TUTORAGGIO FISICA III

2

TUTORAGGI : 4, 5
ESERCIZI : 25 - 40

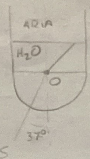
ES 29 PUNTO 6
32 RISULTATO T_1
PUNTO C
33 PUNTO C
(USA TRUCCO ES 14)

25

$n = 1,333$

$\theta = 37°$

HO RIFLESSIONE TOTALE SU ARIA - ACQUA



— O —

a)

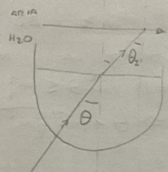
$\theta_r = \frac{\pi}{2}$

$\sin\theta_2 = \frac{n_2}{n_1} = \frac{n_{ARIA}}{n_{H2O}} = \frac{1}{n}$

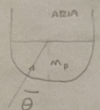
SO SNELL $n_p \sin\theta = n \sin\theta_2$

$\Rightarrow n_p \sin\theta = n \frac{1}{n}$

$\Rightarrow n_p = \frac{1}{\sin\theta} = 1.66$



b)



MA CON L'ACQUA

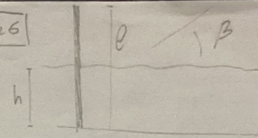
CALCOLO $\theta_{LIM} = \theta_2$

$\sin\theta_2 = \frac{1}{n} \Rightarrow \theta_2 = \sin^{-1}\left(\frac{1}{n}\right) = 48.6$

CON IL PLEXIGLASS

$\theta_{LIM} = \sin^{-1}\left(\frac{1}{n_p}\right) = 37.04 > \theta \Rightarrow$ NON ESCE LUCE

26



$h = 2\ m$

$\ell = 5\ m$

$\beta = 60°$

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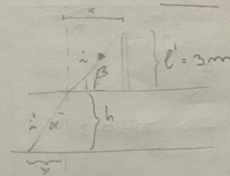


$i \sin\beta = \ell'$

$\Rightarrow i = \frac{\ell'}{\sin\beta} = 3,46\ m$

$x = i \cos\beta = 1.73\ m$

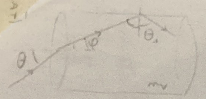
CERCO α

$n \sin\left(\frac{\pi}{2} - \beta\right) = n_{H2O} \sin\alpha \Rightarrow \alpha = \sin^{-1}\left(\frac{\sin\left(\frac{\pi}{2} - \beta\right)}{n_{H2O}}\right) = 22,08°$

$h = i' \cos\alpha \rightarrow i' = \frac{h}{\cos\alpha} = 2,158\ m$

$y = i' \sin\alpha = 0,81\ m$

$g = x + y = 2,54\ m$



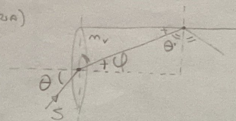
$n_{ARIA} = n = 1$

SUP. LATERALE HO RIFLESSIONE TOTALE $\forall\theta$

$n_g = 1.52$

a) APPLICO SNELL ($n_A = n$ ARIA)

$n_A \sin\theta = n_v \sin\varphi$ (1)



SO DI AVERE RIFLESSIONE TOTALE

$n_v \sin\theta' = n \sin\theta_t$

MA VOGLIO CHE θ_t NON CI SIA $\sin\theta_t \geq 1$

$\Rightarrow n_v \sin\theta' \geq 1 \Rightarrow \frac{1}{n_v} \leq \sin\theta'$

MA $\theta' = \pi/2 - \varphi$

$\Rightarrow \frac{1}{n_v} \leq \cos\varphi = \sqrt{1-\sin^2\varphi}$

MA DA (1) $\Rightarrow \sin\varphi = \frac{n_A}{n_v}\sin\theta$

$\frac{1}{n_v} \leq \sqrt{1-\left(\frac{n_A}{n_v}\sin\theta\right)^2}$ $\quad \frac{1}{n_v^2} + \frac{n_A^2}{n_v^2}\sin^2\theta \leq 1$

$\frac{1}{n_v^2}(1+\sin^2\theta) \leq 1 \Rightarrow \frac{1}{n_v^2} \leq \frac{1}{1+\sin^2\theta}$

$\Rightarrow n_v^2 \geq 1+\sin^2\theta$ MA $\sin^2\theta \in [0,1]$

$\Rightarrow n_v^2 \geq 2 \Rightarrow n_v \geq \sqrt{2}$

b) SNELL $n_v \sin\theta' = n_g \sin\theta_t$

DI NUOVO $\sin\theta_t \geq 1$

$\Rightarrow \frac{1}{n_v} \leq \frac{\sin\theta'}{n_g} \Rightarrow \frac{1}{n_v} \leq \frac{\cos\varphi}{n_g}$

$\frac{1}{n_v} \leq \frac{1}{n_g}\sqrt{1-\sin^2\varphi}$

(1) $\Rightarrow \sin\varphi = \frac{\sin\theta}{n_v}$

$\frac{1}{n_v} \leq \frac{1}{n_g}\sqrt{1-\frac{\sin^2\theta}{n_v^2}} \Rightarrow \frac{1}{n_v^2}\left(1+\frac{\sin^2\theta}{n_g^2}\right) \leq \frac{1}{n_g^2}$

$\Rightarrow n_v^2 \geq n_g^2 + \underbrace{\sin^2\theta}_{\in[0,1]}$

$\Rightarrow n_v^2 \geq (1.52)^2 + 1 = 3,31$

$\Rightarrow n_v \geq 1,82$

c) PARTO DA $n_v^2 \geq n_g^2 + \sin^2\theta$

$\Rightarrow \sin^2\theta \leq n_v^2 - n_g^2$

$\Rightarrow \theta \leq \sin^{-1}\sqrt{n_v^2 - n_g^2} = 41,85°$

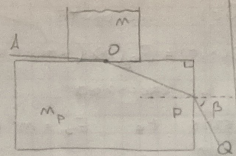
28 RIFRATTOMETRO DI PULFRICH

$n_p > n$

$n_p = 1.62 \quad , \quad n = ?$

$\theta_{AO} \sim \pi/2$

$\beta = 58°$



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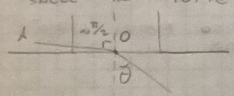
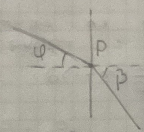
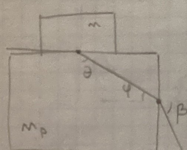
USO SNELL IN TUTTE LE SUP.





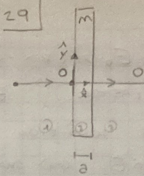
- $n \sin\frac{\pi}{2} = n_p \sin\theta$
- $n_p \sin\varphi = n_{aria} \sin\beta \Rightarrow \varphi = \arcsin\left(\frac{\sin\beta}{n_p}\right) = 31{,}6°$

MA ORA VEDO $\theta = \pi/2 - \varphi$



$$n = n_p \sin\left(\frac{\pi}{2} - \varphi\right) = n_p \cos\varphi = 1{,}38$$

29 S COERENTE, MONOCROMATI



INCIDENZA NORMALE $\Rightarrow \theta = \pi/2$

$$E_i = E_{0i} e^{i(k_i x - \omega_i t)}$$
$$E'_i = E'_{0i} e^{i(k'_i x - \omega'_i t + \varphi)}$$
$$E_r = E_{0r} e^{i(k_r x - \omega_r t + \varphi)}$$

VETRO È UN DIELETTRICO $n \sim \sqrt{\varepsilon_r}$

IMPONGO LE CONDIZIONI DI RACCORDO

$\underbrace{E_{1t} = E_{2t}}_{\text{CON INCIDENZA NORMALE NON HO QUESTA COMPONENTE}} \qquad \underset{\text{ARIA}}{n_A^2} E_{1n} = n^2 E_{2n}$

$n_A^2 E_i = n^2 E_r$ QUESTO ACCADE SE:

$E_{0i} = n^2 E_{0r}$ E FASI UGUALI $\forall t, x$ $(x=0, t) \Rightarrow \varphi = 0$

POI $n^2 E_r = E_{r_2}$ → RIFLESSO FUORI DAL VETRO VERSO $2 \to 3$

$\Rightarrow n^2 E_{0r} = E_{0r_2} \Rightarrow n^2 \frac{E_{0i}}{n^2} = E_{0r_2} = E_{0i}$

$\Rightarrow$ NON È CAMBIATA AMPIEZZA E_0

(VEDI DIETRO)

29 | a) $E(x,t) = E_0 \cos(kx - \omega t)$ INCIDENTE

DOVUTA ALL'ATTRAVERSAMENTO DELLA LASTRA

PERCORSO NEL VUOTO

NELLA LASTRA $E(x,t) = E_{0t} \cos(kna + k(x-a) - \omega t)$

USO LE CONDIZIONI DI RACCORDO

$n_A^2 E_i = n^2 E_t \Leftrightarrow$ AMPIEZZE TC $E_0 = n^2 E_{0t}$

E LE FASI AVRANNO $\Delta\varphi = kna + k(x-a) - \cancel{\omega t} - kx - \cancel{\omega t} = \Delta\varphi$

$\Delta\varphi = kx + k(n-1)a = kx \Rightarrow k(n-1)a = \Delta\varphi$

PASSANDO DA DENTRO LA LASTRA VERSO FUORI AVRÒ GLI STESSI CAMPI E TROVO SEMPRE

$$\Delta\varphi = k(n-1)a = \frac{\omega}{c}(n-1)a$$

b) $E_1 = E_0 \cos(\underbrace{kx - \omega t}_{\xi})$

$E_2 = E_0 \frac{\omega}{c}(n-1)a \cos(kx - \omega t - \pi/2)$

$\quad = E_0 \frac{\omega}{c}(n-1)a \sin(kx - \omega t)$

$E = E_1 + E_2 = E_0\left(1 + \frac{\omega}{c}(n-1)a\right)\left(\cos\xi + \cos(\xi - \pi/2)\right)$

$\quad = E_0\left(1 + \frac{\omega}{c}(n-1)a\right)\left(2\cos\left(\xi - \frac{\pi}{2}\right)\cos\left(\frac{\pi}{4}\right)\right)$

$\quad = 2E_0\left(1 + \frac{\omega}{c}(n-1)a\right)\frac{\sqrt{2}}{2}\sin\xi \sim \sqrt{2}E_0 \sin\xi$

?

CONTINUAZIONE 29

HO $E_1 = E_0 e^{i(k_1 x - \omega_1 t)}$

$$E_2 = \left[E_0 \frac{\omega a}{c}(n-1)\right] e^{i(k_2 x - \omega_2 t + \pi/2)}$$

LA LASTRA DI VETRO NON C'È IN O

$$E_1 + E_2 = E_0 e^{i(kx - \omega t)}\left(\frac{\omega a}{c}(n-1) + e^{i\pi/2}\right)$$

$$= E_1\left(\frac{\omega a}{c}(n-1) + \right.$$

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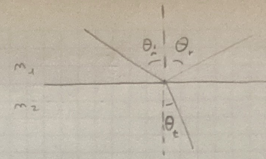
I_i, I_r, I_t



SE IMPONGO LA CONSERVAZIONE DELL'ENERGIA SU UN ELEMENTO dS DI SUPERFICIE

$$I_i dA = I_r dA + I_t dA$$

$$I_i \cos\theta_i A = I_r \cos\theta_i + I_t \cos\theta_t A$$

$$I_i = I_r + \frac{\cos\theta_t}{\cos\theta_i} I_t$$

USO SNELL

$$n_1 \sin\theta_i = n_2 \sin\theta_t \Rightarrow \sin\theta_t = \frac{n_1}{n_2}\sin\theta_i$$

MA

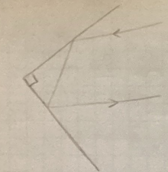
$$\cos\theta_t = \sqrt{1 - \sin^2\theta_t} = \sqrt{1 - \frac{n_1^2}{n_2^2}\sin^2\theta_i}$$

$$= \frac{1}{n_2}\sqrt{n_2^2 - n_1^2 \sin^2\theta_i}$$

QUINDI

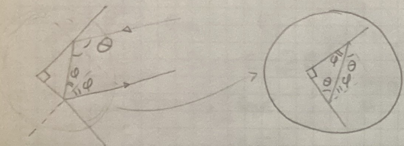
$$I_i = I_r + \frac{\sqrt{n_2^2 - n_1^2 \sin^2\theta_i}}{n_2 \cos\theta_i} I_t$$

31



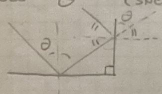
SONO 2 SPECCHI

DIMOSTRA CHE RAGGIO INCIDENTE E USCENTE SONO PARALLELI



$\frac{\pi}{2} = \varphi + \theta \longrightarrow \varphi = \frac{\pi}{2} - \theta$

IL RAGGIO INCIDENTE ARRIVA CON UN ANGOLO θ E VERRÀ RIFLESSO CON UN ANGOLO θ (SNELL)



QUANDO ARRIVA SUL SECONDO SPECCHIO AVRÀ UN ANGOLO RISPETTO LA SUA NORMALE

$\frac{\pi}{2} - \theta$

E DOPO CHE ESCE DAL SECONDO SPECCHIO AVRÀ ANCORA UN ANGOLO DI $\frac{\pi}{2} - \theta = \varphi$

CHE SE GUARDO RISPETTO LA NORMALE DEL PRIMO SPECCIO FORMA UN ANGOLO

$\frac{\pi}{2} - \varphi = \theta$

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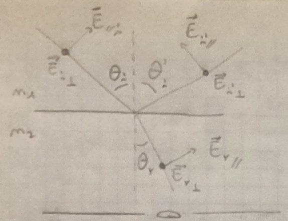


VALGONO LE DEFINIZIONI

$$R = \left(\frac{E'_i}{E_i}\right)^2$$

$$T = \left(\frac{E_r}{E_i}\right)^2$$

APPLICO $E_{1t} = E_{2t}$ $\quad E_i + E'_i = E_r$

so $B = \mu H$, $B = vE = \frac{n}{c} E$

$$\Rightarrow H = \frac{n}{c\mu} E \rightarrow \vec{H} = \frac{n}{c\mu} \hat{k} \times \vec{E} \leftarrow \vec{B} = \frac{n}{c} \hat{k} \times \vec{E}$$

$$\Rightarrow \frac{n_1}{c\mu_1} E_i + \frac{n_1}{c\mu_1} E'_i = \frac{n_2}{c\mu_2} E_r$$

IMPONENDO TUTTE LE EQUAZIONI DI RACCORDO

$\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}$ $\qquad H_{1T} = H_{2T}$

$E_{1T} = E_{2T}$ $\qquad B_{1n} = B_{2n}$

OTTENGO

$$\begin{cases} \left[\varepsilon_1 (\vec{E}_i + \vec{E}'_i) - \varepsilon_2 \vec{E}_r\right] \cdot \hat{n} = 0 \\ \left[\vec{E}_i + \vec{E}'_i - \vec{E}_r\right] \times \hat{n} = 0 \end{cases}$$

$$\begin{cases} \left[n_1 \hat{k}_i \times \vec{E}_i + n_1 \hat{k}'_i \times \vec{E}'_i - n_2 \hat{k}_r \times \vec{E}_r\right] \cdot \hat{n} = 0 \\ \left[\frac{n_1}{\mu_1} \hat{k}_i \times \vec{E}_i + \frac{n_1}{\mu_1} \hat{k}'_i \times \vec{E}'_i - \frac{n_2}{\mu_2} \hat{k}_r \times \vec{E}_r\right] \times \hat{n} = 0 \end{cases}$$

a)

ORA CONSIDERO SOLO LA COMPONENTE $\perp$ (LA OMETTO NELLE FORMULE) RIMANGONO SOLO $(\times)$

$$\perp \begin{cases} E_i + E'_i - E_r = 0 \\ \left[\frac{n_1}{\mu_1}(E_i + E'_i) - \frac{n_2}{\mu_2} E_r\right] \times \hat{n} = 0 \end{cases}$$

$$\frac{n_1}{\mu_1}(E_i - E'_i) \sin\left(\frac{\pi}{2} - \theta_i\right) - \frac{n_2}{\mu_2} E_r \sin\left(\frac{\pi}{2} - \theta_r\right) = 0$$

$$\frac{n_1}{\mu_1}(E_i - E'_i) \cos\theta_i - \frac{n_2}{\mu_2} E_r \cos\theta_r = 0$$

APPROSSIMO IN UN MEZZO TRASPARENTE $\mu \sim 1$

$$\perp \begin{cases} E_i + E'_i - E_r = 0 \quad \Rightarrow E_i = E_r - E'_i \\ n_1 (E_i - E'_i) \cos\theta_i - n_2 E_r \cos\theta_r = 0 \end{cases}$$

$$\Rightarrow n_1 (E_r - 2E'_i) \cos\theta_i - n_2 E_r \cos\theta_r = 0$$

$$E_r (n_1 \cos\theta_i - n_2 \cos\theta_r) = 2 E'_i \cos\theta_i n_1$$

$$E_r = \frac{2 E'_i \cos\theta_i n_1}{n_1 \cos\theta_i - n_2 \cos\theta_r} \quad \circledast$$

$$\Rightarrow E_i = E'_i \frac{2n_1 \cos\theta_i - n_1 \cos\theta_i + n_2 \cos\theta_r}{n_1 \cos\theta_i - n_2 \cos\theta_r}$$

$$\Rightarrow \frac{E'_i}{E_i} = \frac{n_1 \cos\theta_i - n_2 \cos\theta_r}{n_1 \cos\theta_i + n_2 \cos\theta_r}$$

$$\Rightarrow R_\perp = \left(\frac{E'_i}{E_i}\right)^2 \longrightarrow 1 \quad \text{se} \quad \theta_i \rightarrow \pi/2$$

INVECE TROVO ANCHE DA ⊛

$$E_i' = E_r \frac{m_1\cos\theta_i - m_2\cos\theta_r}{2m_1\cos\theta_i} \Rightarrow E_i = E_r \frac{2m_1 c\theta_i - m_1 c\theta_i + m_2 c\theta_r}{2m_1 c\theta_i}$$

$$\Rightarrow \frac{E_r}{E_i} = \frac{2m_1 c\theta_i}{m_1\cos\theta_i + m_2\cos\theta_r}$$

QUINDI

$$T_\perp = \left(\frac{E_r}{E_i'}\right)^2 \longrightarrow 0 \qquad \theta_i \to \pi/2$$

CON STESSI IDENTICI PASSAGGI, MA RIMANENDO CON LA 1° E 3° EQ. DI RACCORDO, SI TROVA PER LA COMPONENTE // : (CON $\mu \sim 1$)

$$// \begin{cases} \dfrac{E_i'}{E_i} = \dfrac{m_2\cos\theta_i - m_1\cos\theta_r}{m_2\cos\theta_i + m_1\cos\theta_r} \\[2ex] \dfrac{E_r}{E_i} = \dfrac{2m_1\cos\theta_i}{m_2\cos\theta_i + m_1\cos\theta_r} \end{cases}$$

QUINDI

$$R_{//} \to 1 \qquad \theta_i \to \pi/2$$

$$T_{//} \to 0 \qquad \theta_i \to \pi/2$$

b) $m_2 < m_1 \qquad \theta_i \to \theta_{LIM}$

$$\Downarrow$$

$$\theta_r > \theta_i \Rightarrow \theta_r = \pi/2$$

VERSO

$$R_{//} = \left(\frac{m_2\cos\theta_i}{m_2\cos\theta_i}\right)^2 \to 1 \quad , \theta_i \to \theta_{LIM}$$

$$T_{//} \to 0 \quad , \theta_i \to \theta_{LIM}$$

IDEM PER $R_\perp$ E $T_{//}$

c) $$R_\perp = \left(\frac{m_1\cos\theta_i - m_2\cos\theta_r}{m_1\cos\theta_i + m_2\cos\theta_r}\right)^2$$

SE $\theta_i \to 0°$ $\quad R_\perp \to \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$

DA SNELL $m_1 \sin\theta_i = m_2 \sin\theta_r \quad , \theta_i \to 0$

$\Rightarrow m_1\theta_i \sim m_2\theta_r \Rightarrow \theta_r = \frac{m_1}{m_2}\theta_i \quad , \theta_i \to 0$

$$R_{//} = \left(\frac{m_2 c\theta_i - m_1 c\theta_r}{m_2 c\theta_i + m_1 c\theta_r}\right)^2$$

SE $\theta_i \to 0$ $\quad R_{//} = \left(\frac{m_2 - m_1}{m_2 + m_1}\right)^2 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 = R_\perp$

$$T_\perp = \left(\frac{2m_1 c\theta_i}{m_1 c\theta_i + m_2 c\theta_r}\right)^2 \to \frac{4m_1^2}{(m_1+m_2)^2} \quad , \theta_i \to 0$$

?

È ANALOGA A $T_{//}$

d) $R = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 = \left(\frac{E_i'}{E_i}\right)^2 \qquad$ SO $m_2 > m_1$

$$E_i'(m_1 + m_2) = E_i(m_1 - m_2)$$

$$m_1(E_i' - E_i) = -m_2(E_i' + E_i)$$

$$\frac{m_1}{m_2} = \frac{E_i + E_i'}{E_i - E_i'} < 1 \Rightarrow E_i + E_i' < E_i - E_i'$$

$$\Rightarrow 2E_i' < 0 \Rightarrow E_i' < 0$$

33

$\alpha = 45°$

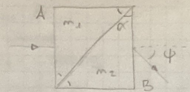
$n_1 = 1,4$

$n_2 = 1,7$

$n = 1$ ARIA

LUCE MONOCROMATICA

$\perp$ AD A

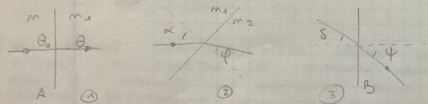


a) so $r = \left(\frac{E'_i}{E_i}\right)^2 = \left(\frac{n - n_1}{n + n_1}\right)^2$ NEL CASO $\perp$

$\Rightarrow \frac{E'_i}{E_i} = \frac{n - n_1}{n + n_1} = -1/6 = -0.166$

$\Rightarrow r = 0,0278$

b) PER TROVARE ψ DEVO APPLICARE SNELL 3 VOLTE

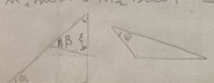


① $n \sin\theta_0 = n_1 \sin\theta \Rightarrow n_1 \sin\theta = 0 \Rightarrow \theta = 0$

② PER EQUIVALENZA DEI TRIANGOLI VEDO CHE IL RAGGIO INCIDENTE ARRIVA CON α

$n_1 \sin\alpha = n_2 \sin\varphi \Rightarrow \varphi = \sin^{-1}\left(\frac{n_1 \sin\alpha}{n_2}\right) = 35,6°$

③



USO $\beta = \pi/2 - \varphi$

POI $\pi = \beta + \alpha + \varepsilon$

$\Rightarrow \varepsilon = \pi - \beta - \alpha = \pi - \pi/2 + \varphi - \frac{\pi}{4} = \varphi + \pi/4$

ALLORA $\delta = \frac{\pi}{2} - \varepsilon = \frac{\pi}{2} - \varphi - \pi/4 = \pi/4 - \varphi = 9,4°$

$n_2 \sin\delta = n \sin\psi \Rightarrow \psi = \sin^{-1}(n_2 \sin\delta) = 16,12°$

c) $n_2 > n_1$ CON n_1 FISSO AFFINCHÉ $\psi = \pi/2$

USO SNELL

$n_2 \sin\delta = \sin\psi$ → NON LO VOGLIO METTO ≥ 1

$n_2 \sin\delta \geq 1 \Rightarrow n_2 \geq \frac{1}{\sin\delta}$

RICOMINCIO DALL'INIZIO DI b

$n_1 \sin\alpha = n_2 \sin\varphi$ ②

$n_2 \sin\delta = 1$ ③

VALE COMUNQUE $\delta = \pi/4 - \varphi$

$n_2 \sin\delta \geq 1 \rightarrow \delta \geq \sin^{-1}\left(\frac{1}{n_2}\right)$

$\frac{\pi}{4} - \varphi \geq \sin^{-1}\frac{1}{n_2}$

$\frac{\pi}{4} - \sin^{-1}\left(\frac{n_1}{n_2}\sin\alpha\right) \geq \sin^{-1}\frac{1}{n_2}$

$\Rightarrow \frac{1}{n_2} \leq \sin\left(\frac{\pi}{4} - \sin^{-1}\left(\frac{n_1}{n_2}\sin\alpha\right)\right)$

$n_2 \geq \frac{1}{\sin\left(\frac{\pi}{4} - \sin^{-1}\left(\frac{n_1}{n_2}\sin\alpha\right)\right)} = 6,1$

$\boxed{34}$ $1 < m < 1{,}00029$ ATMOSFERA (AT)

DALLO SPAZIO $m = 1$ CON θ

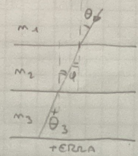
DIVIDIAMO AT IN 3 STRATI

$m_3 > m_2 > m_1$ $\quad m_3 = 1{,}00029$

E TRASCURIAMO LA CURVATURA

— o —

a) APPLICO SNELL



$$m_1 \sin\theta = m_2 \sin\varphi$$

$$m_2 \sin\varphi = m_3 \sin\theta_3$$

$$\Rightarrow \sin\theta_3 = \frac{1}{m_3} m_2 \sin\varphi$$

$$= \frac{1}{m_3} m_1 \sin\theta = \frac{\sin\theta}{m_3}$$

b) SI PERCHÈ COMUNQUE POSSO LEGARE IL TERMINE A SINISTRA (DI INCIDENZA) SEMPRE CON IL TERMINE DI INCIDENZA DELLO STRATO PRECEDENTE. LO FACCIO FINO AD ARRIVARE ALLO STRATO PIÙ ALTO CHE SO $m_1 = 1$ E LO RELAZIONO CON IL TERMINE TRASMESSO (A DESTRA) DELL'ULTIMO STRATO

c) $\sin\theta_3 = \dfrac{\sin\theta}{m_3} \Rightarrow \theta = \sin^{-1}\left(m_3 \sin\theta_3\right)$

CON $\theta_3 = 30°$ $\Rightarrow \theta = 30{,}00959$

$\Rightarrow \Delta\theta = \theta - \theta_3 = (9{,}59 \cdot 10^{-3})°$

$\boxed{35}$ $$n^2 = 1 - \frac{Ne^2}{m_e \varepsilon_0 \omega^2}$$

VALIDA SE $\omega << \omega_0$

—— O ——

$$v_g = \frac{d\omega}{dk}$$

VALE

$$k(\omega) = \frac{\omega}{c} n(\omega)$$

$$\frac{dk}{d\omega} = \frac{n(\omega)}{c} + \frac{\omega}{c}\frac{dn}{d\omega}$$

$$n = \sqrt{1 - \frac{Ne^2}{m_e \varepsilon_0 \omega^2}} \quad , \quad \frac{Ne^2}{m_e \varepsilon_0 \omega^2} \text{ PICCOLO}$$

$$n \simeq 1 - \frac{1}{2}\frac{Ne^2}{m_e \varepsilon_0 \omega^2}$$

$$\frac{dn}{d\omega} = -\frac{1}{2}\frac{-2Ne^2}{m_e \varepsilon_0 \omega^3}$$

$$\frac{dk}{d\omega} = \frac{1}{c}\left(1 - \frac{1}{2}\frac{Ne^2}{m\varepsilon_0\omega^2}\right) + \frac{\omega}{c}\left(+\frac{Ne^2}{m\varepsilon_0\omega^3}\right)$$

$$= \frac{1}{c}\left(1 - \frac{1}{2}\frac{Ne^2}{m\varepsilon_0\omega^2} + \frac{Ne^2}{m\varepsilon_0\omega^2}\right)$$

$$= \frac{1}{c}\left(1 + \frac{1}{2}\frac{Ne^2}{m\varepsilon_0\omega^2}\right)$$

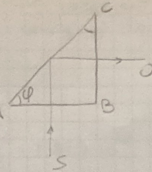
$$v_g = \frac{d\omega}{dk} = \frac{c}{\left(1 + \frac{1}{2}\frac{Ne^2}{m\varepsilon_0\omega^2}\right)} = c\left[1 + \frac{1}{2}\frac{Ne^2}{m\varepsilon_0\omega^2}\right]^{-1}$$

36 LUCE BIANCA

$\lambda_1 < \lambda < \lambda_2$ $\quad \varphi = \pi/4$

$\lambda_1 = 390\ nm$

$\lambda_2 = 780\ nm$



VALE $\quad n(\lambda) = \alpha + \dfrac{\beta}{\lambda^2}$

$\alpha = 1.4 \quad \beta = 4.6 \cdot 10^3\ nm^2$

— o —

b) SULLA FACCIA AC HO RIFLESSIONE TOTALE SE $n(\lambda) > 1 = n_{ARIA}$

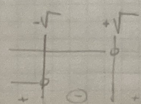
$\sin\theta > \sin\theta_{LIM} = \dfrac{n_{ARIA}}{n(\lambda)}$

$\sin(45) > \dfrac{1}{n(\lambda)} \Rightarrow \alpha + \dfrac{\beta}{\lambda^2} > \dfrac{2}{\sqrt{2}}$

$\sqrt{2}\alpha\lambda^2 + \sqrt{2}\beta > 2\lambda^2 \Rightarrow \lambda^2(\sqrt{2}\alpha - 2) > -\sqrt{2}\beta$

$-\lambda^2(2-\sqrt{2}\alpha) > -\sqrt{2}\beta \Rightarrow \lambda^2 < \dfrac{\sqrt{2}\beta}{2-\sqrt{2}\alpha} = \dfrac{\beta}{\frac{2}{\sqrt{2}} - \alpha}$

$\lambda < \pm\sqrt{\dfrac{\beta}{\sqrt{2}-\alpha}}$
$= \pm 569\ nm$



$\Rightarrow \lambda_1 < \lambda < +\sqrt{\ldots} = 569\ nm$

c) $n(\lambda_1) = 1,4302 \qquad n(\lambda_2) = 1,4075$

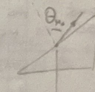
$n(\lambda_2)\sin\varphi = n_{ARIA}\sin\theta$

$\Rightarrow \theta = \sin^{-1}\left(\dfrac{n(\lambda_2)\sin\varphi}{n_{ARIA}}\right) = \theta_{MIN} = 84.4°$

$n(\lambda_1)\sin\varphi = n_{ARIA}\sin\theta_{MAX}$

$\Rightarrow$ SO CHE PER $\lambda_1 < \lambda < \lambda_{MAX} = 569\ nm$ HO RIFLESSIONE TOTALE $\Rightarrow \theta_{MAX} = \pi/2 = 90°$

IL RAGGIO IN USCITA SARÀ DI $\lambda = 780\ nm$ CON $\theta_{MIN} = 84.4°$



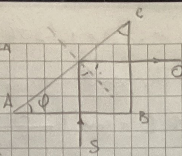
λ_2 = ROSSO

MA ESCONO ANCHE LE $\lambda \geq 569\ nm$ MA CON UN ANGOLO $\theta_{MIN} < \theta < \frac{\pi}{2}$
= GIALLO

d) LE λ USCENTI DA AC SONO $\lambda > 569\ nm \rightarrow$ GIALLO QUINDI LE λ DAL GIALLO AL ROSSO (λ_2) SARANNO PIÙ DEBOLI

$\Rightarrow$ IN O LA LUCE TENDERÀ AL BLU

b) SO CHE CON INCIDENZA NORMALE HO $\theta_r = 0$ $\forall \lambda$ QUINDI TUTTE LE λ DEL FASCIO ENTRANO E LA SEZIONE DEL FASCIO RIMANE UGUALE.



SULLA FACCIA AC CAMBIARE λ MI FA CAMBIARE SOLO θ_r MA LA LEGGE DI SNELL MI DICE $\theta_i = \theta_i'$ QUINDI OGNI λ VERRÀ RIFLESSA IN EGUAL MOTO E TUTTO IL FASCIO MANTERRÀ LA SUA SEZIONE.

DOPODICHÈ HO DI NUOVO $\theta_i = \theta_r = 0$ QUINDI I RAGGI NON VENGONO DEVIATI

IBL spa
Industria Battitura Lamiere

$\boxed{37}$ p_{ATM}, $T = 0°$ MOLECOLE ≡ DIPOLI OSCILLANTI

$1 \cdot e^-$ $K = 3 \cdot 10^2$ Kg/s²

$m_e = 9,11 \cdot 10^{-28}$ g

— o —

a)
$$\omega_0^2 = \frac{K}{m_e} \Rightarrow \omega_0 = \sqrt{\frac{K}{m_e}} = 1,815 \cdot 10^{16} \frac{rad}{s}$$

$$\nu_0 = \frac{\omega_0}{2\pi} = 2,88 \cdot 10^{15} \text{ Hz}$$

HA DETTO OK!

b)
$$n^2 = 1 + \frac{N\alpha}{\varepsilon_0} = 1 + \frac{N}{\varepsilon_0} \frac{e^2}{m(\omega_0^2 - \omega^2)}$$

$$= 1 + \frac{Ne^2}{\varepsilon_0 m_e (\nu_0^2 - \nu^2) 4\pi^2} = 1 + 4,85 \cdot 10^{25} \frac{1}{(\nu_0^2 - \nu^2)}$$

c) PRESO $n^2 = 1 + \frac{1,1 \cdot 10^{27}}{(\nu_0^2 - \nu^2)}$ $\nu_0 = 2,88 \cdot 10^{16}$ Hz

$\nu = 6 \cdot 10^{14}$ Hz $n = 1,0001$

$\nu = 3 \cdot 10^{15}$ Hz $n = 0,9992$

$\boxed{38}$ $n = 1 + 1,4 \cdot 10^{-4}$ PER $\omega_1 = 3,45 \cdot 10^{15}$ rad/s

$n = 1 + 1,547 \cdot 10^{-4}$ PER $\omega_2 = 7,42 \cdot 10^{15}$ rad/s

SUPPONENDO ω_0 UNICA E ATTENUAZIONE TRASCURABILE

— o —

$$\begin{cases} n^2 = 1 + \overbrace{1,4 \cdot 10^{-4}}^{\alpha} = 1 + \dfrac{Ne^2}{m_e\varepsilon_0(\omega_0^2 - \omega_1^2)} \\ n^2 = 1 + \underbrace{1,547 \cdot 10^{-4}}_{\beta} \end{cases}$$

$$\begin{cases} N = \dfrac{\alpha m_e \varepsilon_0 (\omega_0^2 - \omega_1^2)}{e^2} \\ \beta(\omega_0^2 - \omega_2^2) = \dfrac{Ne^2}{m_e\varepsilon_0} \Rightarrow \omega_0 = \left(\dfrac{Ne^2}{m_e\varepsilon_0}\dfrac{1}{\beta} + \omega_2^2\right)^{1/2} \end{cases}$$

$$\Rightarrow \omega_0 = \left(\frac{\alpha \cancel{m_e \varepsilon_0} (\omega_0^2 - \omega_1^2) \cancel{e^2}}{\cancel{e^2} \cancel{m_e \varepsilon_0} \beta} + \omega_2^2\right)^{1/2}$$

$$\Rightarrow \omega_0^2 = \frac{\alpha}{\beta}(\omega_0^2 - \omega_1^2) + \omega_2^2$$

$$\Rightarrow \omega_0^2\left(1 - \frac{\alpha}{\beta}\right) = \omega_2^2 - \frac{\alpha}{\beta}\omega_1^2$$

$$\Rightarrow \omega_0 = \left(\frac{\beta}{\beta - \alpha}\left(\omega_2^2 - \frac{\alpha}{\beta}\omega_1^2\right)\right)^{1/2} \sim 2,158 \cdot 10^{16} \frac{rad}{s}$$

$$\Rightarrow N = \frac{\alpha m_e \varepsilon_0 (\omega_0^2 - \omega_1^2)}{e^2}$$

VA BENE (ERRORI CONTI)

$\boxed{39}$ $\nu = 22\ GHz$

$\nu_0 = \nu$ LUNGO $\hat{x}$

$P = P_0 e^{-bx}$ $\quad b = 0{,}001\ 1/m$

$\varepsilon = \varepsilon_0 + i\tilde{\varepsilon}$

———— o ————

$\nu = 22 \cdot 10^9\ Hz$

VALE $\quad n^2 = 1 + \dfrac{Ne^2}{m_e \varepsilon_0} \dfrac{1}{\omega_0^2 - \omega^2 - i\gamma\omega}$

$n \simeq 1 + \dfrac{1}{2} \dfrac{Ne^2}{m_e\varepsilon_0} \dfrac{1}{\omega_0^2 - \omega^2 - i\gamma\omega}$

$\omega_0 = 2\pi\nu = 1{,}38 \cdot 10^{11}\ \dfrac{rad}{s}$

CON $\omega \sim \omega_0$

$n \simeq 1 + i \dfrac{1}{2} \dfrac{Ne^2}{\gamma\omega_0} = 1 + i \dfrac{A}{2\omega_0}$

CON $A = \dfrac{Ne^2}{\gamma}$

HO PROPAGAZIONE LUNGO x

$E = E_0 e^{i\omega_0\left(\frac{nx}{c} - t\right)} = E_0 e^{i(kx - \omega_0 t)}$

$= E_0 \exp\left\{ i\omega_0 \left(\frac{x}{c}\left(1 + i \frac{1}{2} \frac{Ne^2}{\gamma\omega_0}\right) - t \right)\right\}$

$= E_0 \exp\left\{ i\omega_0 \left(\frac{x}{c} - t\right) + i\omega_0 \left(\frac{x}{c} i \frac{1}{2} \frac{Ne^2}{\gamma\omega_0}\right)\right\}$

$= E_0 \exp\left\{ i\omega_0 \left(\frac{x}{c} - t\right) - \frac{Ax}{2c}\right\}$

QUINDI SI VEDE CHE $E^2 \propto \left(e^{-Ax/2c}\right)^2 = e^{-Ax}$ SI ATTENUA

$\Rightarrow P = P_0 e^{-bx} \Rightarrow A/c = b \Rightarrow A = bc$

QUINDI

$\varepsilon = \varepsilon_0 \varepsilon_r \simeq \varepsilon_0 n^2 \simeq \varepsilon_0 \left(1 + i \dfrac{A}{2\omega_0}\right)^2$

$\sim \varepsilon_0 \left(1 + i\, 2 \dfrac{A}{2\omega_0}\right)$

$= \varepsilon_0 \left(1 + i \dfrac{A}{\omega_0}\right)$

MA ALLORA

$\varepsilon = \varepsilon_0 + i\, \varepsilon_0 \dfrac{A}{\omega_0} = \varepsilon_0 + i\tilde{\varepsilon}$

$\Rightarrow \tilde{\varepsilon} = \varepsilon_0 \dfrac{A}{\omega_0} = \varepsilon_0 \dfrac{cb}{\omega_0} = 1{,}92 \cdot 10^{-17}\ \dfrac{F}{m}$

$\boxed{40}$ $I_i = 10\ W/m^2$ $\qquad \theta_i = 60°$

MATERIALE TRASPARENTE

RAGGIO i POL. LINEARE

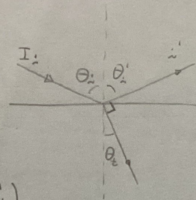
$n_{ARIA} = 1$ $\qquad A = 4\ cm^2 = 4\cdot10^{-4}\ m^2$

— o —

SE HO i' POLARIZZATA HO L'ANGOLO DI BREWSTER

⇓

$\pi/2 = \theta_i' + \theta_t$



USO SNELL TRA i E t

$$n_{ARIA}\sin\theta_i = n\sin\theta_t = n\sin\left(\frac{\pi}{2}-\theta_i'\right)$$

$$\Rightarrow n = \frac{\sin\theta_i}{\sin\left(\frac{\pi}{2}-\theta_i'\right)} = \frac{\sin\theta_i}{\cos\theta_i'} = tg\,\theta_i = 1,73$$

b) $P_i = \int_S I_i\, ds \cos\theta_i = I_i \cos\theta_i A = 2\cdot10^{-3}\ W = 2\ mW$

$$tg(\theta_i+\theta_r) = \infty \Rightarrow r_{//} = \left(\frac{tg(\theta_i-\theta_t)}{tg(\theta_i+\theta_t)}\right)^2 = 0$$

$$\left(\frac{E_i'}{E_i}\right)_\perp = -\frac{\sin(\theta_i-\theta_t)}{\sin(\theta_i+\theta_t)} = -1/2 \Rightarrow r_\perp = 1/4$$

$$t_\perp = \left(\frac{E_t}{E_i}\right)^2_\perp = \left(\frac{2\cos\theta_i\sin\theta_t}{\sin(\theta_i+\theta_t)}\right)^2 = 1/4$$

$$t_{//} = \left(\frac{E_t}{E_i}\right)^2_{//} = \left(\frac{2\cos\theta_i\sin\theta_t}{\sin(\theta_1+\theta_2)\cos(\theta_1-\theta_2)}\right)^2 = 1/3$$

PER UN ONDA NON POLARIZZATA VALE

$$r = \frac{1}{2}(r_\perp + r_{//}) = \frac{1}{2}\left(\frac{1}{4}+0\right) = \frac{1}{8}$$

$$t_\perp = 0,4325 = \frac{n_2}{n_1}\frac{4\sin^2\theta_t\cos^2\theta_i}{\sin^2(\theta_i+\theta_t)}$$

$$t_{//} = 0,5766 = \frac{n_2}{n_1}\frac{4\sin^2\theta_t\cos^2\theta_i}{\sin^2(\theta_i+\theta_t)\cos^2(\theta_i-\theta_t)}$$

$$t = \frac{1}{2}(t_\perp + t_{//}) = 0,50455$$

$$t = \frac{I_t}{I_i} \Rightarrow I_t = 0,50455\, I_i = 5,045\ W/m^2$$

$$P_t = I_t A\cos\theta_t = 1,75\cdot10^{-3}\ W$$

$$r_\perp = \frac{1}{4} = R_\perp = R$$

$$R = \frac{I_i' A\cos\theta_i}{I_i A\cos\theta_i} \Rightarrow I_i' = R I_i = 1,25\ W/m^2$$

$$P_i' = A\cos\theta_i I_i' = 2,5\cdot10^{-4}\ W$$

NOTA $\quad T = t\frac{\cos\theta_t}{\cos\theta_i} \Rightarrow P_t = P_i\, t\frac{\cos\theta_t}{\cos\theta_i} = 1,75\cdot10^{-3}$

$\boxed{n = 1,7}$ $\boxed{P_i = I_i\, A \cos\theta_i = 2 \cdot 10^{-3}\ W = 2\ mW}$

$$r_\perp = \left(\frac{E_i'}{E_i}\right)^2_\perp = \left(\frac{n_A \cos\theta_i - n\cos\theta_t}{n_A\cos\theta_i + n\cos\theta_t}\right)^2_\perp = 0,24$$

$r_{//} = 0$

$$t_\perp = \left(\frac{E_t}{E_i}\right)^2_\perp = \left(\frac{2\, n_A \cos\theta_i}{n_A\cos\theta_i + n\cos\theta_t}\right)^2 \frac{n_2}{n_1} = 0,442$$

$$t_{//} = \left(\frac{2 n_A \cos\theta_i}{n\cos\theta_i + n_A\cos\theta_t}\right)^2 \frac{n_2}{n_1} = 0,578$$

i e t NO POL $\quad t = \frac{1}{2}(t_{//} + t_\perp) = 0,51 \quad r = \frac{1}{2}(r_{//} + r_\perp) = 0,12$

$$r = \frac{I_i'}{I_i} \longrightarrow \boxed{I_i' = r I_i = 1,2\ W/m^2}$$

$$t = \frac{I_t}{I_i} \longrightarrow \boxed{I_t = t I_i = 5,1\ W/m^2}$$

$$R = r = \frac{P_i'}{P_i} \longrightarrow \boxed{P_i' = r P_i = 0,24\ mW}$$

$$T = t\frac{\cos\theta_t}{\cos\theta_i} = \frac{P_t}{P_i} \longrightarrow \boxed{P_t = P_i\, t\, \frac{\cos\theta_t}{\cos\theta_i} = 1,76\ mW}$$