{1 - \frac{1}{4}} c = c \frac{-\frac{5}{12}}{\frac{3}{4}} = -\frac{5}{12} \frac{4}{3} c$
 $= -\frac{5}{9} c$

$v_{By}' = \frac{v_{By}}{\gamma_A (1 - \frac{v_A v_{Bx}}{c^2})} = \frac{c/3}{\gamma_A (1 - 1/4)} = \frac{1}{3} \frac{\sqrt{7}}{4} \frac{4}{3} c$
 $(\frac{1}{1 - \frac{9}{16}} = \frac{4}{\sqrt{7}} = \frac{\sqrt{7}}{9} c$

QUINDI NEL SR DI A $\vec{v}_B' = (-\frac{5}{9}c, \frac{\sqrt{7}}{9}c, 0, 0)$

E $|\vec{v}_B'| = \sqrt{\frac{25}{81}c^2 + \frac{7}{81}c^2} = \frac{\sqrt{32}}{9}c = \frac{4\sqrt{2}}{9}c$

b) $P_A^\mu = (m\gamma_A c, m\gamma_A \vec{v}_A)$ $P_B^\mu = (m\gamma_B c, m\gamma_B \vec{v}_B)$

CALCOLO $\gamma_B = \frac{1}{\sqrt{1 - v_B^2}} = \frac{1}{\sqrt{1 - \frac{2}{9}}} = \frac{3}{\sqrt{7}}$

$P_A^\mu = m \frac{4}{\sqrt{7}} (c, \frac{3}{4}c, 0, 0)$ $P_B^\mu = m \frac{3}{\sqrt{7}} (c, \frac{c}{3}, \frac{c}{3}, 0)$

$P_{TOT}^\mu = mc (\frac{4}{\sqrt{7}} + \frac{3}{\sqrt{7}}, \frac{3}{\sqrt{7}} + \frac{1}{\sqrt{7}}, \frac{1}{\sqrt{7}}, 0)$
 $= mc (\frac{7}{\sqrt{7}}, \frac{4}{\sqrt{7}}, \frac{1}{\sqrt{7}}, 0)$

COSI $E_{TOT} = \sqrt{7} mc^2$ $\vec{P} = mc (\frac{4}{\sqrt{7}}, \frac{1}{\sqrt{7}}, 0)$

c) SO CHE CA NORHA QUADRO
CONSERVA DI QUADRIVETTO

$P_{TOT}^\mu P_{TOT\mu} = -(\sqrt{7} mc)^2 + (\frac{4}{\sqrt{7}} mc)^2 + (\frac{1}{\sqrt{7}} mc)^2$
 $= -7 m^2 c^2 + \frac{16}{7} m^2 c^2 + \frac{1}{7} m^2 c^2$
 $= \frac{-49 + 16 + 1}{7} m^2 c^2$
 $= -\frac{32}{7} m^2 c^2$

NEL CM HO $P_{CM}^\mu = (\frac{E^*}{c}, \vec{0}) \in$

ANALOGO DI B

$P_{CM}^\mu = (2 \frac{E^*}{c}, \vec{0})$

$P_{CM}^\mu P_{CM\mu} = -4 \frac{E^{*2}}{c^2}$

$\Rightarrow 4 \frac{E^{*2}}{c^2} = \frac{32}{7} m^2 c^2$

$\Rightarrow E^2 = \frac{8}{7} m^2 c^4 \Rightarrow E^* = \frac{2\sqrt{2}}{\sqrt{7}} mc^2$

SO $E^* = \gamma(v) mc^2$

$\gamma(v) = \frac{2\sqrt{2}}{\sqrt{7}} \Rightarrow \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{2\sqrt{2}}{\sqrt{7}} \Rightarrow$

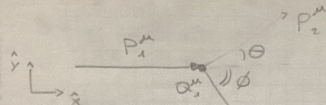
$\frac{c^2 - v^2}{c^2} = \frac{7}{8} \Rightarrow v = c (1 - \frac{7}{8})^{1/2} = \frac{c}{2\sqrt{2}}$

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COLLISIONE ELASTICA

B: M FERRA (SR LAB)

A: 31 NUOVE LUNGO X



$$P_1^M = \left(\frac{E_1}{c}, p_1, 0, 0 \right)$$

$$Q_1^M = (Mc, 0, 0, 0)$$

$$P_2^M = \left(\frac{E_2}{c}, p_2 \cos \theta, p_2 \sin \theta, 0 \right)$$

$$Q_2^M = \left(\frac{E_2}{c}, q_2 \cos \phi, -q_2 \sin \phi, 0 \right)$$

VALE $P_1^M + Q_1^M = P_2^M + Q_2^M$

$$\Rightarrow Q_2^M = P_1^M + Q_1^M - P_2^M$$

o) A: m DIMOSTRA $Mc, -E_2 = \frac{E_1 E_2}{c^2} - p_1 p_2 \cos \theta - m^2 c^2$

CALCOLO

$$\begin{aligned} Q_2^M Q_{2M} &= (Q_2^M)^2 = (P_1^M + Q_1^M - P_2^M)(P_{1M} + Q_{1M} - P_{2M}) \\ &= (P_1^M)^2 + P_1^M Q_{1M} - P_1^M P_{2M} + Q_1^M P_{1M} + (Q_1^M)^2 - Q_1^M P_{2M} - P_2^M P_{1M} \\ &\quad - P_2^M Q_{1M} + (P_2^M)^2 \\ &= (P_1^M)^2 + 2P_1^M Q_{1M} - 2P_1^M P_{2M} - 2Q_1^M P_{2M} + (Q_1^M)^2 + (P_2^M)^2 \\ &= \left[-\left(\frac{E_1}{c}\right)^2 + p_1^2 \right] + 2 \left[-\frac{E_1}{c} Mc \right] - 2 \left[-\frac{E_1 E_2}{c^2} + p_1 p_2 \cos \theta \right] - \\ &\quad - 2 \left[-\frac{E_2}{c} Mc \right] + [-(Mc)^2] + \left[-\left(\frac{E_2}{c}\right)^2 + p_2^2 \cos^2 \theta + p_2^2 \sin^2 \theta \right] \end{aligned}$$

$$\begin{aligned} &= -\frac{E_1^2}{c^2} + p_1^2 - 2McE_1 + 2\frac{E_1 E_2}{c^2} - 2p_1 p_2 \cos \theta + 2McE_2 \\ &\quad - Mc^2 - \frac{E_2^2}{c^2} + p_2^2 \end{aligned}$$

$$(Q_2^M)^2 = -\frac{E_2^2}{c^2} + q_2^2$$

SI CONSERVA IL QUADRIKORRENTO

$$\frac{E_2}{c} = \frac{E_1}{c} + Mc - \frac{E_2}{c} \quad \text{COMPONENTE 0}$$

$$\begin{aligned} \frac{E_2^2}{c^2} &= \frac{E_1^2}{c^2} + Mc^2 + \frac{E_2^2}{c^2} + 2\frac{E_1}{c} Mc - 2\frac{E_1 E_2}{c^2} - 2\frac{E_2}{c} Mc \\ &= \frac{E_1^2}{c^2} + Mc^2 + \frac{E_2^2}{c^2} + 2McE_1 - 2\frac{E_1 E_2}{c^2} - 2McE_2 \end{aligned}$$

$$\begin{aligned} -\frac{E_1^2}{c^2} - Mc^2 - \frac{E_2^2}{c^2} - 2McE_1 + 2\frac{E_1 E_2}{c^2} + 2McE_2 &= -\frac{E_1^2}{c^2} + p_1^2 - 2McE_1 \\ + 2\frac{E_1 E_2}{c^2} - 2p_1 p_2 \cos \theta + 2McE_2 - Mc^2 - \frac{E_2^2}{c^2} + p_2^2 \end{aligned}$$

$$\Rightarrow p_1^2 + p_2^2 - 2p_1 p_2 \cos \theta = 0$$

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$$S = \begin{pmatrix} 0 & -E^x & -E^y & -E^z \\ E^x & 0 & CB^z & -CB^y \\ E^y & -CB^z & 0 & CB^x \\ E^z & CB^y & -CB^x & 0 \end{pmatrix}$$

$$S' = (\Lambda^{-1})^\mu_\alpha S_{\mu\beta} (\Lambda^{-1})^\beta_\nu$$

$$S' = \begin{pmatrix} 0 & -E'^x & -E'^y & -E'^z \\ E'^x & 0 & CB'^z & -CB'^y \\ E'^y & -CB'^z & 0 & CB'^x \\ E'^z & CB'^y & -CB'^x & 0 \end{pmatrix}$$

$$(\Lambda^{-1})^\mu_\alpha S_{\mu\beta} (\Lambda^{-1})^\beta_\nu = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -E^x & - \\ & & & \end{pmatrix} \Lambda^{-1}$$

$$= \begin{pmatrix} -\gamma\beta E^x & -\gamma E^x & -\gamma E^y - \gamma\beta CB^z & -\gamma E^z + \gamma\beta CB^y \\ \gamma E^x & \gamma\beta E^x & \gamma\beta E^y + \gamma\beta^2 C & \gamma\beta E^z - \gamma CB^y \\ E^y & -CB^z & 0 & CB^x \\ E^z & CB^y & -CB^x & 0 \end{pmatrix} \Lambda^{-1}$$

$$\begin{array}{|c|c|} \hline \text{PRIMA} & \text{DOPO} \\ \hline \begin{vmatrix} \hat{u} & \hat{v} & \hat{w} \\ \gamma & 0 & 0 \\ 0 & 0 & 0 \\ \gamma\beta\gamma & \gamma\beta\gamma & \gamma\beta\gamma \end{vmatrix} = \begin{vmatrix} 0 \\ -\gamma\beta E^x \\ \gamma\beta E^y \\ \gamma\beta E^z \end{vmatrix} & \begin{vmatrix} \hat{u} & \hat{v} & \hat{w} \\ \gamma & \gamma & \gamma \\ \gamma\beta\gamma & \gamma\beta\gamma & \gamma\beta\gamma \\ E^x & E^y & E^z \end{vmatrix} = \begin{vmatrix} 0 \\ \gamma E^x \\ \gamma E^y \\ \gamma E^z \end{vmatrix} \\ \hline \end{array}$$

$$= \begin{pmatrix} -\gamma\beta E^x + \gamma\beta E^x & \gamma\beta E^x - \gamma E^x & -\gamma E^y - \gamma\beta CB^z & -\gamma E^z + \gamma\beta CB^y \\ \gamma E^x + \gamma\beta CB^z & \gamma\beta E^x + \gamma E^x & \gamma\beta E^y + \gamma\beta^2 C & \gamma\beta E^z - \gamma CB^y \\ \gamma E^y + \gamma\beta CB^z & \gamma\beta E^y - \gamma CB^z & 0 & CB^x \\ \gamma E^z - \gamma\beta CB^y & \gamma\beta E^z + \gamma\beta^2 C & -CB^x & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -\gamma E^x (1-\beta^2) & -\gamma E^y - \gamma\beta CB^z & -\gamma E^z + \gamma\beta CB^y \\ \gamma E^x (1-\beta^2) & 0 & \gamma\beta E^y + \gamma\beta^2 C & \gamma\beta E^z - \gamma CB^y \\ \gamma E^y + \gamma\beta CB^z & \gamma\beta E^y - \gamma CB^z & 0 & CB^x \\ \gamma E^z - \gamma\beta CB^y & \gamma\beta E^z + \gamma\beta^2 C & -CB^x & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -E^x & -E^y & -E^z \\ E^x & 0 & \gamma\beta E^y + \gamma\beta^2 C & \gamma\beta E^z - \gamma CB^y \\ \gamma E^y + \gamma\beta CB^z & \gamma\beta E^y - \gamma CB^z & 0 & CB^x \\ \gamma E^z - \gamma\beta CB^y & \gamma\beta E^z + \gamma\beta^2 C & -CB^x & 0 \end{pmatrix}$$

QUINDI

$$E'^x = E^x$$

$$E'^y = \gamma E^y + \gamma\beta CB^z = \gamma(E^y + (v \times B)_y)$$

$$E'^z = \gamma E^z - \gamma\beta CB^y = \gamma(E^z - (v \times B)_z)$$

$$CB'^x = CB^x$$

$$CB'^y = -\gamma\beta E^z + \gamma CB^y = \gamma C(B^y + (\frac{v}{c^2} \times E)_y)$$

$$CB'^z = \gamma\beta E^y + \gamma\beta^2 C = \gamma C(B^z + (\frac{v}{c^2} \times E)_z)$$

$$\beta = \frac{v}{c}$$