



Esercizi

MECCANICA

PROVE D'ESAME

VENGONO RACCOLTI GLI ESERCIZI DI PROVE D'ESAME SPARSE. SE NON
SPECIFICATO SONO PRESI DAL FILE "exam_mech".

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exe 2 : 1 30/8/2017
ES 1.7 (ARTIGLIERE) LIBRO
1 7/4/2017
exe 3 : 1 15/9/2017
exe 5 : 2 2/7/2018
1 23/6/2022
exe 6 : 2 18/9/2018
exe 7 : 2 22/06/2020
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exe 13 : 1 16/7/2020

ANNOTAZIONI

- 1, 18/07/2022 - RIVEDERE PUNTO C
- 2, 13/06/2023 - RIVEDERE PUNTO b, c
- 2, 14/09/2022 - RIVEDERE
- 1, 15/02/2021 - PUNTO v_{max}

21/03/2017

2



m, v_0

$$F_{\text{att}} = kv$$

SCRIVO L'EQUAZIONE DEL MOTO IN UN SR SOLIDALE CON LA DISCESA

$$\begin{cases} \hat{x} & m a_x = mg \sin \theta - kv \\ \hat{y} & m a_y = 0 = N - mg \cos \theta \end{cases}$$

$$\Rightarrow \begin{cases} \frac{dv}{dt} = g \sin \theta - \frac{k}{m} v \\ N = mg \cos \theta \end{cases}$$

$$1^{\circ} \text{ RGA} \Rightarrow \int_0^v \frac{dv}{g \sin \theta - \frac{k}{m} v} = \int_0^t dt$$

$$\Rightarrow -\frac{m}{k} \ln \left(\frac{g \sin \theta - \frac{k}{m} v}{g \sin \theta - \frac{k}{m} v_0} \right) = t$$

$$\frac{g \sin \theta - \frac{k}{m} v}{g \sin \theta - \frac{k}{m} v_0} = e^{-\frac{k}{m} t}$$

$$\Rightarrow g \sin \theta - \frac{k}{m} v = (g \sin \theta - \frac{k}{m} v_0) e^{-\frac{k}{m} t}$$

$$-\frac{k}{m} v = -\frac{k}{m} v_0 e^{-\frac{k}{m} t} - g \sin \theta (1 - e^{-\frac{k}{m} t})$$

$$\Rightarrow v(t) = v_0 e^{-\frac{k}{m} t} + \frac{mg}{k} \sin \theta (1 - e^{-\frac{k}{m} t})$$

LA v_{lim} È $v(t \rightarrow \infty)$: $v_{\text{lim}} = \frac{mg}{k} \sin \theta$

E POSSO DESCRIVERE

$$v(t) = v_0 e^{-\frac{k}{m}t} + v_{\text{lim}} (1 - e^{-\frac{k}{m}t})$$

USO IL TEOREMA DELLE FORZE VIVE

$$W = W_c + W_{\text{ATT}} = \Delta K, \quad W_c = \Delta U = mgl \sin \theta$$

$$\Rightarrow W_{\text{ATT}} = \frac{m}{2} (v_{\text{lim}}^2 - v_0^2) - mgl \sin \theta$$

SO CHE VALE $P = \vec{F} \cdot \vec{v}$

$$\Rightarrow P_{\text{ATT}} = -F_{\text{ATT}} v_{\text{lim}} \Rightarrow P_{\text{cic}} = k v_{\text{lim}} \cdot v_{\text{lim}} = k v_{\text{lim}}^2 = \frac{m^2 g^2}{k} \sin^2 \theta$$

07/04/2017

1

$$x(0) = x_0$$

$$v(x) = kx^2$$

$$HO \quad v = \frac{dx}{dt}$$

$$\frac{dx}{dt} = kx^2 \Rightarrow \frac{dx}{x^2} = k dt \Rightarrow -\frac{1}{x} \Big|_{x_0}^x = kt$$

$$\Rightarrow \frac{1}{x_0} - \frac{1}{x} = kt \Rightarrow \frac{1}{x} = \frac{1}{x_0} - kt \Rightarrow \frac{1}{x} = \frac{1 - x_0 kt}{x_0}$$

$$\Rightarrow x(t) = \frac{x_0}{1 - kx_0 t}$$

DERIVO DUE VOLTE $x(t)$ PER TROVARE $a(t)$:

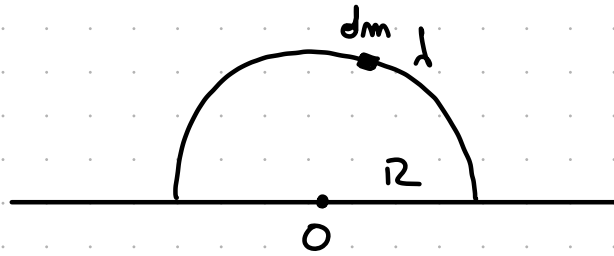
$$v(t) = \frac{-kx_0^2}{(1 - kx_0 t)^2}$$

$$a(t) = \frac{2k^2 x_0^3}{(1 - kx_0 t)^3}$$

→ NELLE SOLUZIONI MANCA UN k E PENSO
SIA SBAGLIATO.
DIMENSIONALMENTE $[k] = m^{-1} s^{-1}$ E LA MIA $a(t)$:
$$[a] = \frac{m^{-2} s^{-2} m^3}{m^{-1} s^{-1} m s} = m s^{-2}$$

06/07/2017

3 m, R, λ



$$m = \int dm = \int \lambda dl = \int \lambda R d\theta = \lambda R \pi$$

$$x_{cm} = \frac{\int x dm}{\lambda R \pi} = \frac{\lambda \int (R \cos \theta) R d\theta}{\lambda R \pi} = \frac{R}{\pi} \int_0^\pi \cos \theta d\theta = \frac{R}{\pi} \sin \theta \Big|_0^\pi = 0$$

$x = R \cos \theta$

$$y_{cm} = \frac{\int y dm}{\lambda R \pi} = \frac{\lambda \int R^2 \sin \theta d\theta}{\lambda R \pi} = \frac{R}{\pi} \int_0^\pi \sin \theta d\theta = \frac{R}{\pi} \cos \theta \Big|_\pi^0 = \frac{2R}{\pi}$$

$y = R \sin \theta$

$$I_0 = \int R^2 dm = R^2 \int_0^\pi \lambda R d\theta = \lambda R^3 \pi = m R^2$$

$$I_0 = I_{cm} + m d^2 = I_{cm} + m (x_{cm}^2 + y_{cm}^2) = I_{cm} + m \frac{4R^2}{\pi^2}$$

$$\Rightarrow I_{cm} = I_0 - m \frac{4R^2}{\pi^2} = m R^2 - \frac{4m R^2}{\pi^2} = m R^2 \left(1 - \frac{4}{\pi^2} \right)$$

30/08/2017

1

$$a = -kv^2$$

$$v_0, t_0 = 0$$

$$a = \frac{dv}{dt}$$

$$\Rightarrow dv = -kv^2 dt$$

$$\Rightarrow \frac{dv}{v^2} = -k dt$$

$$\Rightarrow -\frac{1}{v} \Big|_{v_0}^v = -kt$$

$$\Rightarrow \frac{1}{v_0} - \frac{1}{v} = -kt$$

$$\Rightarrow \frac{1}{v} = \frac{1}{v_0} + kt$$

$$\Rightarrow v(t) = \frac{v_0}{1 + v_0 kt}$$

$$\omega \quad [a] = [kv^2] \Rightarrow \frac{m}{s^2} = [k] \frac{m^2}{s^2} \Rightarrow [k] = \frac{1}{m}$$

$$v_0 = 10 \quad \tau \quad t_c \quad v(\tau) = \frac{v_0}{10} = \frac{v_0}{1 + kv_0 \tau} \Rightarrow 1 + kv_0 \tau = 10$$

$$\Rightarrow \tau = \frac{9}{kv_0}$$

$$x_0 \quad v(t) = \frac{dx}{dt} \Rightarrow x(t) = x_0 + \int_0^t \frac{v_0}{1 + kv_0 t} dt$$

$$\Rightarrow x(t) = x_0 + \frac{1}{k} \int_0^t \frac{kv_0}{1 + kv_0 t} dt \Rightarrow x(t) = x_0 + \frac{1}{k} \ln(1 + kv_0 t)$$

2

$$\frac{dm}{dt} = \lambda$$

HO $\frac{dp}{dt} = \frac{dm}{dt} v + m \frac{dv}{dt}$, MA VOGLIO $v = \text{cost} = v_0$

$$\Rightarrow \frac{dp}{dt} = \lambda v_0$$

$$\Rightarrow F = \frac{dp}{dt} = \lambda v_0$$

IL LAVORO SARÀ

$$dW = \vec{F} \cdot d\vec{r} = \lambda v_0 dr = \lambda v_0^2 dt$$

$$\Rightarrow W_{0,\tau} = \lambda v_0^2 \tau$$

SO CHE $\frac{dm}{dt} = +\lambda \Rightarrow m(t) = m_0 + \lambda t$

IN QUEL CASO AVREI $\frac{dp}{dt} = 0 = \lambda v + m(t) \frac{dv}{dt}$

$$\Rightarrow m(t) \frac{dv}{dt} = -\lambda v \Rightarrow \frac{dv}{v} = -\frac{\lambda}{m} \Rightarrow \ln\left(\frac{v}{v_0}\right) = -\ln\left(\frac{m_0 + \lambda t}{m_0}\right)$$

$$\Rightarrow \boxed{v(t) = \frac{m_0}{m_0 + \lambda t} v_0}$$

10 VOGLIO t^* TC $m(t^*) = 0,1 \cdot m_0 \Rightarrow \frac{11}{10} m_0 = m_0 + \lambda t^*$

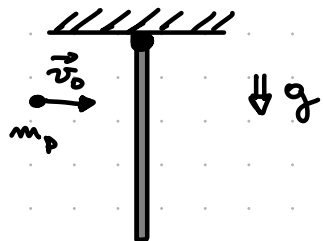
$$\Rightarrow t^* = \frac{m_0}{\lambda} \left(\frac{11}{10} - 1 \right) = \frac{1}{10} \frac{m_0}{\lambda}$$

COSI' TROVO

$$v(m) = \frac{m_0}{m_0 + \frac{m_0}{10}} v_0 = \boxed{\frac{10}{11} v_0}$$

30/08/2017

113



URTO SOLO LA QUANTITÀ ANGOLARE
COMPLETAMENTE ANELASTICO
POSSO CONSERVARE IL MOMENTO

$$m_p d v_0 = I \omega \quad , \quad I = I_{ASTA} + m_p d^2$$

$$\Rightarrow m_p d v_0 = \left(\frac{1}{3} m l^2 + m_p d^2 \right) \omega_0$$

$$\Rightarrow \omega_0 = \frac{3 m_p d v_0}{m l^2 + 3 m_p d^2}$$

$$U_{EXT} = -I \ddot{\theta}$$

$$U_{EXT} = m g \frac{l}{2} \sin \theta + m_p g d \sin \theta$$

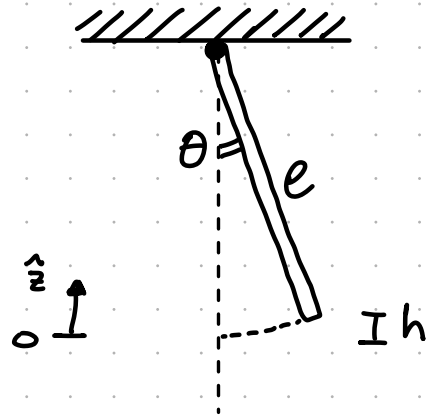
$$\Rightarrow m g \frac{l}{2} \sin \theta + m_p g d \sin \theta = -I \ddot{\theta}$$

$$\sin \theta \sim \theta \Rightarrow \ddot{\theta} = - \frac{m g \frac{l}{2} + m_p g d}{I} \theta \Rightarrow \kappa = \sqrt{\frac{m g l + 2 m_p g d}{2 I}}$$

$$T = \frac{2\pi}{\kappa} = 2\pi \sqrt{\frac{2 m l^2 + 6 m_p d^2}{(3 m l + 6 m_p d) g}}$$

HO K E TRAVO

PER TROVARE $\Theta_{\max}$ CONSERVO L'ENERGIA MECCANICA



$$\frac{1}{2} I \omega_0^2 + m_p g (\ell - d) = \frac{1}{2} I \omega^2(\Theta_{\max}) + m_p g (\ell - d - \ell \cos \Theta) + m g (\ell - \ell \cos \Theta)$$

MA $\omega(\Theta_{\max}) = 0$

$$\Rightarrow \frac{1}{2} I \omega_0^2 + m_p g (\ell - d) = m_p g (\ell - d \cos \Theta) + m g \frac{\ell}{2} (1 - \cos \Theta)$$

$$\Rightarrow I \omega_0^2 - 2 m_p g d = - 2 m_p d \cos \Theta + m g \ell (1 - \cos \Theta)$$

$$\Rightarrow I \omega_0^2 = 2 m_p g d (1 - \cos \Theta) + m g \ell (1 - \cos \Theta)$$

$$\Rightarrow (1 - \cos \Theta) = \frac{I \omega_0^2}{2 m_p g d + m g \ell}$$

RICORDO $\omega_0 = \frac{m_p d v_0}{I}$

$$\Rightarrow \Theta = \arccos \left\{ 1 - \frac{\frac{m_p^2 d^2 v_0^2}{I}}{(2 m_p d + m \ell) g} \right\}$$

$$\Rightarrow \Theta = \arccos \left\{ 1 - \frac{3m_p^2 d^2 v_0^2}{(m\ell^2 + 3m_p d^2)(2m_p d + m\ell)g} \right\}$$

15/09/2017

1

$$\omega_1 = \omega_0, \quad \omega_2(t) = \alpha t$$

DALLE $\omega(t)$ HO

$$\begin{cases} \theta_1(t) = \omega_0 t \\ \theta_2(t) = 2\pi - \frac{1}{2} \alpha t^2 \end{cases}$$

METTENDO $\theta_{01} = 0, \quad \theta_{02} = 2\pi$

SI INCONTRANO QUANDO $\theta_1(\tau) = \theta_2(\tau)$

$$\Rightarrow \omega_0 \tau = 2\pi - \frac{1}{2} \alpha \tau^2$$

$$\Rightarrow -\frac{1}{2} \alpha \tau^2 - \omega_0 \tau + 2\pi = 0$$

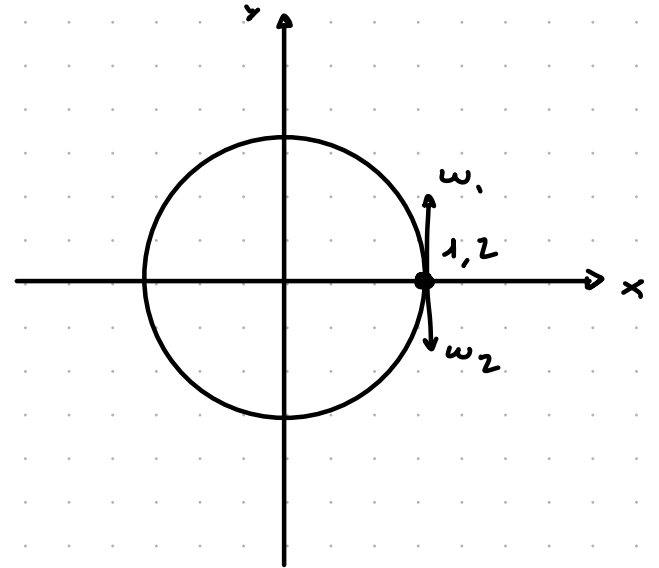
$$\tau_{1,2} = \frac{\omega_0 \pm \sqrt{\omega_0^2 + 4\pi\alpha}}{-\alpha} = -\frac{\omega_0}{\alpha} \pm \frac{\sqrt{\omega_0^2 + 4\pi\alpha}}{\alpha}$$

SCELGO IL +

HO $\tau = 0,67 \text{ s}$

$$\begin{cases} \theta_1(\tau) = 5,36 \text{ rad} \\ \theta_2(\tau) = 5,38 \text{ rad} \end{cases}$$

I CICLISTI SI TROVANO OUNQUE A $\theta \sim 307,1^\circ$



DUNQUE 1 HA PERCORSO $\Theta_1 = 307^\circ$ E 2 $\Theta_2 = 360 - 307 \sim 53^\circ$

HO LE ESPRESSIONI (MODULI)

$$\begin{aligned} a_t &= \dot{\omega} R \\ a_c &= \omega^2 R \end{aligned} \Rightarrow \begin{cases} a_{t1} = 0 \\ a_{c1} = \omega_0^2 R \end{cases} ; \begin{cases} a_{t2} = \alpha R \\ a_{c2} = \alpha^2 R t^2 \end{cases}$$

2 m, r_0

CONSERVO L'ENERGIA

$$-G \frac{Mm}{r_0} = -G \frac{Mm}{r_T} + \frac{m}{2} v_T^2$$

$$\Rightarrow v_T^2 = 2GM \left(\frac{1}{r_T} - \frac{1}{r_0} \right) \quad \Rightarrow \quad v_T = \sqrt{2GM \left(\frac{1}{r_T} - \frac{1}{r_0} \right)}$$

LA VELOCITÀ ORBITALE LA TROVO CON

FORZA CENTRIFUGA = ATTRAZIONE GRAVITAZIONALE

$$m r_0 \frac{v_0^2}{r_0^2} = G \frac{Mm}{r_0^2} \quad \Rightarrow \quad v_0 = \sqrt{\frac{GM}{r_0}}$$

PRENDO ANGOLARE $v_1 = v_0/2$ E CONSERVO ENERGIA E MOMENTO

$$\begin{cases} -G \frac{Mm}{r_0} + \frac{m}{2} v_1^2 = -G \frac{Mm}{r_1} + \frac{m}{2} v_2^2 \\ m v_1 r_0 = m v_2 r_1 \end{cases}$$

$$\begin{cases} GM\left(\frac{1}{r_1} - \frac{1}{r_0}\right) + \frac{1}{2}v_1^2 = \frac{1}{2}\left(\frac{r_0}{r_1}v_1\right)^2 \\ v_2 = \frac{r_0}{r_1}v_0 \end{cases}$$

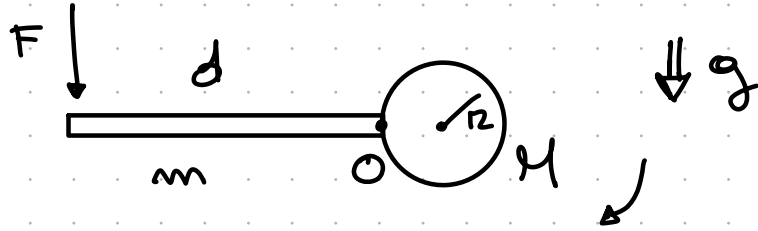
$$\begin{cases} GM\left(\frac{1}{r_1} - \frac{1}{r_0}\right) = \frac{1}{2}v_1^2\left(\frac{r_0^2}{r_1^2} - 1\right) \\ v_2 = \frac{r_0}{r_1}v_0 \end{cases}$$

$$\begin{aligned} \stackrel{1^o \text{ EGA}}{\Rightarrow} GM\left(\frac{1}{r_1} - \frac{1}{r_0}\right) &= \frac{v_1^2}{2} r_0^2 \underbrace{\left(\frac{1}{r_1^2} - \frac{1}{r_0^2}\right)}_{\left(\frac{1}{r_1} - \frac{1}{r_0}\right)\left(\frac{1}{r_1} + \frac{1}{r_0}\right)} \\ & \Rightarrow \frac{2GM}{v_1^2 r_0^2} = \frac{1}{r_1} + \frac{1}{r_0} \end{aligned}$$

$$\Rightarrow \frac{1}{r_1} = \frac{2GM}{v_1^2 r_0^2} - \frac{1}{r_0}$$

$$\Rightarrow \frac{1}{r_1} = \frac{8GM}{v_0^2 r_0^2} - \frac{1}{r_0}$$

3

SBARRA m, d + DISCO M, R 

HO EQUILIBRIO, UTILIZZO LE EQUAZIONI CARDINALI

$$\text{POLO } O \left\{ \begin{array}{l} 0 = -F - mg - Mg + N \\ 0 = +Fd + mg \frac{d}{2} - MgR \end{array} \right.$$

2° RIGA
 $\Rightarrow$

$$Fd = MgR - mg \frac{d}{2} \Rightarrow F = Mg \frac{R}{d} - \frac{mg}{2}$$

$$\text{SAPEVDO } M = 3m \quad R = \frac{d}{4}$$

$$\Rightarrow F = \frac{3}{4}mg - \frac{mg}{2} = \frac{mg}{4}$$

1° RIGA
 $\Rightarrow$

$$N = F + mg + Mg = \frac{mg}{4} + mg + 3mg = \frac{17}{4}mg$$

PER

IL

SISTEMA

HO

$$I = I_{\text{DISCO}} + I_{\text{ASTA}} \quad \text{PER } O$$

$$I_{ASTA} = \frac{1}{3} m d^2$$

PER IL DISCO OMogeneo DEVO USARE E' HUYGENS-STEINER AL CENTRO DEL DISCO SAPENDO CHE ESSENDO

$$I_{DISCO}^0 = I_{cm} + M R^2$$

$$= \frac{1}{2} M R^2 + M R^2 = \frac{3}{2} M R^2 = \frac{9}{2} m \frac{d^2}{16} = \frac{9}{32} m d^2$$

$$\Rightarrow I = \frac{1}{3} m d^2 + \frac{9}{32} m d^2 = \frac{59}{96} m d^2$$

CON LA 2° EQ. CARDINALE HO

$$M_e = I \alpha \Rightarrow \alpha = \frac{M_e}{I} = \frac{m g \frac{d}{2} - M g R}{I} = \frac{\frac{m g d}{2} - \frac{3 m g d}{4}}{\frac{59}{96} m d^2}$$

$$\Rightarrow \alpha = \frac{96}{59} \frac{g}{d} \left(\frac{1}{2} - \frac{3}{4} \right) = \frac{96}{59} \left(-\frac{1}{4} \right) \frac{g}{d} = -\frac{24}{59} \frac{g}{d}$$

DATO α TROVO $\omega(t) = -\frac{24}{59} \frac{g}{d} t \Rightarrow \theta(t) = -\frac{1}{2} \frac{24}{59} \frac{g}{d} t^2$

volgio $\theta(\tau) = -\frac{\pi}{2} \Rightarrow \tau = \sqrt{\frac{59\pi}{24} \frac{d}{g}}$

$\Rightarrow \omega(\tau) = -\frac{24}{59} \frac{g}{d} \tau = -\sqrt{\frac{24\pi}{59} \frac{g}{d}}$

NON HO UN FATTORE $\sqrt{2}$

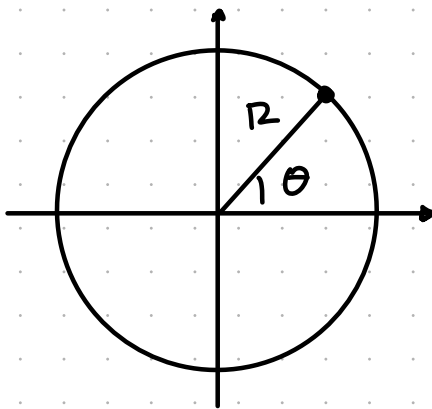
21/12/2017

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$$v(t) = v_0 e^{-t/\tau}$$

$$g(t) \equiv R \quad \forall t$$

$$\theta(t), \quad \theta(0) = 0$$



SO IN GENERALE IN COORDINATE POLARI

$$v(t) = \frac{dr}{dt} \hat{r} + r \frac{d\theta}{dt} \hat{\theta} = R \frac{d\theta}{dt} \hat{\theta}$$

QUINDI $v_r(t) = 0$

$$v_\theta(t) = v(t) = R \frac{d\theta}{dt} \hat{\theta} = v_0 e^{-t/\tau}$$

POSSO TROVARE $\theta(t)$:

$$\theta(t) = \int_0^t \frac{v(t)}{R} dt = \frac{v_0}{R} \int_0^t e^{-t/\tau} dt = -\frac{v_0 \tau}{R} e^{-t/\tau} \Big|_0^t = \frac{v_0 \tau}{R} (1 - e^{-t/\tau})$$

CHE SE $t \rightarrow \infty$ $\theta(t) \rightarrow \frac{v_0 \tau}{R}$

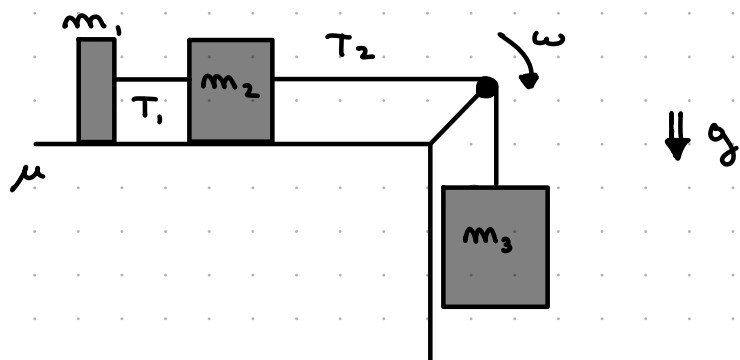
PER $\vec{a}$ SO:

$$a_\theta = R \ddot{\theta} = v_0 \tau \frac{d}{dt} \left(\frac{1}{\tau} e^{-t/\tau} \right) = -\frac{v_0}{\tau} e^{-t/\tau}$$

$$a_r = -R \dot{\theta}^2 = -R \frac{v_0^2 \tau^2}{R^2} \frac{1}{\tau^2} e^{-2t/\tau} = -\frac{v_0^2}{R} e^{-2t/\tau}$$

??

2



DETERMINA :

- a , T_1
- μ T.C. m_3 con KRU
- $m_1, m_2 \rightarrow m_1 + m_2$

SCRIVO LE EQUAZIONI DEL MOTO DEI 3 CORPI

$$\begin{cases} m_1 a = T_1 - m_1 g \mu \\ m_2 a = T_2 - m_2 g \mu - T_1 \\ m_3 a = m_3 g - T_2 \end{cases}$$

$$\Rightarrow \begin{cases} T_1 = m_1 a + m_1 g \mu \\ m_2 a = m_3 g - m_3 a - m_2 g \mu - m_1 a - m_1 g \mu \\ T_2 = m_3 g - m_3 a \end{cases}$$

2° RIGA

$$\Rightarrow (m_1 + m_2 + m_3) a = (m_3 - \mu(m_1 + m_2)) g$$

$$\Rightarrow a = \frac{m_3 - \mu(m_1 + m_2)}{m_1 + m_2 + m_3} g$$

TROVO LA T_1 :

$$T_1 = \frac{m_3 - \mu m_1 - \mu m_2}{m_1 + m_2 + m_3} m_1 g + m_1 g \mu$$

$$= \frac{m_1 m_3 - \mu m_1^2 - \mu m_1 m_2}{m_1 + m_2 + m_3} g + \frac{\mu m_1^2 + m_1 m_2 \mu + m_1 m_3 \mu}{m_1 + m_2 + m_3} g$$

$$= \frac{m_1 m_3 (1 + \mu)}{m_1 + m_2 + m_3} g$$

TROVO T_2 :

$$T_2 = m_3 (g - a) = \frac{m_3}{m_1 + m_2 + m_3} (m_1 + m_2 + m_3 - m_3 - \mu m_1 - \mu m_2) g$$

$$= \frac{m_3 g}{m_1 + m_2 + m_3} (m_1 + m_2)(1 - \mu)$$

CERCO μ T.C. m_3 KZU, OSSIA, T.C. $a = 0$

$$a = 0 \Rightarrow m_3 - \mu(m_1 + m_2) = 0 \Rightarrow \mu = \frac{m_3}{m_1 + m_2}$$

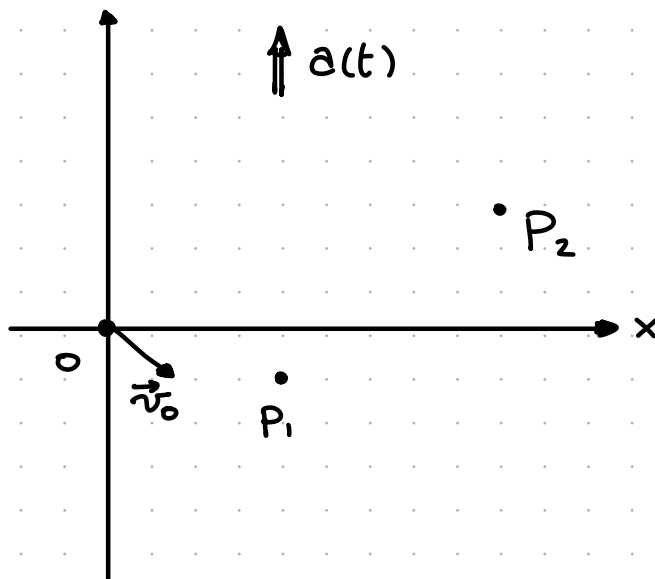
SE $m_1, m_2 \rightarrow m_1 + m_2$ $\vec{a}$ NON CAMBIA, $\vec{T}_2$ NESSUNO

20/03/2018

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$$\vec{v}_0 = (v_{0x}, v_{0y})$$

$$\vec{a}(t) = \gamma t \hat{y}$$



HO MOTO ACCELERATO LUNGO y E UNIFORME SU $\hat{x}$:

$$\begin{cases} a_x = 0 \\ a_y = \gamma t \end{cases} \quad \begin{cases} v_x = v_{0x} \\ v_y = v_{0y} + \frac{1}{2} \gamma t^2 \end{cases} \Rightarrow \begin{cases} x(t) = v_{0x} t \\ y(t) = v_{0y} t + \frac{1}{6} \gamma t^3 \end{cases}$$

HO DUNQUE $t = \frac{x}{v_{0x}}$

$$\Rightarrow y(x) = \frac{v_{0y}}{v_{0x}} x + \frac{\gamma}{6 v_{0x}^3} x^3$$

$$v_0 = (3, -2) \frac{\text{km}}{\text{h}}$$

$$\Rightarrow \gamma = 2$$

$$y(x) = -\frac{2}{3} x + \frac{x^3}{81}$$

INSIEME

A

$$\vec{v}(t) = (v_{0x}, v_{0y} + \frac{1}{2} \gamma t^2)$$

→ QUELLA NELLE
SOLUZIONI KI
SEMBRA SIA
(x, y)

VEDO P_1 : (uso $\gamma(x)$)

$$-1 = -\frac{2}{3} \cdot 4 + \frac{64}{81} \Rightarrow -1 = -\frac{8}{3} + \frac{64}{81} \Rightarrow -1 = \frac{-8 \cdot 27 + 64}{81}$$

$\Rightarrow$ NON URTA P_1

PER P_2 :

$$3 = -\frac{2}{3} \cdot 9 + \frac{9^3}{81} \Rightarrow 3 = -6 + 9 \Rightarrow \boxed{\text{URTA } P_2}$$

VEDO

$$x(\tau) = 9 = v_{0x} \tau \Rightarrow \boxed{\tau = \frac{9}{v_{0x}} = 3 \text{ s}}$$

2

$$m, v_0 \quad v(t) = \frac{v_0}{1 + \frac{t}{\tau}}$$

POSSO TROVARE $a(t) = \frac{dv}{dt} = \frac{d}{dt} \left(\frac{\tau v_0}{\tau + t} \right) = \frac{\tau v_0}{(\tau + t)^2} = \frac{v(t)}{\tau + t} = \frac{v^2}{\tau v_0}$

$$F = -m a(t) \Rightarrow F(v) = - \frac{m v(t)^2}{v_0 \tau}$$

NELLE SOLUZIONI MANCA IL QUADRATO (DIMENSIONALMENTE NON TORNA SENZA)

USO LA DEFINIZIONE DI POTENZA

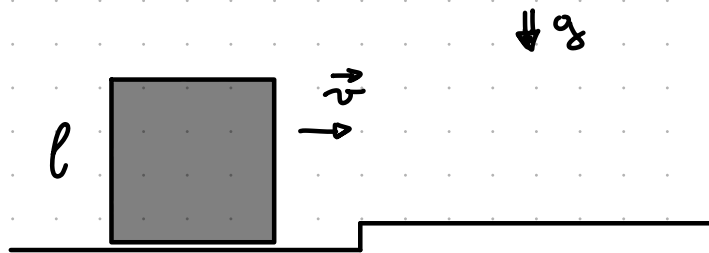
$$P = \vec{F} \cdot \vec{v} = F(v) \cdot \frac{v_0}{2} = - \frac{m \left(\frac{v_0}{2} \right)^3}{v_0 \tau} = - \frac{m v_0^2}{8 \tau} \quad \rightarrow \text{È QUELLA RESISTENTE}$$

$$\Rightarrow P_{\text{TOT}} = \frac{m v_0^2}{8 \tau}$$

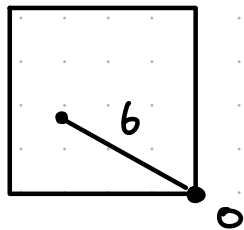
SO

$$dW = P_{\text{TOT}} dt \Rightarrow W_{0,4\tau} = P_{\text{TOT}} \int_0^{4\tau} dt = \frac{m v_0^2}{2}$$

3



HO



$$b^2 = x^2 + y^2 + z^2$$

$$\begin{aligned} I &= \int b^2 dm = \int (x^2 + y^2) \rho dx dy dz \\ &= \rho \left\{ \int_0^l x^2 dx \int_0^l dy \int_0^l dz + \int_0^l dx \int_0^l y^2 dy \int_0^l dz \right\} \\ &= \rho \left\{ \frac{1}{3} l^3 + \frac{1}{3} l^3 \right\} = \frac{2}{3} \rho l^3 \end{aligned}$$

HA

$$m = \rho V = \rho l^3$$

$$\Rightarrow I = \frac{2}{3} m l^2$$

PER LA
È SULLA
APPLICANDO

CONDIZIONE
VERTICALE
SUBITO LA

SCRIVETE
IL CUBO NON
CONSERVAZIONE

CHIEDO
CHE QUANDO
RUOTA.
DELL'ENERGIA

IL CU
SI TROVA

$$\frac{m}{2} v^2 + mg \frac{l}{2} = mg \sqrt{2} \frac{l}{2} \Rightarrow v = \sqrt{(\sqrt{2} - 1) gl}$$

CHE È SBAGLIATO PERCHÉ NON SI STA CONSIDERANDO IL FATTO CHE QUANDO URTA CON LO SCALINO SI HA URTO COMPLETAMENTE ANELASTICO E QUINDI DISSIPAZIONE DI ENERGIA. CONSERVO L PRIMA E DOPO L'URTO PER TROVARE ω IMMEDIATAMENTE DOPO L'URTO

$$m v_0 \frac{l}{2} = I \omega_0 \Rightarrow \omega_0 = \frac{m v_0 l}{2I} = \frac{3}{4} \frac{v_0}{l}$$

ORA IMPONGO LA CONSERVAZIONE DELL'ENERGIA IMMEDIATAMENTE DOPO L'URTO E LA VERTICALE

$$\frac{1}{2} I \omega_0^2 + m g \frac{l}{2} = m g \sqrt{2} \frac{l}{2} \Rightarrow \omega_0^2 = \frac{2}{I} m g \frac{l}{2} (\sqrt{2} - 1)$$

$$\Rightarrow \frac{9}{16} \frac{v_0^2}{l^2} = \frac{3}{2} \frac{g}{l} (\sqrt{2} - 1) \Rightarrow \frac{3}{8} \frac{v_0^2}{l} = (\sqrt{2} - 1) g$$

$$\Rightarrow v_0 = \sqrt{\frac{8(\sqrt{2} - 1) l g}{3}}$$

CALCOLO

$$\begin{aligned} \Delta K &= \frac{1}{2} I \omega_0^2 - \frac{1}{2} m v_0^2 = \frac{1}{3} m l^2 \frac{9}{16} \frac{v_0^2}{l^2} - \frac{1}{2} m v_0^2 = \frac{3}{16} m v_0^2 - \frac{1}{2} m v_0^2 \\ &= \frac{1}{2} m v_0^2 \left(\frac{3}{8} - 1 \right) = -\frac{5}{16} m v_0^2 \Rightarrow \frac{\Delta K}{K_{in}} = -\frac{\frac{5}{16} m v_0^2}{\frac{1}{2} m v_0^2} = -\frac{5}{8} \end{aligned}$$

06/04/2018

1 → (+ 28/10/2019)

$$x(t) = A \ln \left(1 + \frac{t^2}{\tau^2} \right)$$

TROVO

$$v(t) = \frac{dx}{dt} = A \frac{1}{1 + \frac{t^2}{\tau^2}} \frac{2t}{\tau^2} = \frac{2At}{\tau^2 + t^2}$$

$$\begin{aligned} a(t) &= \frac{dv}{dt} = \frac{2A}{\tau^2 + t^2} + \frac{2At}{(\tau^2 + t^2)^2} (-2t) = \frac{2A}{\tau^2 + t^2} \left(1 - \frac{2t^2}{\tau^2 + t^2} \right) \\ &= \frac{2A}{\tau^2 + t^2} \left(\frac{\tau^2 + t^2 - 2t^2}{\tau^2 + t^2} \right) = \frac{2A(\tau^2 - t^2)}{(\tau^2 + t^2)^2} \end{aligned}$$

IL TEMPO t^* LO TROVO IMPONENDO

$$x^* \equiv 3 \text{ m} = A \ln \left(1 + \frac{t^{*2}}{\tau^2} \right) \quad \Rightarrow \quad t^* = \tau \sqrt{e^{x^*/A} - 1}$$

LA VELOCITÀ MEDIA PER DEFINIZIONE È

$$v_{AB} = \frac{1}{t_B - t_A} \int_{t_A}^{t_B} v(t) dt = \frac{A}{t_B - t_A} \int_{t_A}^{t_B} \frac{2t}{\tau^2 + t^2} dt = \frac{A}{t_B - t_A} \ln \left(\frac{\tau^2 + t_B^2}{\tau^2 + t_A^2} \right)$$

QUINDI

$$v_{0,\tau} = \frac{A}{\tau} \ln\left(\frac{\tau^2 + \tau^2}{\tau^2}\right) = \frac{A}{\tau} \ln(2)$$

$$v_{\tau,2\tau} = \frac{A}{\tau} \ln\left(\frac{\tau^2 + 4\tau^2}{\tau^2 + \tau^2}\right) = \frac{A}{\tau} \ln\left(\frac{5}{2}\right)$$

LA VELOCITÀ MASSIMA LA HO PER $\tilde{t}$ T.C. $\frac{dv}{dt} = 0 \Rightarrow a = 0$:

$$a(t) = 0 \Rightarrow \tilde{t}^2 = \tau^2 \Rightarrow \tilde{t} = \tau$$

PER CUI

$$v_{\max} = v(\tilde{t} = \tau) = \frac{2A\tau}{\tilde{t}^2 + \tau^2} = \frac{2A\tau}{2\tau^2} = \frac{A}{\tau}$$

2 CIRCOLARE $a_0 \Rightarrow m \frac{v_0^2}{a_0} = G \frac{M_T m}{a_0^2} \Rightarrow v_0 = \sqrt{\frac{GM_T}{a_0}}$

$$v_m = v_0 - \Delta v_m \quad \Delta v_m = \frac{2}{5} \sqrt{\frac{GM}{a_0}} = \frac{2}{5} v_0$$

CONSERVO LA QUANTITÀ DI MOTO NEL DISTACCO (URTO COMPLETAMENTE ANELASTICO)

$$m v_0 = \frac{2m}{3} v_a + \frac{m}{3} v_m$$

$$\Rightarrow_{so \ v_m} v_0 = \frac{2}{3} v_a + \frac{1}{3} \left(v_0 - \frac{2}{5} v_0 \right)$$

$$\Rightarrow \frac{2}{3} v_a = v_0 - \frac{1}{3} \frac{3}{5} v_0 = \frac{4}{5} v_0 \Rightarrow \begin{cases} v_a = \frac{6}{5} v_0 \\ v_m = \frac{3}{5} v_0 \end{cases}$$

NEL SR DELL'ASTRONAVE

$$v = v' + v_0 \Rightarrow v'_m = v_m - v_0$$



$$\Rightarrow v'_m = \frac{3}{5} v_0 - \frac{6}{5} v_0 = -\frac{3}{5} v_0 = -v_m$$

VISTA DALL'ASTRONAVE

APPLICO LA CONSERVAZIONE DELL'ENERGIA AI DUE RAGGI
E LA CONSERVAZIONE DEL MOMENTO ANGOLARE

$$\begin{cases} \frac{1}{2} \frac{2m}{3} v_o^2 - G \frac{M_T 2m}{3 a_o} = \frac{1}{2} \frac{2m}{3} v_H^2 - G \frac{M_T 2m}{3 r_H} \\ \frac{2m}{3} v_o a_o = \frac{2m}{3} v_H r_H \end{cases}$$

$$\begin{cases} v_o^2 - 2 \frac{G M_T}{a_o} = v_H^2 - 2 \frac{G M_T}{r_H} \\ v_H = \frac{v_o a_o}{r_H} \end{cases}$$

$$\Rightarrow v_o^2 - \frac{a_o^2}{r_H^2} v_o^2 = 2 G M_T \left(\frac{1}{a_o} - \frac{1}{r_H} \right)$$

$$\Rightarrow a_o^2 v_o^2 \left(\frac{1}{a_o^2} - \frac{1}{r_H^2} \right) = 2 G M_T \left(\frac{1}{a_o} - \frac{1}{r_H} \right)$$

$$\underbrace{\left(\frac{1}{a_o} - \frac{1}{r_H} \right) \left(\frac{1}{a_o} + \frac{1}{r_H} \right)}$$

$$\Rightarrow \left(\frac{1}{a_o} + \frac{1}{r_H} \right) = \frac{2 G M_T}{a_o^2 v_o^2} \Rightarrow \frac{1}{r_H} = \frac{2 G M_T - a_o v_o^2}{a_o^2 v_o^2}$$

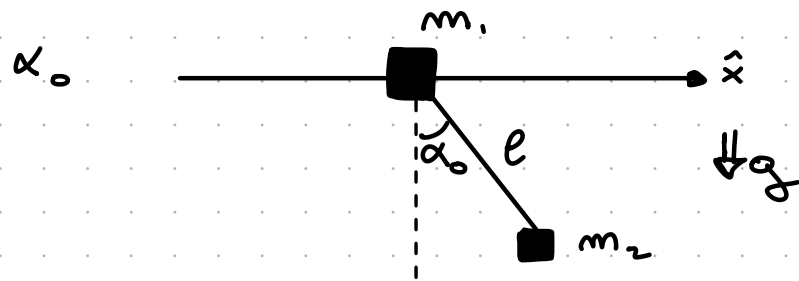
$$\Rightarrow \gamma_r = \frac{a_0^2 v_a^2}{2GM_r - a_0 v_a^2}$$

IL LAVORO DELLE FORZE NEL VIVO DISTACCO LO TROVO CON IL TEOREMA

$$\begin{aligned} W = \Delta K &= \frac{1}{2} \frac{2m}{3} v_a^2 + \frac{1}{2} \frac{m}{3} v_m^2 - \frac{1}{2} m v_0^2 \\ &= \frac{m}{3} \frac{36}{25} \frac{GM_r}{a_0} + \frac{m}{6} \frac{9}{25} \frac{GM_r}{a_0} - \frac{m}{2} \frac{GM_r}{a_0} \\ &= \frac{12}{25} G \frac{M_r m}{a_0} + \frac{3}{50} G \frac{M_r m}{a_0} - \frac{1}{2} G \frac{M_r m}{a_0} \\ &= \frac{24 + 3 - 25}{50} G \frac{M_r m}{a_0} \end{aligned}$$

$$\Rightarrow W = - \frac{GM_r m}{25 a_0}$$

3



CONSERVO L'ENERGIA TRA L'ISTANTE INIZIALE E QUANDO m_2 È SULLA VERTICALE (CONSERVO LA QUANTITÀ DI MOTO) LUNGO $\hat{x}$

$$\begin{cases} m_2 g (l - l \cos \alpha_0) = \frac{m_1}{2} v_1^2 + \frac{m_2}{2} v_2^2 \\ 0 = m_1 v_1 + m_2 v_2 \end{cases}$$

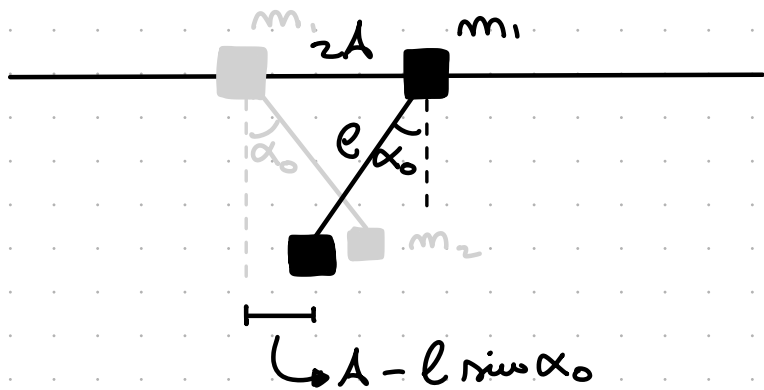
$$\begin{cases} 2m_2 g l (1 - \cos \alpha_0) = \frac{m_2^2}{m_1} v_2^2 + m_2 v_2^2 \\ v_1 = -\frac{m_2}{m_1} v_2 \end{cases}$$

$$\Rightarrow v_2^2 \left(1 + \frac{m_2}{m_1} \right) = 2g l (1 - \cos \alpha_0)$$

$$\Rightarrow v_2 = \sqrt{\frac{2g m_1 l (1 - \cos \alpha_0)}{m_1 + m_2}}$$

PER TROVARE L'ARPIEZZA DELLA QUANTITÀ DI MOTO LUNGO L'ASSE x , DUNQUE x_{cm} NON CAMBIA, QUINDI POSSO RICAVARE L'ARPIEZZA. IL CENTRO DI MASSA. x_{cm} ALL'INIZIO,

COME ORIGINE DEL SR PRENDENDO LA POSIZIONE INIZIALE DI m_1 .



$$x_{cm}^0 = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{m_2 l \sin \alpha_0}{m_1 + m_2}$$

$$x_{cm}^1 = \frac{m_1 zA + m_2 (zA - l \sin \alpha_0)}{m_1 + m_2}$$

$$\Rightarrow x_{cm}^0 = x_{cm}^1$$

$$\Rightarrow m_2 l \sin \alpha_0 = (m_1 + m_2) zA - m_2 l \sin \alpha_0$$

$$\Rightarrow zA = \frac{2m_2 l \sin \alpha_0}{m_1 + m_2}$$

$$\Rightarrow A = \frac{m_2 l \sin \alpha_0}{m_1 + m_2}$$

2/07/2018

1

2

$$b = \alpha \pi R^2$$

$$F_{\text{ext}} = -bv = -\alpha \pi R^2 v$$

HO 2 RAGGI R_1, R_2

PER OGNI SFERA

$$m_i \frac{dv}{dt} = -\alpha \pi R^2 v + m_i g$$

$$\frac{dv}{g - \frac{\alpha \pi R^2}{m_i} v} = dt \quad \Rightarrow \quad -\frac{m_i}{\alpha \pi R^2} \ln \left(1 - \frac{\alpha \pi R^2}{m_i g} v \right) = t$$

$$\Rightarrow \ln \left(1 - \frac{\alpha \pi R^2}{m_i g} v \right) = -\frac{\alpha \pi R^2}{m_i} t$$

$$\Rightarrow \frac{\alpha \pi R^2}{m_i g} v = 1 + e^{-\frac{\alpha \pi R^2}{m_i} t} \quad \Rightarrow \quad v_i(t) = \frac{m_i g}{\alpha \pi R_i^2} \left(1 + e^{-\frac{\alpha \pi R_i^2}{m_i} t} \right)$$

$$\text{con } m = \frac{4}{3} \pi R^3, \quad \text{VEDO } v_{\text{lim}} = \frac{m_i g}{\alpha \pi R_i^2}$$

$$\Rightarrow v_i(t) = v_{\text{lim},i} \left(1 + e^{-\frac{\alpha \pi R_i^2}{m_i} t} \right)$$

trovo

$$\begin{aligned}v_r(t) &= v_1(t) - v_2(t) \\&= v_{Lm,1} - v_{Lm,2} + v_{Lm,1} e^{-\frac{b}{m_1}t} - v_{Lm,2} e^{-\frac{b}{m_2}t}\end{aligned}$$

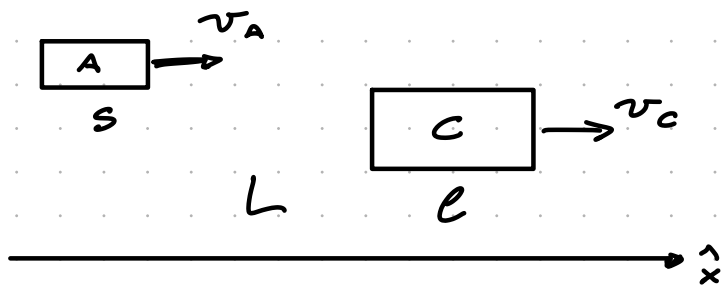
e

$$g = \frac{v_{Lm,1}}{v_{Lm,2}} = \frac{m_1 g}{\alpha \pi R_1^2} \frac{\alpha \pi R_2^2}{m_2 g} = \frac{m_1}{m_2} \frac{R_2^2}{R_1^2} = \frac{g \frac{4}{3} \pi R_1^3}{g \frac{4}{3} \pi R_2^3} \frac{R_2^2}{R_1^2}$$

$$\Rightarrow g = \frac{R_1}{R_2}$$

18/09/2018

1



SCRIVO LE EQUAZIONI DEL MOTO RIFERITE AL CULO DEI MEZZI

$$\begin{aligned} a) \quad x_A(t) &= v_A t \\ x_C(t) &= (s + L) + v_C t \end{aligned}$$

HO IL SORPASSO QUANDO

$$x_A(\tau) = \Delta x_C(\tau) + s + L + l$$

$\xrightarrow{\text{SPAZIO PERCORSO DA C}}$

$$\Rightarrow v_A \tau = (x_C(\tau) - x_C(0)) + s + L + l$$

$$\Rightarrow v_A \tau = s + L + v_C \tau - s - L + s + L + l$$

$$\Rightarrow \tau = \frac{s + L + l}{v_A - v_C}$$

b) Ho $x_A(t) = v_A t + \frac{1}{2} a t^2$
 $x_C(t) = s + L + v_C t$

SEMPRE LA CONDIZIONE $x_A(\tau) = s + L + \ell + \Delta x_C(\tau)$

$$\Rightarrow v_A \tau + \frac{1}{2} a \tau^2 = s + L + \ell + (s + L + v_C \tau - s - L)$$

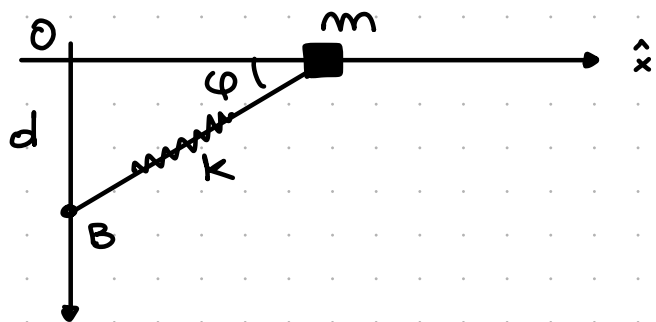
$$\Rightarrow \frac{1}{2} a \tau^2 + (v_A - v_C) \tau - (s + \ell + L) = 0$$

$$\Rightarrow \tau_{1,2} = \frac{v_C - v_A \pm \sqrt{(v_A - v_C)^2 + 2a(s + \ell + L)}}{a}$$

→ NELLE SOLUZIONI
LA RADICE DOVREBBE
ESSERE PIÙ LUNGA

2

$k, m, x_0 = x(t=0)$



LUNGO y AVREI

$$m a_y = 0 = mg + F_{Ey} - N$$

$$\Rightarrow N = mg + kd$$

HO L'EQUAZIONE DEL MOTO (LUNGO x)

$$m a_x = F_{ELx} = (\vec{F}_{EL}) \cdot \hat{x} = -k \vec{r} \cdot \hat{x} = -k(x, d) \cdot \hat{x} = -kx$$

$$\Rightarrow \frac{d^2 x}{dt^2} = -\frac{k}{m} x$$

$$\Rightarrow \omega = \sqrt{\frac{k}{m}} \quad \Rightarrow x(t) = A \sin(\omega t + \phi)$$

FISSO $x(0) = x_0 \Rightarrow A \sin \phi = x_0$

TROVO $v(t) = \omega A \cos(\omega t + \phi) \Rightarrow v(0) = 0 \Rightarrow \omega A \cos \phi = 0$

HO QUINDI

$$\phi = \frac{\pi}{2}, \quad A = x_0$$

PER CUI

$$x(t) = x_0 \cos(\omega t), \quad v(t) = -\omega x_0 \sin(\omega t)$$

LA VELOCITÀ MASSIMA È QUINDI

$$v_{\max} = -x_0 \sqrt{\frac{k}{m}}$$

L'ACCELERAZIONE È

$$a(t) = -\omega^2 x_0 \cos(\omega t)$$

ASSUMENDO CHE LA MOLLA ABBA LUNGHEZZA A RIPOSO d HO
L'EQUAZIONE DEL MOTO

$$m \ddot{x} = F_{\text{el } x} = \vec{F}_{\text{el } x} \cdot \hat{x} = (-k(\vec{r} - d)) \cdot \hat{x}$$

$$= -k(\sqrt{d^2 + x^2} - d) \cos \varphi$$

$$= -k(\sqrt{d^2 + x^2} - d) \frac{x}{\sqrt{d^2 + x^2}}$$

$$= -k \left(x - \frac{d}{\sqrt{d^2 + x^2}} x \right)$$

$$\Rightarrow m \ddot{x} = -kx + \frac{kd}{\sqrt{d^2 + x^2}} x$$

CHE NON È L'EQUAZIONE DI UN OSCILLATORE ARMONICO.

PER TROVARE $v'_{\max}$ USO LA CONSERVAZIONE DELL'ENERGIA

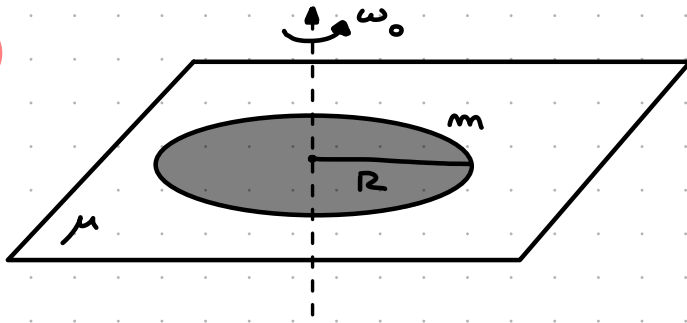
$$\frac{1}{2} k r_0^2 = \frac{1}{2} k r_{\min}^2 + \frac{m}{2} v_{\max}^2$$

IN CUI $r_0 = \sqrt{d^2 + x_0^2} - d$, $r_{\min} = 0$ OSSIA IL PUNTO IN CUI
LA MOLLA È PIÙ MENO ALLUNGATA
QUINDI QUANDO m È IN O

$$\frac{1}{2} k \left(\sqrt{d^2 + x_0^2} - d \right)^2 = \frac{m}{2} v_{\max}^2$$

$$\Rightarrow v'_{\max} = \sqrt{\frac{k}{m}} \left(\sqrt{d^2 + x_0^2} - d \right)$$

3



$$\text{HO } \sigma = \text{OMOG}$$

$$m = \int dm = \int \sigma ds = \pi \sigma R^2$$

OGNI ELEMENTINO DEL DISCO SENTE SIA LA FORZA CENTRIFUGA CHE L'ATTRITO.

CALCOLO dM_{ATT} PER OGNI dm E POI INTEGRATO SUL DISCO.

$$dM_{\text{ATT}} = \mathbf{r} \times \mathbf{F}_{\text{ATT}} = -v dm g \mu$$

$\mathbf{v} \perp \mathbf{F}_{\text{ATT}}$

$$\Rightarrow M_{\text{ATT}} = -g \mu \int r dm = -g \mu \int r^2 dr d\theta \sigma = -\frac{1}{3} g \mu 2\pi R^3 \sigma = -\frac{2}{3} \mu g m R$$

USO L'EQUAZIONE CARDINALE

$$M^E = I \alpha \Rightarrow \frac{d\omega}{dt} = \frac{M^E}{I} \Rightarrow \frac{d\omega}{dt} = -\frac{2}{3} \mu g m R \cdot \frac{2}{m R^2} = -\frac{4}{3} \mu g \frac{1}{R}$$

$$I = \frac{1}{2} m R^2$$

$$\Rightarrow \omega(t) = \omega_0 - \frac{4}{3} \frac{\mu g}{R} t$$

$$\text{HO } \omega(\tau) = 0 \Rightarrow \tau = \frac{3 R \omega_0}{4 \mu g}$$

DA $\omega(t)$:

$$\Theta(t) = \Theta_0 + \omega_0 t - \frac{2}{3} \frac{\mu g}{l^2} t^2$$

$$\theta_0 = 0$$

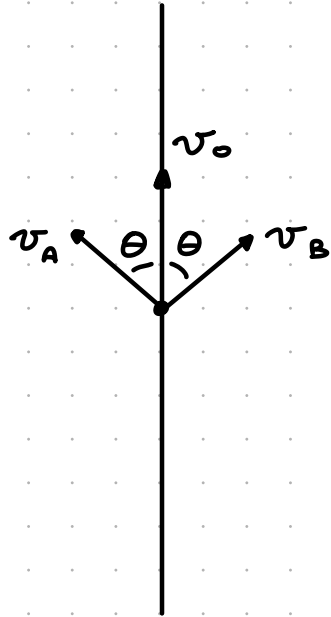
$$\Rightarrow \Theta(\tau) = \omega_0 \tau - \frac{2}{3} \frac{\mu g}{l^2} \tau^2 = \frac{3 l^2 \omega_0^2}{4 \mu g} - \frac{2}{3} \frac{\mu g}{l^2} \frac{9 l^2 \omega_0^2}{16 \mu^2 g^2}$$

$$\Rightarrow \Theta(\tau) = \frac{3 l^2 \omega_0^2}{4 \mu g} - \frac{3}{8} \frac{l^2 \omega_0^2}{\mu g} = \frac{3}{8} \frac{l^2 \omega_0^2}{\mu g}$$

19/03/2019

1 $m, d, m_A = \frac{3}{4}m, m_B = \frac{1}{4}m, \theta = \frac{\pi}{3}$

DEVO CONSERVARE LA Q.TÀ DI MOTO



$$\begin{cases} m v_0 = \frac{3}{4} m v_A \cos \theta + \frac{1}{4} m v_B \cos \theta \\ 0 = -\frac{3}{4} m v_A \sin \theta + \frac{1}{4} m v_B \sin \theta \end{cases}$$

$$\begin{cases} 4 v_0 = v_B \cos \theta + v_B \cos \theta \\ 3 v_A = v_B \end{cases}$$

$$\Rightarrow \begin{cases} v_B = \frac{2 v_0}{\cos \theta} = 4 v_0 \\ v_A = \frac{2 v_0}{3 \cos \theta} = \frac{4}{3} v_0 \end{cases}$$

v_0 È LA VELOCITÀ ORBITALE

$$m \frac{v_0^2}{d} = G \frac{M_T m}{d^2} \Rightarrow v_0 = \sqrt{\frac{G M_T}{d}}$$

DEVO TROVARE L'ENERGIA DELL'URTO

$$\Delta E = \Delta K = \frac{m_A}{2} v_A^2 + \frac{m_B}{2} v_B^2 - \frac{m}{2} v_0^2$$

PER LE ORBITE CALCOLO (DOVREI METTERE I NUMERI)

$$\begin{cases} E_A = \frac{m_A}{2} v_A^2 - G \frac{M_T m_A}{d} \\ E_B = \frac{m_B}{2} v_B^2 - G \frac{M_T m_B}{d} \end{cases}$$

PER $r_{\min}$ E $r_{\max}$ USO LA CONSERVAZIONE DELL'ENERGIA E DEL MOMENTO ANGOLARE

$$\begin{cases} \frac{m_A}{2} v_A^2 - G \frac{M_T m_A}{r_{\max}} = \frac{m_A}{2} \tilde{v}_A^2 - G \frac{M_T m_A}{r_{\min}} \\ m_A r_{\max} v_A = m_A r_{\min} \tilde{v}_A \end{cases}$$

$$\Rightarrow \tilde{v}_A = \frac{r_{\max}}{r_{\min}} v_A$$

$$\Rightarrow \frac{1}{2} v_A^2 - \frac{GM_T}{r_n} = \frac{1}{2} \frac{r_n^2}{r_m^2} v_A^2 - \frac{GM_T}{r_m}$$

$$\Rightarrow \frac{v_A^2}{2} \left(1 - \frac{r_H^2}{r_m^2} \right) = GM_T \left(\frac{1}{r_H} - \frac{1}{r_m} \right)$$

$$\Rightarrow \frac{v_A^2}{2} r_H^2 \left(\frac{1}{r_H^2} - \frac{1}{r_m^2} \right) = GM_T \left(\frac{1}{r_H} - \frac{1}{r_m} \right)$$

$$\Rightarrow \frac{v_A^2}{2} r_H^2 \left(\frac{1}{r_H} + \frac{1}{r_m} \right) = GM_T$$

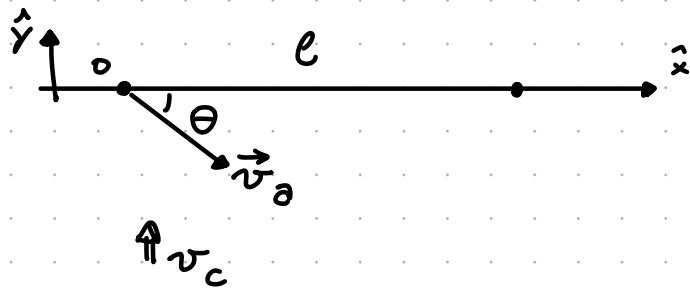
$$\Rightarrow \left(\frac{1}{r_H} + \frac{1}{r_m} \right) = \frac{2GM_T}{v_A^2 r_H^2} \quad \Rightarrow \quad \frac{1}{r_m} = \frac{2GM_T}{v_A^2 r_H^2} - \frac{1}{r_H}$$

$$r_H \equiv d$$

$$\Rightarrow r_m = \frac{v_A^2 d^2}{2GM_T - v_A^2 d}$$

02/04/2019

1



$$\vec{v}_c = 3kx^2 \hat{y}$$

$$\vec{v}_a = v_a \hat{x}$$

PER TROVARE LA LEGGE ORARIA CALCOLO TUTTE LE COMPONENTI DI v

$$\begin{cases} v_x = v_a \cos \theta \\ v_y = v_c - v_a \sin \theta = 3kx^2 - v_a \sin \theta \end{cases} \Rightarrow \begin{cases} v_x = \frac{dx}{dt} \\ v_y = \frac{dy}{dt} \end{cases}$$

$$\Rightarrow \begin{cases} x(t) = v_a \cos \theta t \\ y(t) = 3k \int x^2 dt - v_a \sin \theta t = 3k v_a^2 \cos^2 \theta \int t^2 dt - v_a \sin \theta t = k v_a^2 t^3 \cos^2 \theta - v_a \sin \theta t \end{cases}$$

MAI HO $x(t)$ E POSSO TROVARE LO SPOSTAMENTO D FATTO IN $t = \tau$.

$$\begin{cases} x(\tau) = l = v_a \cos \theta \tau \\ y(\tau) = D = k v_a^2 \tau^3 \cos^2 \theta - v_a \sin \theta \tau \end{cases} \Rightarrow \begin{cases} \tau = l / v_a \cos \theta \\ D = k v_a^2 \cos^2 \theta \frac{l^3}{v_a^3 \cos^3 \theta} - v_a \sin \theta \frac{l}{v_a \cos \theta} \end{cases}$$

$$\Rightarrow D = \frac{k l^3}{v_a \cos \theta} - l \tan \theta$$

02/07/2019

1

$$v(z) = a + bz, \quad v_0 \quad v(h) = v_1 \quad v_p$$

IMPOSTO

$$v(0) = v_0 \Rightarrow a = v_0$$

$$v(h) = v_1 \Rightarrow v_0 + bh = v_1 \Rightarrow b = \frac{v_1 - v_0}{h}$$

DUNQUE

$$v(z) = v_0 - \frac{v_0 - v_1}{h} z$$

IL PALLONE

HA

$$\vec{v} \left(v_0 - \frac{v_0 - v_1}{h} z, v_p \right)$$

$$\begin{cases} v_x = \frac{dx}{dt} = v_0 - \frac{v_0 - v_1}{h} z \end{cases}$$

$$\begin{cases} v_z = \frac{dz}{dt} = v_p \end{cases}$$

$$\begin{cases} dx = \left(v_0 - \frac{v_0 - v_1}{h} z \right) dt \end{cases}$$

$$\begin{cases} z(t) = v_p t \end{cases}$$

$$\Rightarrow x(t) = v_0 t - \frac{v_0 - v_1}{h} v_p \frac{1}{2} t^2$$

RICAVO

$$t = \frac{z}{v_p}$$

$$\Rightarrow x(z) = \frac{v_0}{v_p} z - \frac{v_0 - v_1}{2h} \frac{z^2}{v_p}$$

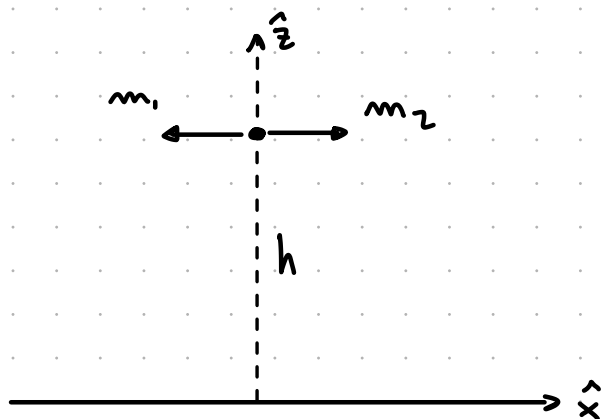
VOGLIO τ T.C. $x(\tau) = \ell$

$$\frac{1}{2} \frac{v_0 - v_1}{h} v_p \tau^2 - v_0 \tau + \ell = 0$$

$$\Rightarrow \tau = \frac{v_0 \pm \sqrt{v_0^2 - 2\ell \frac{v_0 - v_1}{h} v_p}}{\frac{v_0 - v_1}{h} v_p}$$

$$= \frac{h}{(v_0 - v_1)} \frac{v_0}{v_p} \pm \sqrt{\frac{h^2}{(v_0 - v_1)^2} \frac{v_0^2}{v_p^2} - \frac{2\ell h}{v_0 - v_1} \frac{1}{v_p}}$$

2 $m, v_0, h, m_1 = \frac{2}{3}m, m_2 = \frac{m}{3}, E$



URTO ANELASTICO $\Rightarrow$ CONSERVO p (LUNGO $\hat{x}$)

$$0 = m_1 v_{1x} + m_2 v_{2x}$$

$$\Rightarrow v_{1x} = -\frac{m_2}{m_1} v_{2x} = -\frac{1}{2} v_{2x}$$

CONOSCO LA TRASFORMAZIONE DELLE VELOCITÀ

$$v = v' + v_0'$$

$$\begin{cases} v_{1x} = v'_{1x} + v_{cmx} \Rightarrow v_{1x} = v'_{1x} \\ v_{1y} = v'_{1y} + v_{cmy} \Rightarrow v_{1y} = v_{cmy} \end{cases}$$

IDEA $v_{2x,y}$

NEL SR DEL
CM $\hookrightarrow$ VEDO
ANDARE SOLO
LUNGO x

$$v_{cmy} = v(t) = v_0 - gt$$

QUINDI IN UN SR SOLIDALE CON IL TERRENO

$$\begin{cases} v_{1x} = -\frac{m_2}{m_1} v_{2x} \\ v_{1y}, v_{2y} = v_0 - gt \end{cases}$$

L'ENERGIA DISSIPATA È DUNQUE (AL TEMPO τ SONO ALLA QUOTA h)

$$y(t) = v_0 t - \frac{1}{2} g t^2$$

$$\Rightarrow h = v_0 \tau - \frac{1}{2} g \tau^2$$

$$\Rightarrow \frac{1}{2} g \tau^2 - v_0 \tau + h = 0$$

$$\tau = \frac{v_0}{g} \pm \frac{1}{g} \sqrt{v_0^2 - 2hg}$$

$$E = K_{FIN} - K_{IN}$$

$$= \frac{m_1}{2} v_1^2 + \frac{m_2}{2} v_2^2 - \frac{m}{2} v^2(\tau)$$

$$= \frac{m_1}{2} \left(\frac{m_2^2}{m_1^2} v_{2x}^2 + (v_0 - g\tau)^2 \right) + \frac{m_2}{2} \left(v_{2x}^2 + (v_0 - g\tau)^2 \right) - \frac{m}{2} (v_0 - g\tau)^2$$

$$= \frac{m_1}{2} \frac{1}{4} v_{2x}^2 + \frac{m_2}{2} v_{2x}^2 + \frac{1}{2} (m_1 + m_2 - m) (v_0 - g\tau)^2$$

$$= \frac{v_{2x}^2}{2} \left(\frac{m_1}{4} + m_2 \right)$$

$$\Rightarrow 2E = v_{2x}^2 \left(\frac{m_1 + 4m_2}{4} \right)$$

$$\Rightarrow v_{2x} = \sqrt{\frac{8E}{m_1 + 4m_2}}$$

$$\Rightarrow v_{1x} = -\frac{m_2}{m_1} v_{2x} = -\frac{1}{2} v_{2x} = -\sqrt{\frac{2E}{m_1 + 4m_2}}$$

IMMEDIATAMENTE
PARABOLICI

DOPO

L'URTO

PERCHÉ

HO

DUE

MOTI

$$\begin{cases} v_x = v_{1x} \\ v_y = v_{y0} - gt = v_0 - g\tau - gt \end{cases}$$

$$\begin{cases} x(t) = v_x t \\ y(t) = h + v_{0y} t - \frac{1}{2} g t^2 \\ \quad \quad \quad = h + (v_0 - g\tau) t - \frac{1}{2} g t^2 \end{cases}$$

ARRIVANO A TERRA

$$\begin{cases} y_1(t_1) = 0 = h + (v_0 - g\tau)t_1 - \frac{1}{2}gt_1^2 \\ y_2(t_2) = 0 = h + (v_0 - g\tau)t_2 - \frac{1}{2}gt_2^2 \end{cases}$$

$$t_1 = t_2$$

$$\Rightarrow \frac{1}{2}gt_2^2 - (v_0 - g\tau)t_2 - h = 0$$

$$t_{1,2} = \frac{v_0 - g\tau}{g} \pm \sqrt{\frac{(v_0 - g\tau)^2}{g^2} + \frac{2h}{g}}$$

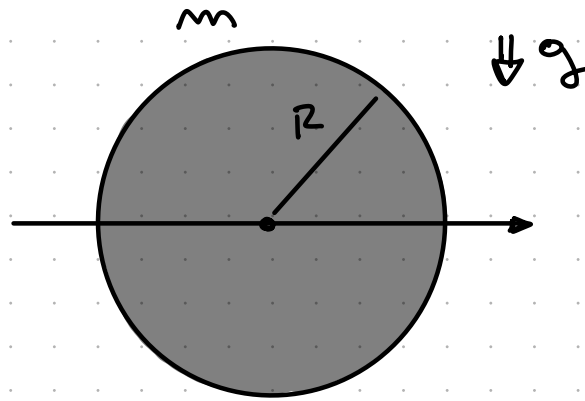
E ARRIVANO A

$$\begin{cases} x_1(t_2) = v_{x1}t_2 \\ x_2(t_2) = v_{2x}t_2 \end{cases}$$

3

DISCO m, R

$$\sigma = \sigma_0 \frac{r}{R}$$



$$m = \int dm = \int \sigma_0 \frac{r}{R} r dr d\theta = \frac{\sigma_0 2\pi}{R} \int_0^R r^2 dr = \frac{2\pi}{3} \frac{\sigma_0}{R} R^3 = \frac{2}{3} \pi \sigma_0 R^2$$

$$I = \int b^2 dm, \quad b = r \cos \theta$$

$$\Rightarrow I = \int r^2 \cos^2 \theta \sigma(r) r dr d\theta = \int r^3 \cos^2 \theta \sigma_0 \frac{r}{R} dr d\theta$$

$$= \frac{\sigma_0}{R} \int_0^{2\pi} \cos^2 \theta d\theta \int_0^R r^4 dr$$

$$= \frac{\sigma_0}{R} \pi \frac{1}{5} R^5 = \frac{1}{5} \pi \sigma_0 R^4 = \frac{3}{2} \frac{1}{5} \frac{2}{3} \pi \sigma_0 R^4 = \frac{3}{10} m R^2$$

$$\text{NO } \mathcal{P}_{\text{ATT}} = -k\omega$$

E' USO L'EQUAZIONE CARDINALE

$$-k\omega = I \frac{d\omega}{dt} \Rightarrow \frac{d\omega}{\omega} = -\frac{k}{I} dt \Rightarrow \omega(t) = \omega_0 e^{-\frac{k}{I} t}$$

25/03/2020

1

$$x(t) = v_0 t + \alpha t^2 + \beta t^3$$

$$\Rightarrow v(t) = \frac{dx}{dt} = v_0 + 2\alpha t + 3\beta t^2$$

$$\Rightarrow a(t) = \frac{dv}{dt} = 2\alpha + 6\beta t$$

TROVO $v_{\max}$:

$$\frac{dv}{dt} = 0 \Rightarrow a(t) = 0 \Rightarrow 2\alpha + 6\beta t = 0 \Rightarrow 3\beta t = -\alpha$$

$$\Rightarrow t_{\max} = -\frac{\alpha}{3\beta}$$

$$\Rightarrow v_{\max} = v(t_{\max}) = v_0 + 2\alpha \left(-\frac{\alpha}{3\beta}\right) + 3\beta \left(-\frac{\alpha}{3\beta}\right)^2$$

$$\Rightarrow v_{\max} = v_0 - \frac{2}{3} \frac{\alpha^2}{\beta} + \frac{1}{3} \frac{\alpha^2}{\beta} = v_0 - \frac{1}{3} \frac{\alpha^2}{\beta}$$

CERCO QUANDO RIPASSA DA $x(t_0)$.
METTO $t_0 = 0 \Rightarrow x(t_0) = x_0 = 0$
CERCO

$$0 = v_0 t + \alpha t^2 + \beta t^3 \Rightarrow t_1 = 0$$

$$\Rightarrow 0 = v_0 + \alpha t + \beta t^2$$

$$t_{2,3} = \frac{-\alpha \pm \sqrt{\alpha^2 - 4v_0\beta}}{2\beta}$$

CERCO

ORA

QUANDO

$$v(t) = 0$$

$$v_0 + 2\alpha t + 3\beta t^2 = 0$$

$$t_{1,2} = \frac{-2\alpha \pm \sqrt{4\alpha^2 - 12\beta v_0}}{6\beta}$$

2 $F = F_0 e^{-kt}$

SCRIVENDO L'EQUAZIONE DEL MOTO

$$ma = F_0 e^{-kt} \Rightarrow \frac{dv}{dt} = \frac{F_0}{m} e^{-kt} \Rightarrow v(t) = \frac{F_0}{mk} (1 - e^{-kt})$$

DUNQUE

$$dW = \vec{F} \cdot d\vec{s} = F ds = F v dt$$

$$\Rightarrow W = \frac{F_0^2}{mk} \int_0^t (e^{-kt} - e^{-2kt}) dt = \frac{F_0^2}{mk} \left(-\frac{1}{k} e^{-kt} + \frac{1}{2k} e^{-2kt} \right) \Big|_0^t$$

$$= \frac{F_0^2}{mk^2} \left(-e^{-kt} + 1 + \frac{1}{2} e^{-2kt} - \frac{1}{2} \right)$$

$$= \frac{F_0^2}{mk^2} (1 - e^{-kt}) - \frac{F_0^2}{2mk^2} (1 - e^{-2kt})$$

$$P = \frac{dW}{dt} = \vec{F} \cdot \vec{v} = \frac{F_0^2}{mk} e^{-kt} (1 - e^{-kt})$$

DATA

 $v(t)$

TROVO

$$\begin{aligned}x(t) &= \int_0^t v(t) dt = \frac{F_0}{k_m} \int_0^t (1 - e^{-kt}) dt = \frac{F_0}{k_m} t - \frac{F_0}{k_m} \left(-\frac{1}{k} e^{-kt} \right) \Big|_0^t \\&= \frac{F_0}{k_m} t - \frac{F_0}{k^2 m} (1 - e^{-kt})\end{aligned}$$

3

DISCO $\Rightarrow I_{\text{DISCO}} = \frac{1}{2} m_0 R^2$, ω_0

APPLICO $\mathcal{M} = \text{cost}$

HO LA SECONDA EQUAZIONE CARDINALE

$$\mathcal{M} = \frac{dL}{dt} = -I \alpha \quad \Rightarrow \quad \mathcal{M} = -I \frac{d\omega}{dt}$$

$$\Rightarrow -I d\omega = \mathcal{M} dt \quad \Rightarrow \quad I(\omega - \omega_0) = -\mathcal{M} t$$

$$\Rightarrow \omega(t) = \omega_0 - \frac{\mathcal{M}}{I} t$$

CERCO t_1 T.C. $\omega(t_1) = 0 \Rightarrow t_1 = \frac{I \omega_0}{\mathcal{M}} = \frac{m_0 \omega_0 R^2}{2\mathcal{M}}$

ORA HO $\frac{dm}{dt} = \kappa$, $\mathcal{M} \sim 0$

$$\mathcal{M} = \frac{dL}{dt} \quad , \quad L = I \omega \Rightarrow \frac{dL}{dt} = \frac{dI}{dt} \omega + I \frac{d\omega}{dt}$$

CONSIDERO $\mathcal{M} = 0$

$$\Rightarrow \frac{dI}{dt} \omega + I \frac{d\omega}{dt} = 0 \quad \frac{d}{dt} I = \frac{d}{dt} \left(\frac{1}{2} m R^2 \right) = \frac{1}{2} \frac{dm}{dt} R^2$$

$$\Rightarrow \frac{1}{2} \frac{dm}{dt} R^2 \omega + \frac{1}{2} m R^2 \frac{d\omega}{dt} = 0$$

$$\Rightarrow k\omega + m \frac{d\omega}{dt} = 0$$

$$\Rightarrow k\omega = -m \frac{d\omega}{dt} \Rightarrow -\frac{k}{m} dt = \frac{d\omega}{\omega}$$

$$HO \quad \frac{dm}{dt} = k \Rightarrow m(t) = m_0 + kt$$

$$\Rightarrow -\frac{k dt}{m_0 + kt} = \frac{d\omega}{\omega} \Rightarrow -\ln\left(\frac{m_0 + kt}{m_0}\right) = \ln\left(\frac{\omega}{\omega_0}\right)$$

$$\Rightarrow \frac{m_0}{m_0 + kt} = \frac{\omega}{\omega_0} \Rightarrow \omega(t) = \frac{m_0 \omega_0}{m_0 + kt}$$

$$VEDO \quad \omega(\tau) = 0 \Rightarrow \tau \rightarrow \infty \quad \underline{\text{NON SI FERMA}}$$

$$TROVO \quad \theta(t) = \int_0^t \omega(t') dt' = \frac{m_0 \omega_0}{k} \int_0^t \frac{k dt'}{m_0 + kt'} = \frac{m_0 \omega_0}{k} \ln\left(\frac{m_0 + kt}{m_0}\right)$$

$$CERCO \quad \tau_1 \quad \text{T.C.} \quad \theta(\tau_1) = 2\pi$$

$$\Rightarrow \frac{2\pi k}{m_0 \omega_0} = \ln\left(\frac{m_0 + k\tau_1}{m_0}\right) \Rightarrow m_0 + k\tau_1 = m_0 e^{\frac{2\pi k}{m_0 \omega_0}}$$

$$\Rightarrow \tau_1 = \frac{m_0}{k} \left(e^{\frac{2\pi k}{m_0 \omega_0}} - 1 \right)$$

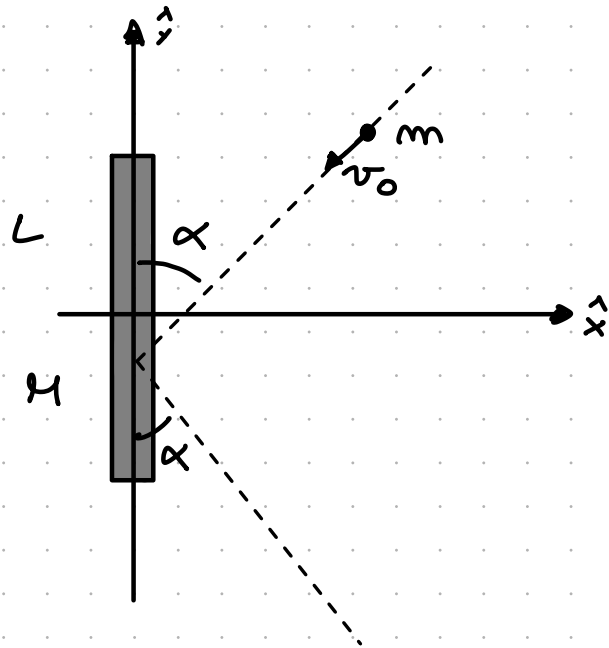
07/09/2020

1

ASTA L, M

$P(0, -\frac{L}{5})$

$$\frac{m}{M} = \frac{4}{5}$$



$$v_1 = \frac{v_0}{4}$$

$$\alpha = 30^\circ$$

HO UN URTO ELASTICO PER CUI POSSO
CONSERVARE L'ENERGIA E L'IMPULSO (MOMENTO ANGOLARE)

$$\begin{cases} \hat{x} & -m v_0 \sin \alpha = M v_{cmx} + m v_1 \sin \alpha \\ \hat{y} & -m v_0 \cos \alpha = M v_{cmy} - m v_1 \cos \alpha \end{cases}$$

$$\begin{cases} -m \sin \alpha \left(v_0 + \frac{v_0}{4} \right) = M v_{cmx} \\ -m \cos \alpha \left(v_0 - \frac{v_0}{4} \right) = M v_{cmy} \end{cases}$$

$$\begin{cases} v_{cmx} = -\frac{m}{M} \sin \alpha \frac{5}{4} v_0 = -\frac{v_0}{2} \\ v_{cmy} = -\frac{m}{M} \cos \alpha \frac{3}{4} v_0 = -\frac{12\sqrt{3}}{20 \cdot 2} v_0 = -\frac{3\sqrt{3}}{10} v_0 \end{cases}$$

CONSERVO IL MOMENTO ANGOLARE (POLO IN O)

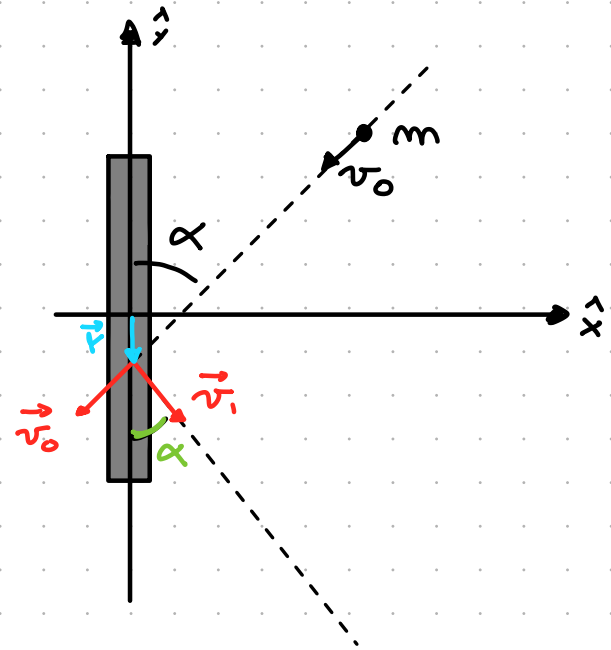
$$m \frac{L}{15} v_0 \sin \alpha = m \frac{L}{15} v_1 \sin \alpha + I \omega$$

$$\Rightarrow I \omega = \frac{m L v_0}{15} \sin \alpha \left(1 - \frac{1}{4} \right)$$

$$\Rightarrow \frac{1}{12} M L^2 \omega = \frac{3}{4} \frac{m L v_0}{15} \sin \alpha$$

$$\Rightarrow \omega = \frac{3}{4} \frac{12}{15} \frac{m}{M} \frac{v_0}{L} \sin \alpha$$

$$\Rightarrow \omega = \frac{3}{5} \frac{4}{5} \frac{v_0}{L} \frac{1}{2} = \frac{6}{25} \frac{v_0}{L}$$



DALLA SOLUZIONE $\omega = \frac{2v_0}{5l}$

$$K_{in} = \frac{m}{2} v_0^2$$

$$K_{FIN} = \frac{m}{2} v_1^2 + \frac{1}{2} I \omega^2 = \frac{m}{32} v_0^2 + \frac{1}{24} M L^2 \frac{4 v_0^2}{25 L^2}$$

$$\begin{aligned} \Rightarrow K_{FIN} &= \frac{m}{32} v_0^2 + \frac{1}{12 \cdot 25} \frac{5}{4} m v_0^2 = \frac{m}{32} v_0^2 + \frac{1}{240} m v_0^2 \\ &= \frac{m}{16} v_0^2 \left(\frac{1}{2} + \frac{1}{15} \right) = \frac{m}{16} \frac{17}{30} v_0^2 = \frac{17}{16 \cdot 30} m v_0^2 \end{aligned}$$

QUINDI

$$\frac{K_{FIN}}{K_{in}} = \frac{17}{16 \cdot 30} 2 = \frac{17}{16 \cdot 15} =$$


2

r_0 TROVO LA VELOCITÀ ORBITALE

$$m r_0 \frac{v_0^2}{r_0^2} = G \frac{Mm}{r_0^2} \Rightarrow v_0 = \sqrt{\frac{GM}{r_0}}$$

CORRISPONDE AD UNA CERTA $\omega \Rightarrow \omega_0 = \frac{v_0}{r_0} = \frac{1}{r_0} \sqrt{\frac{GM}{r_0}} = \sqrt{\frac{GM}{r_0^3}}$

DUNQUE IL PERIODO $T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{r_0^3}{GM}}$

HO $v_1 = \frac{v_0}{2} \Rightarrow$ ORBITA ELLITTICA

CERCO LA DISTANZA MINIMA DALLA TERRA

$$\begin{cases} -G \frac{Mm}{r_0} + \frac{m}{2} v_1^2 = -G \frac{Mm}{r_1} + \frac{m}{2} v_2^2 \\ m v_1 r_0 = m v_2 r_1 \end{cases}$$

$$\Rightarrow v_2 = \frac{r_0}{r_1} v_1$$

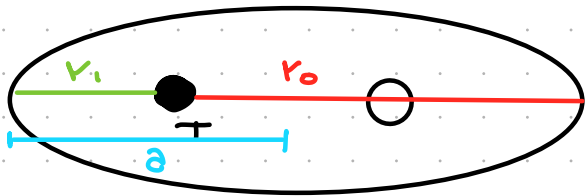
$$GM \left(\frac{1}{r_1} - \frac{1}{r_0} \right) = \frac{1}{2} v_1^2 \left(\frac{r_0^2}{r_1^2} - 1 \right) \quad r_0^2 \left(\frac{1}{r_1^2} - \frac{1}{r_0^2} \right) = r_0^2 \left(\frac{1}{r_1} - \frac{1}{r_0} \right) \left(\frac{1}{r_1} + \frac{1}{r_0} \right)$$

$$\frac{2GM}{v_1^2 r_0^2} = \frac{1}{r_1} + \frac{1}{r_0}$$

$$\Rightarrow \frac{1}{r_1} = \frac{8GM}{v_0^2 r_0^2} - \frac{1}{r_0} \quad \Rightarrow \quad \frac{1}{r_1} = \frac{8GM r_0 - v_0^2 r_0}{v_0^2 r_0^2}$$

$$\Rightarrow r_1 = \frac{v_0^2 r_0^3}{8GM r_0 - v_0^2 r_0^2}$$

DUNQUE



$$\Rightarrow a = r_1 + \frac{r_0 - r_1}{2}$$

$$\Rightarrow a = \frac{v_0^2 r_0^3}{8GM r_0 - v_0^2 r_0^2} + \frac{r_0}{2} - \frac{1}{2} \frac{v_0^2 r_0^3}{8GM r_0 - v_0^2 r_0^2}$$

CON $v_0^2 = \frac{GM}{r_0}$

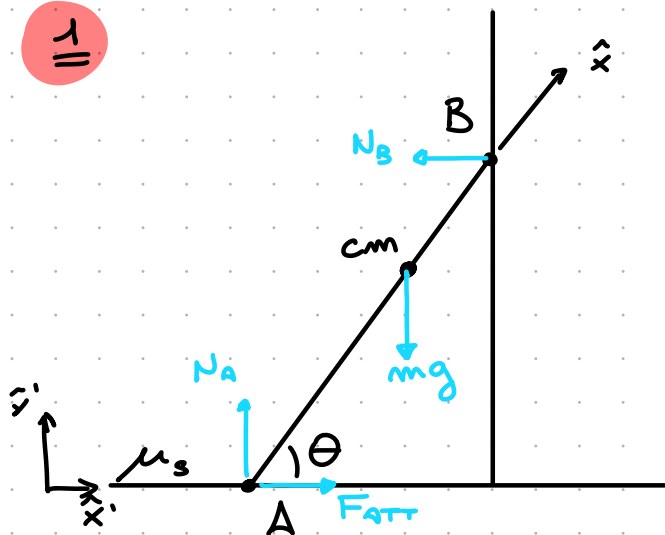
$$\Rightarrow a = \frac{GM r_0^2}{8GM r_0 - GM r_0} + \frac{r_0}{2} - \frac{1}{2} \frac{GM r_0^2}{8GM r_0 - GM r_0} = \frac{r_0}{7} + \frac{r_0}{2} - \frac{1}{2} \frac{r_0}{1}$$

$$\Rightarrow a = \frac{1}{2} \frac{r_0}{7} + \frac{r_0}{2} = \frac{r_0}{2} \left(\frac{1}{7} + 1 \right) = \frac{4r_0}{7}$$

L'ECCENTRICITÀ NON LA
SO

28 / 01 / 2021

1



$$\rho(x) = \rho_0 \left(1 + \frac{x}{l}\right)$$

CALCOLO $m = \int dm = \int_0^l \rho(x) dx = \rho_0 \int_0^l \left(1 + \frac{x}{l}\right) dx$

$$\Rightarrow m = \rho_0 \left(l + \frac{1}{2} l\right) = \frac{3}{2} \rho_0 l$$

$$x_{cm} = \frac{1}{m} \int x dm = \frac{1}{m} \int_0^l \rho_0 \left(x + \frac{x^2}{l}\right) dx = \frac{\rho_0}{m} \left(\frac{1}{2} l^2 + \frac{1}{3} l^2\right)$$

$$\Rightarrow x_{cm} = \frac{\rho_0}{m} \frac{5}{6} l^2 = \frac{1}{\frac{3}{2}} \frac{2}{3} \frac{5}{6} l^2 \rho_0 = \frac{10}{18} l = \frac{5}{9} l$$

USO LE 2 EQUAZIONI CARDINALI E IMponGO L'EQUILIBRIO

$$\begin{cases} 0 = F_{att} - N_B \\ 0 = N_A - mg \\ \text{per } A \quad 0 = -mg x_{cm} \sin\left(\frac{\pi}{2} + \theta\right) + N_B l \sin\theta \end{cases}$$

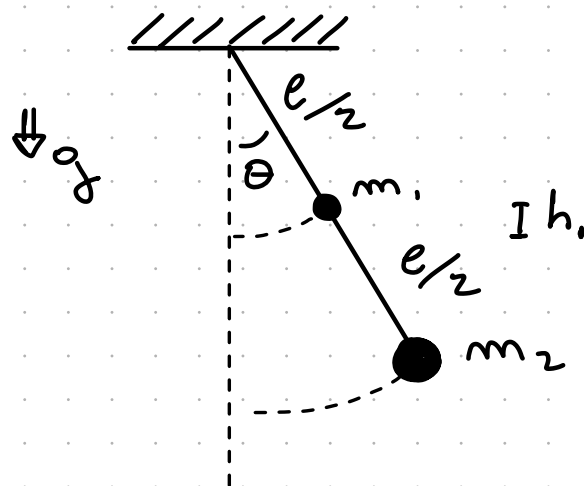
$$\begin{cases} N_B = F_{\text{ATT}} = mg\mu_s \\ N_A = mg \\ mg \frac{s}{g} l \cos\theta = N_B l \sin\theta \end{cases}$$

$$\Rightarrow mg\mu_s l \sin\theta = \frac{s}{g} l mg \cos\theta$$

$$\Rightarrow \tan\theta = \frac{s}{g\mu} \quad \Rightarrow \quad \theta_{\text{EQ MAX}} = \tan^{-1}\left(\frac{s}{g\mu}\right)$$

15/02/2022

1 l . $m_1, \frac{l}{2}$. $m_2 = 2m_1, l$. v_0



DOPO CHE SI STACCA m_2 POSSO USARE
LA CONSERVAZIONE DELL'ENERGIA PER
TROVARE $v_{\max}$

$$m_1 g h_1 = \frac{1}{2} m_1 v_{\max}^2$$

MA SERVE IL DISLIVELLO h_1 CHE
COMPIE m_1 .

TRATTANDO IL SISTEMA NEL SUO INSIEME POSSO CONSERVARE
L'ENERGIA

$$\frac{1}{2} I_1 \omega^2 + \frac{1}{2} I_2 \omega^2 = m_1 g h_1 + m_2 g h_2$$

PERO' NOTO CHE $h_1 = \frac{l}{2}(1 - \cos\theta)$, $h_2 = l(1 - \cos\theta)$

$$\begin{aligned} \frac{1}{2} m_1 \left(\frac{l}{2} \right)^2 \frac{v_0^2}{(\frac{l}{2})^2} + \frac{1}{2} m_2 l^2 \frac{v_0^2}{l^2} &= m_1 g \frac{l}{2} (1 - \cos\theta) + m_2 g l (1 - \cos\theta) \\ &= l(1 - \cos\theta) g \left(\frac{m_1}{2} + m_2 \right) \end{aligned}$$

$$\Rightarrow \frac{1}{2} m_1 v_0^2 + \frac{1}{2} m_2 v_0^2 = \frac{1}{2} \log(m_1 + 2m_2)(1 - \cos\theta)$$

$$\Rightarrow v_0^2 \frac{(m_1 + m_2)}{\log(m_1 + 2m_2)} = 1 - \cos\theta$$

DUNQUE TRAVO

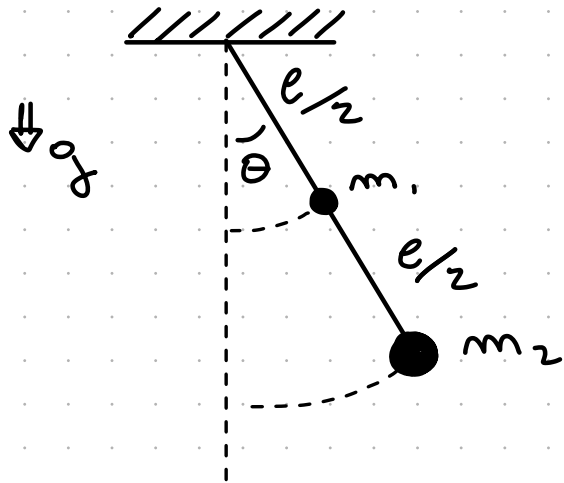
$$m_1 g h_1 = \frac{m}{2} v_{\max}^2 \Rightarrow m_1 g \frac{l}{2} (1 - \cos\theta) = \frac{m_1}{2} v_{\max}^2$$

$$g l \frac{m_1 + m_2}{m_1 + 2m_2} \frac{v_0^2}{\log} = v_{\max}^2$$

$$\Rightarrow v_{\max} = \sqrt{\frac{m_1 + m_2}{m_1 + 2m_2}} v_0 = \sqrt{\frac{3}{5}} v_0^2$$

15/02/2022

1 l . $m_1, \frac{l}{2}$. $m_2 = 2m_1, l$. v_0



CALCOLO

$$r_{cm} = \frac{m_1 \frac{l}{2} + m_2 l}{m_1 + m_2} = \frac{1}{2} \frac{m_1 l + 2m_2 l}{m_1 + m_2}$$

PER

UN

PENDOLO

FISICO

SO

$$T = 2\pi \sqrt{\frac{I}{(m_1 + m_2) g r_{cm}}}$$

con $I = m_1 \frac{l^2}{4} + m_2 l^2 = \frac{1}{4} l^2 (m_1 + 4m_2)$

$$\Rightarrow T = 2\pi \sqrt{\frac{\frac{1}{4} l^2 (m_1 + 4m_2)}{(m_1 + m_2) g} \cdot \frac{2(m_1 + m_2)}{l(m_1 + 2m_2)}}$$

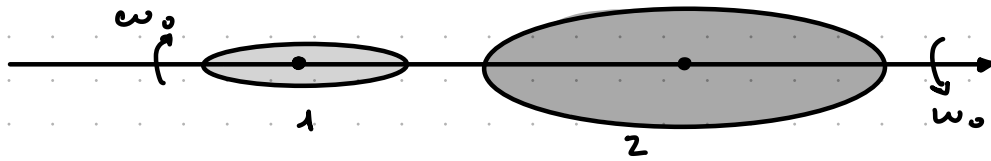
$$= 2\pi \sqrt{\frac{\frac{1}{2} l \frac{m_1 + 4m_2}{m_1 + 2m_2}}{g}} = 2\pi \sqrt{\frac{l}{2g} \frac{m_1 + 8m_2}{m_1 + 4m_2}}$$

$$\Rightarrow T = 2\pi \sqrt{\frac{gl}{10g}}$$

IL PRIMO PUNTO NON LO SO.

2 $m, R, 3R, \omega_0$

$$\text{HO} \quad I_{\text{disco}} = \frac{1}{4} m r^2$$



PARTE DAL PUNTO 6.

$$\begin{aligned} \Delta K &= \frac{1}{2} I_1 \omega_1^2 + \frac{1}{2} I_2 \omega_1^2 - \frac{1}{2} I_1 \omega_0^2 - \frac{1}{2} I_2 \omega_0^2 \\ &= \frac{1}{2} \cdot \frac{1}{4} m R^2 (9\omega_1^2 + \omega_1^2 - 9\omega_0^2 - \omega_0^2) \\ &= \frac{1}{8} m R^2 (10\omega_1^2 - 10\omega_0^2) \end{aligned}$$

$$\Rightarrow \Delta K = \frac{5}{4} m R^2 (\omega_1^2 - \omega_0^2)$$

$$K_{i,\omega} = \frac{1}{2} I_1 \omega_0^2 + \frac{1}{2} I_2 \omega_0^2 = \frac{1}{8} m R^2 (9\omega_0^2 + \omega_0^2) = \frac{5}{4} m R^2 \omega_0^2$$

$$\Rightarrow f = \frac{\Delta K}{K_{i,\omega}} = \frac{\omega_1^2 - \omega_0^2}{\omega_0^2}$$

$$K_{FIN} = \frac{1}{2} I_1 \omega_1^2 + \frac{1}{2} I_2 \omega_1^2 = \frac{1}{8} m l^2 \omega_1^2 (9+1) = \frac{5}{4} m l^2 \omega_1^2$$

$$\Rightarrow f = \frac{K_{FIN}}{K_{IN}} = \frac{\omega_1^2}{\omega_0^2}$$

LA FORZA D'ATTRITO $\vec{L}$ NON COMPIE LAVORO, PER CUI POSSO CONSERVARE

$$I_1 \omega_0 + I_2 \omega_0 = I_1 \omega_1 + I_2 \omega_1$$

$$\frac{1}{4} m l^2 \omega_0 - \frac{1}{4} m g l^2 \omega_0 = \frac{1}{4} m l^2 \omega_1 + \frac{1}{4} m g l^2 \omega_1$$

$$-8 \omega_0 = 10 \omega_1 \quad \Rightarrow \quad \boxed{\omega_1 = -\frac{4}{5} \omega_0}$$

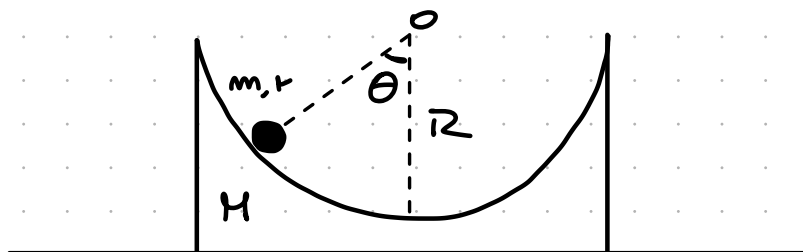
DUNQUE

$$f = \frac{16}{25}$$

$$\phi = -\frac{9}{25}$$

22/06/2021

1. $R, H, m, r \ll R, \theta_0$



HO $I_{\text{SFERA}} = \frac{2}{5} m r^2$

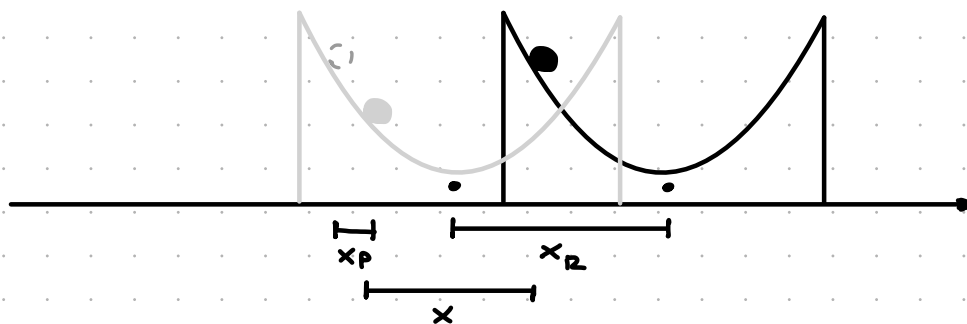
E DAL MOTO DI PURO ROTOLAMENTO

$v_{\text{cm}}^s = \omega_s r$

NON HO FORZE ORIZZONTALI $\Rightarrow$ CONSERVO LA QUANTITÀ
E POSSO TRATTARE IL PROBLEMA COME UN URTO
CONSERVARE L'ENERGIA.

MOMENTO ANGOLARE
DI MOTO LUNGO $\hat{x}$
ELASTICO E

PER TROVARE LO SPOSTAMENTO DELLA RAMPA



$x = x_p - x_R$ MA $x_R < 0 \Rightarrow x = x_R + x_p$

CONSERVANDOSI $p_x \Rightarrow x_{\text{cm}}$ NON CAMBIA E PONENDO L'ORIGINE
 $\equiv$ CON IL CM DELLA RAMPA SI HA

$$x_{cm}^0 = \frac{m R \sin \theta_0}{m+M}$$

$$x_{cm}^1 = \frac{M s + m(s - R \sin \theta)}{m+M}$$

$$\Rightarrow \frac{m R \sin \theta_0}{m+M} = \frac{M s + m(s - R \sin \theta_0)}{m+M}$$

$$\Rightarrow \frac{m R \sin \theta_0}{m+M} = \frac{M s + m s}{m+M} - \frac{m R \sin \theta_0}{m+M}$$

$$\Rightarrow s = \frac{2 m R \sin \theta_0}{m+M}$$

ORA CONSERVO ENERGIA (MOMENTO ANGOLARE) ED IMPULSO LUNGO X:

$$\begin{cases} m g (R - R \cos \theta_0) = \frac{1}{2} I \omega^2 + \frac{M}{2} V^2 \\ 0 = m v + M V \end{cases}$$

PERO' I DELLA SFERETTA NON LO CALCOLO RISPETTO UN ASSE PASSANTE PER IL CM MA RISPETTO O DISTANTE $R+r$ DAL CM

$$\Rightarrow I = I_{cm} + m(R+r)^2 = \frac{2}{5} m r^2 + m(R+r)^2$$

$$\Rightarrow \begin{cases} mgR(1 - \cos\theta_0) = \frac{1}{2} I \left(\frac{v_{cm}}{r} \right)^2 + \frac{M}{2} v^2 \\ v = - \frac{m}{M} v \end{cases}$$

$$\Rightarrow mgR(1 - \cos\theta_0) = \frac{1}{2} \left(\frac{2}{5} mr^2 + mR^2 + mr^2 + 2mRr \right) \frac{v^2}{r^2} + \frac{m^2}{2M} v^2$$

$$\Rightarrow 2gR(1 - \cos\theta_0) = \frac{2}{5} v^2 + \frac{R^2}{r^2} v^2 + v^2 + 2 \frac{R}{r} v^2 + \frac{m}{M} v^2$$

$$\Rightarrow 10gR(1 - \cos\theta_0) \sim 2v^2 + 5v^2 + 5 \frac{m}{M} v^2$$

$$\Rightarrow 10gR(1 - \cos\theta_0) = \left(7 + 5 \frac{m}{M} \right) v^2$$

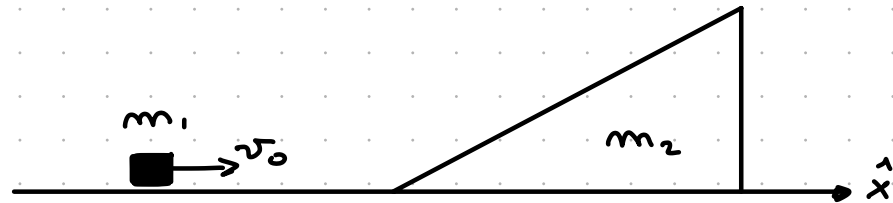
$$\Rightarrow v^2 \frac{7M + 5m}{M} = 10gR(1 - \cos\theta_0)$$

$$\Rightarrow v = \sqrt{\frac{10MRgR(1 - \cos\theta_0)}{7M + 5m}}$$

15/07/2021

1

m_1, v_0, m_2



(i) LE SITUAZIONE A È ASSIMILABILE AD UN URTO ELASTICO, I DUE CORPI URTANO E PROCEDONO UNO SULL'ALTRO CON VELOCITÀ RELATIVA NON UGUALE PER LA SITUAZIONE b.

ESSENDO APPUNTO GLI URTI ELASTICI POSSO CONSERVARE L'ENERGIA E LA QUANTITÀ DI MOTO (LUNGO X)

$$a) \begin{cases} \frac{m_1}{2} v_0^2 = m_1 g h + \frac{m_1 + m_2}{2} v_a^2 \\ m_1 v_0 = (m_1 + m_2) v_a \end{cases}$$

PERCHÉ QUANDO m_1 RAGGIUNGE h È FERMO E $m_1 + m_2$ SI MUOVONO INSIEME A VELOCITÀ v

$$\Rightarrow v_a = \frac{m_1}{m_1 + m_2} v_0$$

$$\frac{m_1}{2} v_0^2 = m_1 g h + \frac{m_1 + m_2}{2} \frac{m_1^2}{(m_1 + m_2)^2} v_0^2$$

$$\frac{m_1}{2} v_0^2 = m_1 g h + \frac{m_1^2}{2(m_1 + m_2)} v_0^2$$

$$\frac{v_0^2}{2} \left(1 - \frac{m_1}{m_1 + m_2} \right) = g h$$

$$\Rightarrow \boxed{h = \frac{m_2 v_0^2}{2g(m_1 + m_2)}}$$

b) RIFACCIO LA STESSA COSA

$$\begin{cases} \frac{m_1}{2} v_0^2 = \frac{m_1}{2} v_{1b}^2 + \frac{m_2}{2} v_{2b}^2 \\ m_1 v_0 = m_1 v_{1b} + m_2 v_{2b} \end{cases}$$

$$\begin{cases} \frac{m_1}{2} v_0^2 = \frac{m_1}{2} \left(v_0^2 + \frac{m_2^2}{m_1^2} v_{2b}^2 - 2 \frac{m_2}{m_1} v_0 v_{2b} \right) + \frac{m_2}{2} v_{2b}^2 \\ v_{1b} = v_0 - \frac{m_2}{m_1} v_{2b} \end{cases}$$

$$\Rightarrow \frac{m_2^2}{2m_1} v_{2b}^2 - m_2 v_0 v_{2b} + \frac{m_2}{2} v_{2b}^2 = 0$$

$$\Rightarrow \frac{m_2}{2m_1} v_{2b}^2 - v_0 + \frac{1}{2} v_{2b} = 0$$

$$\Rightarrow \frac{1}{2} v_{2b} \left(1 + \frac{m_2}{m_1} \right) = v_0$$

$$\Rightarrow \boxed{v_{2b} = \frac{2m_1 v_0}{m_1 + m_2}}$$

DONQUE

$$v_{1b} = v_0 - \frac{m_2}{m_1} v_{2b} = v_0 - \frac{2m_2}{m_1 + m_2} v_0 = \boxed{\frac{m_1 - m_2}{m_1 + m_2} v_0}$$

25/01/2022

1

ORBITA $r_1 \Rightarrow$ VELOCITÀ ORBITALE

$$v = \sqrt{\frac{GM}{r_1}}$$

ORBITA GEOSTAZIONARIA $\Rightarrow \omega = \omega_{\text{TERRA}}$

$$\Rightarrow v_3 = \omega r_3 = \omega_T r_3 = \frac{2\pi}{T} r_3$$

MA QUANDO SONO IN ORBITA $r_2 \Rightarrow v_3 = \sqrt{\frac{GM}{r_2}}$

$$\Rightarrow \sqrt{\frac{GM}{r_2}} = \frac{2\pi}{T} r_2$$

$$\Rightarrow \frac{GM}{r_2} = \frac{4\pi^2}{T^2} r_2^2 \Rightarrow r_2^3 = \frac{GMT^2}{4\pi^2} \Rightarrow r_2 = \sqrt[3]{\frac{GMT^2}{4\pi^2}}$$

QUELLO CHE SUCCEDDE È

v
ORBITA CIRCOLARE
 r_1

$\Rightarrow$ v_1, v_2
ORBITA ELLITTICA
 r_1, r_2
($v_1 \neq v$)

$\Rightarrow$ v_3
ORBITA CIRCOLARE
= ORBITA GEOSTAZIONARIA
 r_2

SULL'ORBITA ELLITTICA

$$\begin{cases} m v_1 r_1 = m v_2 r_2 \\ \frac{m}{2} v_1^2 - G \frac{Mm}{r_1} = \frac{m}{2} v_2^2 - G \frac{Mm}{r_2} \end{cases}$$

MI SERVE v_2 PER TROVARE ΔK

$$\begin{cases} v_1 = \frac{r_2}{r_1} v_2 \\ \frac{m}{2} \left(\frac{r_2^2}{r_1^2} - 1 \right) v_2^2 = G M m \left(\frac{1}{r_1} - \frac{1}{r_2} \right) \end{cases}$$

$$v_2^2 \left(\frac{1}{r_1} - \frac{1}{r_2} \right) \left(\frac{1}{r_1} + \frac{1}{r_2} \right) r_2^2 = 2 G M \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

$$v_2^2 \left(\frac{r_1 + r_2}{r_1 r_2} \right) r_2^2 = 2 G M$$

$$v_2 = \sqrt{\frac{2 G M r_1}{r_2 (r_1 + r_2)}}$$

$$\Rightarrow \frac{\Delta K}{m} = \frac{1}{2} (v_3^2 - v_2^2)$$

2

 l , UNIFORME,

LA POSIZIONE DEL CM LUNGO L'ASTA È AD $\frac{l}{2}$ E SE IPOTIZZO IL PERNO O AD UN'ALTEZZA h DA TERRA CONSERVANDO L'ENERGIA HO

$$mgh = mg\left(h - \frac{l}{2}\right) + \frac{1}{2} I \omega_0^2$$

$$\Rightarrow \frac{1}{2} I \omega_0^2 = mg \frac{l}{2} \quad \text{con} \quad I = I_{\text{ASTA}} = \frac{1}{3} m l^2$$

$$\Rightarrow \omega_0 = \sqrt{\frac{mgl}{I}} = \sqrt{\frac{mgl}{\frac{1}{3} m l^2}} = \sqrt{\frac{3g}{l}}$$

SI SPEZZA L'ASTA (URTO ANELASTICO) CONSERVO $\vec{p}, \vec{L}$ IMMEDIATAMENTE PRIMA E DOPO

$$\begin{cases} m v_{\text{cm}} = \frac{m}{2} v_1 + \frac{m}{2} v_2 \\ I \omega_0 = I_1 \omega_1 + I_2 \omega_2 \end{cases}$$

17/02/2022

1 $\ell, \lambda = cx$

$$m = \int dm = \int \lambda dx = c \left. \frac{x^2}{2} \right|_0^\ell = c \frac{\ell^2}{2}$$

$$I = \int b^2 dm \quad \text{con} \quad b = x$$

$$\Rightarrow I = \int x^2 dm = \int_0^\ell cx^3 dx = \frac{1}{4} c \ell^4 = \frac{1}{2} m \ell^2$$

$$x_{\text{cm}} = \frac{1}{m} \int x dm = \frac{1}{m} \int_0^\ell cx^2 dx = \frac{1}{3} \frac{c}{m} \ell^3 = \frac{2}{3} \frac{1}{m} \frac{1}{2} c \ell^3 = \frac{2}{3} \ell$$

$$I = I_{\text{cm}} + m \frac{4}{9} \ell^2 \Rightarrow I_{\text{cm}} = I - \frac{4}{9} m \ell^2$$

$$\Rightarrow I_{\text{cm}} = \frac{1}{2} m \ell^2 - \frac{4}{9} m \ell^2 = \left(\frac{1}{2} - \frac{4}{9} \right) m \ell^2 = \underline{\underline{\frac{1}{18} m \ell^2}}$$

OGNI dm SENTE UNA FORZA
CENTRIFUGA

$$dF_c = dm \times \omega^2$$

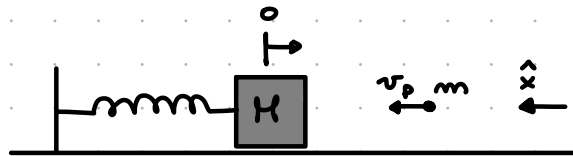
$$\Rightarrow F_c = \omega^2 \int x dm = \omega^2 c \frac{\ell^3}{3} \Rightarrow N = -\frac{2}{3} m \ell \omega^2$$

DALLA TEORIA SO CHE
 I_{cm} È IL MINIMO PER
CUI SI SEMBRA CORRETTO
IL RISULTATO

1

M, A, ω

URTO ISTANTANEO E COMPLETAMENTE ANELASTICO.



- DETERMINA:
- MODULO DELLA VELOCITÀ FINALE
 - ENERGIA DISSIPATA
 - L'AMPIEZZA DELLE NUOVE OSCILLAZIONI

URTO COMPLETAMENTE ANELASTICO CONSERVO LA QUANTITÀ DI MOTO ($v(t)$ È LA VELOCITÀ DI M)

$$x(t) = A \sin(\omega t) \Rightarrow v(t) = \omega A \cos(\omega t)$$

$$M v(t) + m v_p = (m + M) \tilde{v}$$

$$\Rightarrow \tilde{v} = \frac{m v_p + M \omega A \cos(\omega t)}{m + M}$$

L'URTO AVVIENE NELLA POSIZIONE DI RIPOSO $\Rightarrow x(t) = 0 \Rightarrow \sin(\omega t) = 0$
 $\Rightarrow \cos(\omega t) = \pm 1 \Rightarrow$ HO $\tilde{v}(t) = -v(t) \hat{x} \Rightarrow \cos(\omega t) = -1$

$$\Rightarrow v(t) = -A\omega$$

QUINDI

$$\tilde{v} = \frac{m v_p - A \omega M}{m + M}$$

L'ENERGIA MECCANICA PRIMA E DOPO L'URTO È (IN UN GENERICO t)

$$E_0 = \frac{1}{2} M v(t)^2 + \frac{1}{2} k x(t)^2 + \frac{1}{2} m v_p^2$$

$$E_1 = \frac{1}{2} (M+m) \tilde{v}^2 + \frac{1}{2} k \tilde{x}(t)^2$$

NEL PUNTO DI RIPOSO ($x(t)=0$, $v(t) = -A\omega$)

$$\begin{cases} E_0 = \frac{1}{2} M A^2 \omega^2 + \frac{1}{2} m v_p^2 \\ E_1 = \frac{1}{2} (m+M) \frac{(m v_p - M A \omega)^2}{(m+M)^2} = \frac{1}{2} \frac{m^2 v_p^2 + M^2 A^2 \omega^2 - 2 M m A \omega v_p}{m+M} \end{cases}$$

L'ENERGIA DISSIPATA È

$$\begin{aligned} \Delta E = E_1 - E_0 &= \frac{1}{2} \frac{1}{m+M} (m^2 v_p^2 + M^2 A^2 \omega^2 - 2 M m A \omega v_p) - \frac{1}{2} M A^2 \omega^2 - \frac{1}{2} m v_p^2 \\ &= \frac{1}{2(m+M)} (m^2 v_p^2 + M^2 A^2 \omega^2 - 2 M m A \omega v_p) - \frac{1}{2(m+M)} (M^2 A^2 \omega^2 + m M A^2 \omega^2 + m^2 v_p^2 + m M v_p^2) \end{aligned}$$

$$\Delta E = \frac{-mM}{2(m+M)} (2A\omega v_p + A^2 \omega^2 + v_p^2)$$

POSSO USARE LA CONSERVAZIONE DELL'ENERGIA PER TROVARE $\tilde{A}$

$$\frac{1}{2} (M+m) \tilde{v}^2 = \frac{1}{2} k \tilde{A}^2$$

$$\Rightarrow \tilde{A} = \sqrt{\frac{M+m}{k}} \cdot \tilde{v}$$

$$\Rightarrow \tilde{A} = \sqrt{\frac{M+m}{k}} \frac{mv_p - A\omega M}{M+m} = \boxed{\frac{mv_p - A\omega M}{\sqrt{k(M+m)}}}$$

NOTA

PUOI SCRIVERE $k = m\omega^2$ CHE RIMANE COSTANTE DIPENDENDO SOLO DALLE PROPRIETÀ DELLA MOLLA

$$\Rightarrow k = \text{cost} = M\omega^2 = (m+M)\tilde{\omega}^2$$

$$\Rightarrow \tilde{\omega} = \sqrt{\frac{M}{M+m}} \omega$$

HA SO CHE NEL PUNTO DI MASSIMO $v(t) = A\omega$

$$\Rightarrow \tilde{v} = \tilde{A} \tilde{\omega}$$

$$\Rightarrow \frac{mv_p - A\omega M}{M+m} = \sqrt{\frac{M}{M+m}} \omega \tilde{A}$$

$$\Rightarrow \tilde{A} = \frac{mv_p - \hbar \omega}{\sqrt{M+m} \cdot \omega \sqrt{M}}$$

ANALOGA

A

QUELLA

TROVATA

$$(\kappa = M\omega^2)$$

2

$$m_0, R, \omega_0, \frac{dm}{dt} = k e^{t/\tau}$$

DETERMINA :

- $\omega(t)$
- $P(t)$

HO

$$\mathcal{M}_{\text{EXT}} = \frac{dL}{dt} = \frac{d}{dt} (I \omega)$$

E POSSO CONSERVARE IL MOMENTO ANGOLARE

$$I_0 \omega_0 = I(t) \omega(t)$$

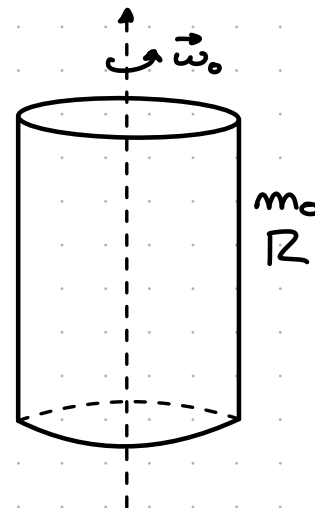
con $I_0 = m_0 R^2$ (CILINDRO CAVO) E $I(t) = I_0 + \frac{1}{2} m(t) R^2$

$$m_0 R^2 \omega_0 = \left(m_0 R^2 + \frac{1}{2} m(t) R^2 \right) \omega(t)$$

con $m(t) = m_0 + \int_0^t k e^{t'/\tau} dt' = m_0 + k\tau (e^{t/\tau} - 1)$

$$\Rightarrow 2m_0 \omega_0 = (2m_0 + k\tau (e^{t/\tau} - 1)) \omega(t)$$

$$\Rightarrow \omega(t) = \frac{2m_0 \omega_0}{2m_0 + k\tau (e^{t/\tau} - 1)}$$



PER LA POTENZA USO

$$P = \vec{M}_{\text{EXT}} \cdot \vec{\omega}(t) \quad \text{MA VOGLIO } \omega(t) \equiv \omega_0 \quad \forall t$$

$$= M_{\text{EXT}} \omega_0 = \frac{d}{dt} (I \omega) \omega_0 = \frac{dI}{dt} \omega_0^2$$

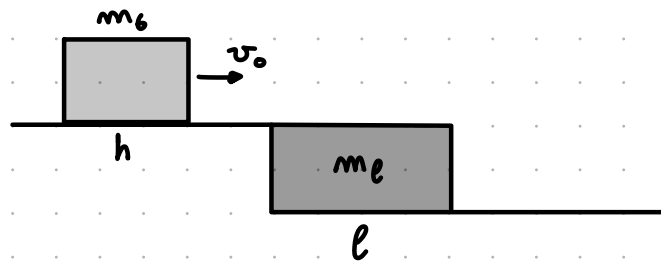
con

$$I(t) = m_0 R^2 + \frac{1}{2} m(t) R^2 \quad \Rightarrow \quad \frac{dI}{dt} = \frac{1}{2} \frac{dm}{dt} R^2$$

$$\Rightarrow P = \frac{1}{2} \frac{dm}{dt} R^2 \omega_0^2 = \boxed{\frac{\kappa R^2 \omega_0^2}{2} e^{t/\tau}}$$

18/07/2022

111



HO UN URTO $\rightarrow$ CONSERVO L'IMPULSO TRA PRIMA E QUANDO SI MUOVONO INSIEME

$$m_b v_0 = (m_e + m_b) v \Rightarrow v = \frac{m_b}{m_e + m_b} v_0$$

PER ΔK HO

$$\begin{aligned} \Delta K &= \frac{m_b + m_e}{2} v^2 - \frac{m_b}{2} v_0^2 = \frac{m_b + m_e}{2} \frac{m_b^2}{(m_e + m_b)^2} v_0^2 - \frac{m_b}{2} v_0^2 \\ &= \frac{m_b^2}{2(m_e + m_b)} v_0^2 - \frac{m_b}{2} v_0^2 = \frac{m_b}{2} v_0^2 \left(\frac{m_b}{m_e + m_b} - 1 \right) = - \frac{m_b m_e}{2(m_e + m_b)} v_0^2 \end{aligned}$$

PER TROVARE μ FACCIO:

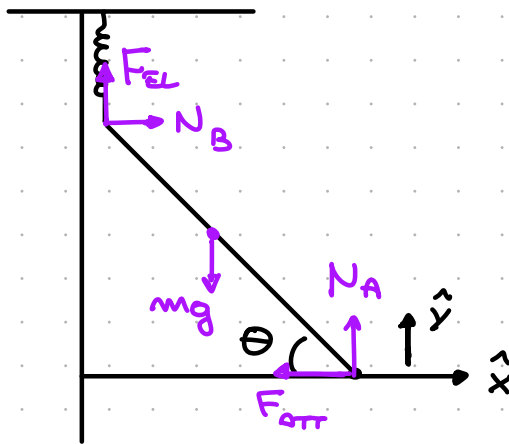
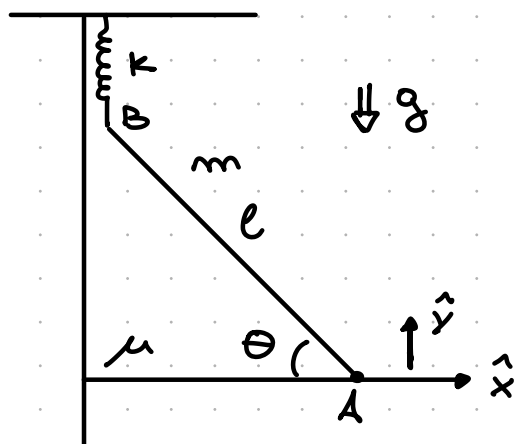
$$W = W_c + W_{nc} = \Delta K \Rightarrow W_{nc} = \Delta K, \quad W_{nc} = -F_{att} l$$

$$\text{con } F_{att} = m_b g \mu \Rightarrow -\mu m_b g l = \Delta K$$

$$\Rightarrow \mu = - \frac{1}{m_b g l} \frac{-m_b m_e}{2(m_e + m_b)} v_0^2 = \frac{m_e v_0^2}{2 g l (m_e + m_b)}$$

!!

2



ESSENDO UN PROBLEMA DI STATICA DEVO IMPORRE CHE LE
 RISULTANTI DELLE FORZE SIANO NULLE E CHE IL
 MOMENTO RISULTANTE SIA NULLO.

$$\begin{aligned} \hat{x} & \left\{ \begin{aligned} -F_{ATT} + N_B &= 0 \\ k\delta - mg + N_A &= 0 \end{aligned} \right. \\ \text{Polo A: } \hat{y} & \left\{ \begin{aligned} mg \frac{l}{2} \sin\left(\frac{\pi}{2} - \theta\right) - N_B l \sin\theta - k\delta l \sin\left(\frac{\pi}{2} + \theta\right) &= 0 \end{aligned} \right. \end{aligned}$$

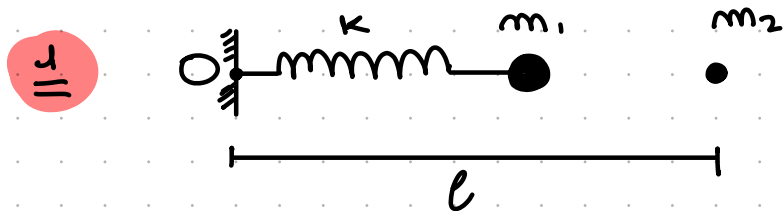
DALLA 2° E 3°: $\left(\sin\left(\frac{\pi}{2} \pm \theta\right) = \cos\theta \right)$

$$\boxed{N_A = mg - k\delta} \quad \Rightarrow \quad N_B = \mu (mg - k\delta)$$

$$\Rightarrow \frac{mg}{2} \cos\theta - N_B \sin\theta - k\delta \cos\theta = 0$$

$$\Rightarrow \boxed{N_B = \left(\frac{mg}{2} - k\delta \right) \cot\theta}$$

14/09/2022



INIZIALMENTE HO $\Delta U_0 = \frac{1}{2} k h^2$
CONSERVO L'ENERGIA (PRIMA DELL'URTO)

$$\frac{1}{2} k h^2 = \frac{m_1}{2} v^2 + \frac{1}{2} k \Delta l^2$$

ALLUNGAMENTO = $(d - l_0)$

$$\Rightarrow m_1 v^2 = k (h^2 - (d - l_0)^2)$$

$$\Rightarrow v = \sqrt{\frac{k}{m_1} (h^2 - (d - l_0)^2)}$$

CON L'URTO CONSERVO LA QUANTITÀ DI MOTO E L'ENERGIA
(IMMEDIATAMENTE DOPO L'URTO)

$$\begin{cases} m_1 v = m_1 \tilde{v} + m_2 v_2 \\ \frac{1}{2} k (d - l_0)^2 + \frac{m_1}{2} v^2 = \frac{1}{2} k (d - l_0)^2 + \frac{m_1}{2} \tilde{v}^2 + \frac{m_2}{2} v_2^2 \end{cases}$$

$$\begin{cases} m_1 v = m_1 \tilde{v} + m_2 v_2 \Rightarrow v_2 = \frac{m_1}{m_2} (v - \tilde{v}) \\ m_1 v^2 = m_1 \tilde{v}^2 + m_2 v_2^2 \end{cases}$$

$$\Rightarrow m_1 v^2 = m_1 \tilde{v}^2 + \frac{m_1^2}{m_2} (v^2 + \tilde{v}^2 - 2v\tilde{v})$$

$$\Rightarrow \frac{m_1}{m_2} (m_1 + m_2) \tilde{v}^2 - 2 \frac{m_1^2}{m_2} v\tilde{v} + \frac{m_1}{m_2} (m_1 - m_2) v^2 = 0$$

$$\Rightarrow (m_1 + m_2) \tilde{v}^2 - 2m_1 v\tilde{v} + (m_1 - m_2) v^2 = 0$$

$$\tilde{v}_{1,2} = \frac{2m_1 v \pm \sqrt{4m_1^2 v^2 - 4(m_1^2 - m_2^2) v^2}}{2(m_1 + m_2)}$$

USO $m_2 = 3m_1$

$$\tilde{v} = \frac{m_1 v \pm \sqrt{m_1^2 v^2 + 8m_1^2 v^2}}{4m_1} = \frac{v \pm 3v}{4} = \begin{cases} v \\ -\frac{v}{2} \end{cases}$$

LA PRIMA $\tilde{v}$ LA AVREI SE NON CI FOSSE URTO.

$$\Rightarrow \tilde{v} = -\frac{v}{2}$$

CONSERVO Δ NUOVO L'ENERGIA SU m_1

$$\frac{m_1}{2} \tilde{v}^2 + \frac{1}{2} K (d - l_0)^2 = \frac{m_1}{2} v_F^2 + \frac{1}{2} K \delta_{\text{MAX}}^2$$

LA VOGLIO
= 0

$$\Rightarrow \delta_{\text{MAX}}^2 = \frac{m_1}{K} \tilde{v}^2 + (d - l_0)^2$$

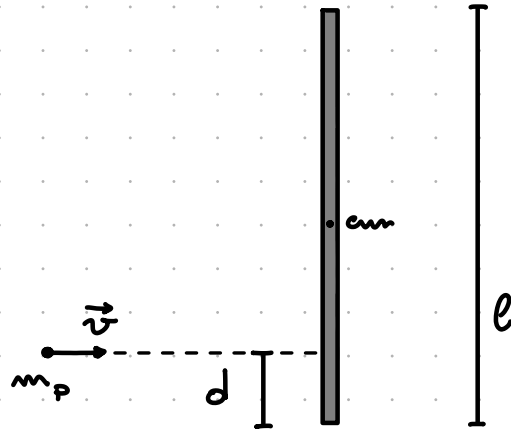
$$\Rightarrow \delta_{\text{MAX}} = \sqrt{\frac{m_1}{K} \frac{v^2}{4} + (d - l_0)^2} = \sqrt{\frac{m_1}{4K} \frac{K}{m_1} (h^2 - (d - l_0)^2) + (d - l_0)^2}$$

$$\delta_{\text{MAX}} = \sqrt{\frac{h^2}{4} + \frac{3}{4} (d - l_0)^2}$$

2

URTO

ELASTICO

CONSERVO
CHIEDENDOENERGIA
CHEE
DOPOMOMENTO
L'URTOANGOLARE
 m_p SIA FERMA

$$\begin{cases} \frac{m_p}{2} v^2 = \frac{1}{2} I_{ASTA} \omega^2 \\ m_p v d = I_{ASTA} \omega \end{cases}$$

(POLO cm)

CON

$$I_{ASTA} = \frac{1}{12} m l^2$$

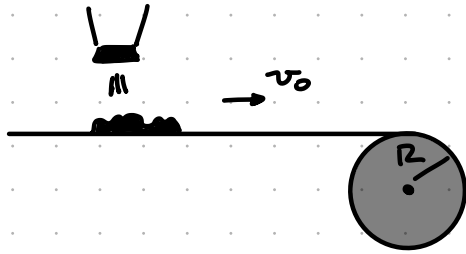
$$\begin{cases} m_p v^2 = I_{ASTA} \frac{m_p^2 v^2 d^2}{I_{ASTA}^2} \\ \omega = \frac{m_p v d}{I_{ASTA}} \end{cases} \Rightarrow 1 = \frac{m_p d^2}{I_{ASTA}}$$

$$\Rightarrow m_p = \frac{I_{ASTA}}{d^2} = \frac{m l^2}{12 d^2}$$

24/01/2023

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$$r = \frac{dm}{dt} \quad , \quad v_0$$



$$H_0 \quad I_{\text{RUOTA}} = \frac{1}{4} m R^2$$

UTILIZZO LA 2° EQUAZIONE CARDINALE

$$\begin{aligned} \vec{M} &= \frac{d}{dt} \vec{L} = \frac{d}{dt} (m v_0 R + I_{\text{DISCO}} \omega) \\ &= \frac{dm}{dt} v_0 R \end{aligned}$$

$$\Rightarrow M_{\text{TOT}} = R F_{\text{TOT}} = \frac{dm}{dt} v_0 R \Rightarrow F_{\text{TOT}} = \frac{dm}{dt} v_0 = r v_0$$

$$P_{\text{DISP}} = P - f P = P(1 - f)$$

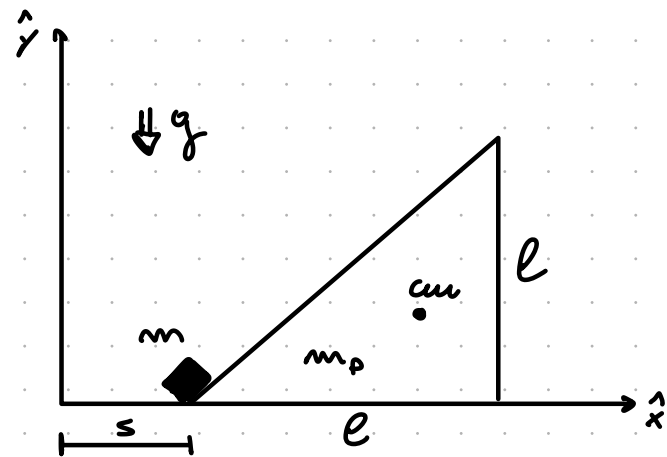
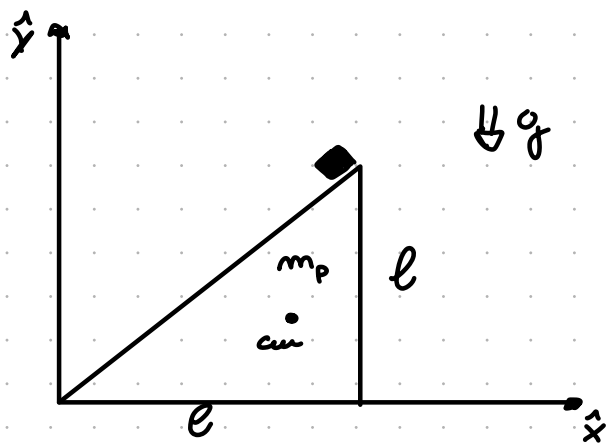
$$\Rightarrow P = \frac{P_{\text{DISP}}}{1 - f}$$

DEVO TROVARE P_{DISP}

$$P_{\text{DISP}} = F_{\text{TOT}} \cdot v = r v_0^2$$

$$\Rightarrow P = \frac{r v_0^2}{1 - f}$$

2



$$x_{cm} = \frac{\int x dm}{m_p}$$

$$\text{con } dm = \sigma dx dy$$

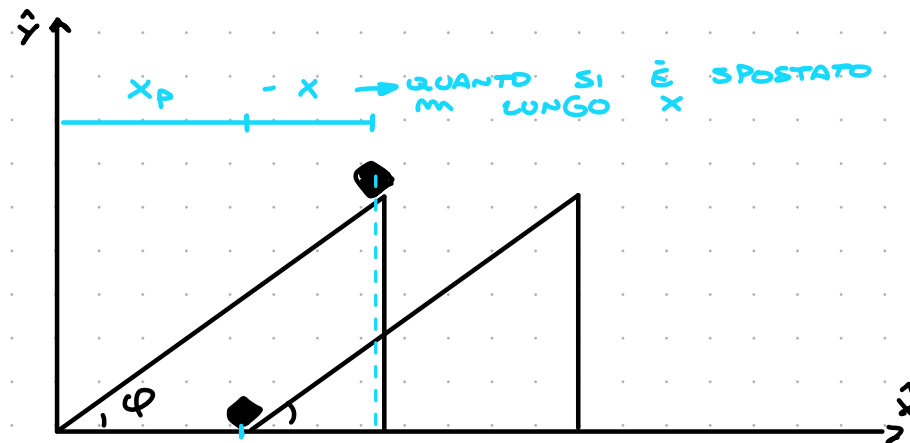
$$x \in [0, l], y \in [0, x]$$

$$\Rightarrow x_{cm} = \frac{\sigma}{m_p} \int_0^l x dx \int_0^x dy = \frac{\sigma}{m_p} \int_0^l x^2 dx = \frac{\sigma l^3}{3m_p}$$

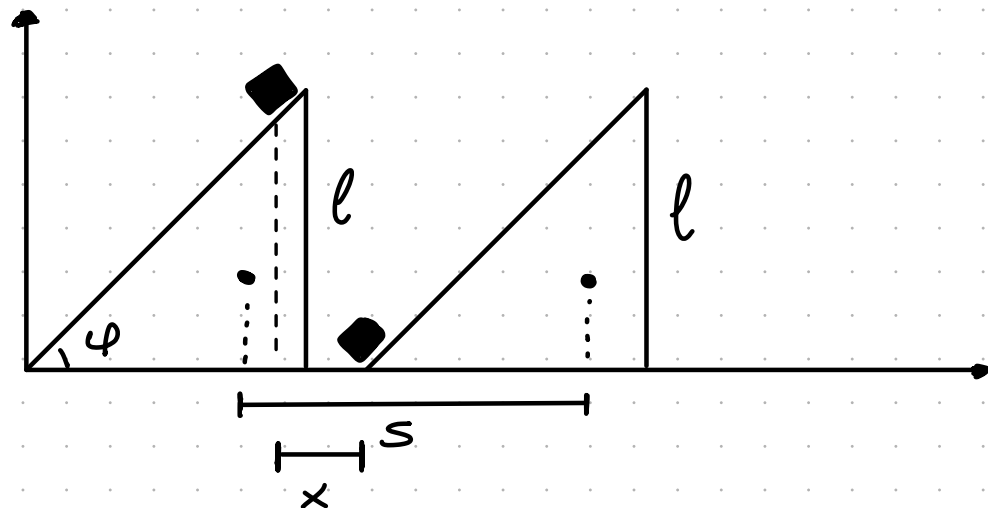
$$\text{HO } m = \sigma \frac{l^2}{2} \Rightarrow x_{cm} = \frac{\sigma l^3}{3\sigma \frac{l^2}{2}} = \frac{2l}{3}$$

QUANDO MUOVERSI, CONSERVO (LUNGO x)
 m POSSO TRATTARE ED ENERGIA
 SCENDERE IL TUTTO
 IL PIANO COME UN URTO ELASTICO,
 COMINCIA A

$$\begin{cases} 0 = m_p v_p - m v_x \\ mgl = \frac{m}{2} v^2 + \frac{m_p}{2} v_p^2 \end{cases}$$



POTENDO CONSERVARE p_x SO CHE x_{cm} NON CAMBIA E LO POSSO USARE PER TROVARE S.



$$x_{cm}^0 = \frac{m(l - \frac{2}{3}l)}{m_p + m} = \frac{l}{3} \frac{m}{m_p + m}$$

$$x_{cm}^1 = \frac{m_p s + m(s + (\frac{l}{3} - l))}{m_p + m}$$

$$\Rightarrow x_{cm}^0 = x_{cm}^1 \Rightarrow \frac{m \frac{l}{3}}{m_p + m} = \frac{m \frac{l}{3} - m l + s(m_p + m)}{m_p + m}$$

$$\Rightarrow \boxed{s = \frac{m}{m_p + m} l}$$

CONTINUO 1 CONTI PER v_x :



$$\begin{cases} v_p = \frac{m}{m_p} v_x \\ mgl = \frac{m}{2} v^2 + \frac{m_p}{2} v_p^2 \end{cases}$$

HA HO $v_x = v_y = \frac{\sqrt{2}}{2} v$

$$\begin{cases} v_p = \frac{m}{m_p} \frac{\sqrt{2}}{2} v \\ 2mgl = mv^2 + m_p \frac{m^2}{m_p^2} \frac{2}{4} v^2 \end{cases}$$

$$\Rightarrow 2mgl = mv^2 + \frac{m^2}{2m_p} v^2$$

$$\Rightarrow v^2 \underbrace{\left(1 + \frac{m}{2m_p} \right)}_{\frac{2m_p + m}{2m_p}} = 2gl$$

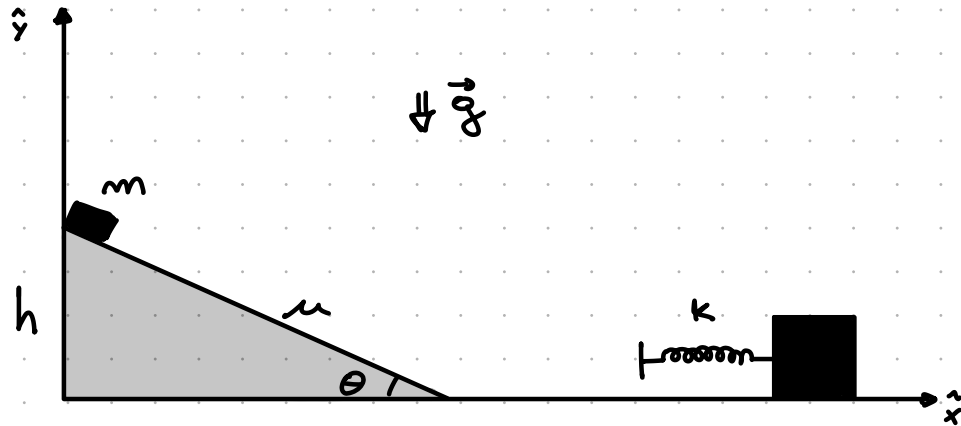
$$\Rightarrow v = \sqrt{\frac{4m_p gl}{2m_p + m}}$$

avina

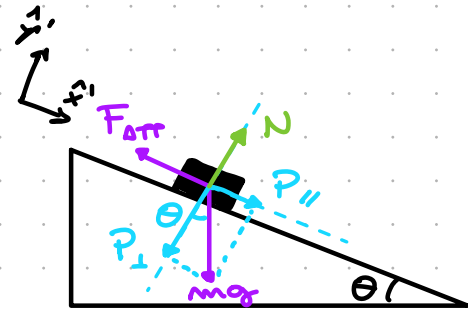
$$v_x = \frac{\sqrt{2}}{2} v = \sqrt{\frac{2m_p gl}{2m_p + m}}$$

16/02/2023

1



GUARDO IL PIANO



$$\hat{y}' : -mg \cos \theta + N = 0$$

$$\Rightarrow N = mg \cos \theta$$

$$\Rightarrow F_{att} = \mu N \Rightarrow \vec{F}_{att} = -\mu mg \cos \theta \hat{x}$$

QUINDI

$$W_{att} = \vec{F}_{att} \cdot \vec{\ell} = -\mu mg \cos \theta \ell$$



$$h = \ell \sin \theta$$

$$\Rightarrow W_{att} = -\mu mg h \frac{\cos \theta}{\sin \theta}$$

UNGO $\hat{x}'$ HO

$$\hat{x}' : mg \sin \theta - F_{att} = ma_{x'} \Rightarrow a_{x'} = g \sin \theta - \mu g \cos \theta$$

$$\Rightarrow a_{x'} = \frac{dv_{x'}}{dt} \Rightarrow v_{x'}(t) = (g \sin \theta - \mu g \cos \theta) t$$

$$\Rightarrow x'(t) = (g \sin \theta - \mu g \cos \theta) \frac{1}{2} t^2$$

ARRIVO AL FONDO $\Rightarrow x'(\tau) \equiv \ell = \frac{h}{\sin \theta}$

$$\frac{\ell}{\sin \theta} = (g \sin \theta - \mu g \cos \theta) \frac{1}{2} \tau^2$$

$$\Rightarrow \tau = \sqrt{\frac{2\ell}{g \sin^2 \theta - \mu g \cos \theta \sin \theta}}$$

$$\Rightarrow v(\tau) = (g \sin \theta - \mu g \cos \theta) \sqrt{\frac{2\ell}{(g \sin \theta - \mu g \cos \theta) \sin \theta}}$$

$$= \sqrt{\frac{2\ell(g \sin \theta - \mu g \cos \theta)}{\sin \theta}}$$

QUANDO ARRIVA GIÙ

CONSERVO L'ENERGIA FINO ALLA MASSIMA COMPRESSIONE DELLA MOLLA

$$\frac{m}{2} v^2(\tau) = \frac{1}{2} k \delta^2 \Rightarrow \delta = \sqrt{\frac{m v^2}{k}}$$

$$\Rightarrow \delta_{\mu} = \sqrt{\frac{2ml(g \sin \theta - \mu g \cos \theta)}{k \sin \theta}} = \sqrt{\frac{2mg l}{k} \left(1 - \mu \frac{\cos \theta}{\sin \theta}\right)}$$

CON

ATTRITO

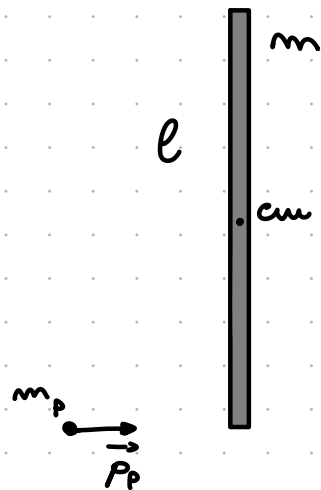
TRASCURABILE

$\mu \rightarrow 0$

$$\Rightarrow \delta_{\mu} \xrightarrow{\mu \rightarrow 0} \delta_0 = \sqrt{\frac{2mg l}{k}}$$

$$\Rightarrow f = \frac{\delta_{\mu}}{\delta_0} = \sqrt{1 - \mu \frac{\cos \theta}{\sin \theta}}$$

2



APPLICO LA PRIMA EQUAZIONE CARDINALE

$$Z^E = m a_{cm} \Rightarrow \frac{dp_p}{dt} = m a_{cm}$$

$$\frac{dp_p}{m} = a_{cm} dt$$

$$\Rightarrow \frac{p_p}{m} = v_{cm}$$

ARRIVARE CON p_p VUOL DIRE IMPRIMERE UN IMPULSO

$$J = \Delta p_p = p_p$$

$$\Rightarrow \boxed{v_{cm} = \frac{J}{m}}$$

HO UN URTO COMPLETAMENTE ANELASTICO, POSSO CONSERVARE IL MOMENTO ANGOLARE

$$p_p \frac{l}{2} = I \omega \Rightarrow \omega = \frac{p_p l}{2 I}$$

$$\text{con } I = I_{\text{ASTA}_{cm}} + m_p \left(\frac{l}{2}\right)^2 = \frac{1}{12} m l^2 + \frac{1}{4} m_p l^2 \sim \frac{1}{12} m l^2 \quad m_p \ll m$$

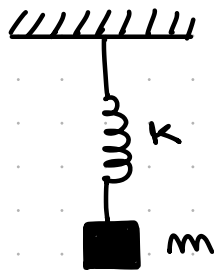
$$\Rightarrow \omega = \frac{p_P l}{2 \frac{1}{12} m l^2} \Rightarrow \omega = \frac{6 p_P}{m l} = \frac{6 J}{m l}$$

HO

$$\begin{aligned} T &= \frac{1}{2} I \omega^2 + \frac{1}{2} m v_{cm}^2 \\ &= \frac{1}{2} I \left(\frac{J l}{2 I} \right)^2 + \frac{m}{2} \frac{J^2}{m^2} \\ &= \frac{1}{2} \frac{J^2 l^2}{4 I} + \frac{m}{2} \frac{J^2}{m^2} \\ &= \frac{1}{8} \frac{J^2 l^2}{\frac{1}{12} m l^2} + \frac{J^2}{2 m} \\ &= \frac{3}{2} \frac{J^2}{m} + \frac{J^2}{2 m} = 2 \frac{J^2}{m} \end{aligned}$$

13/06/2023

111



INIZIALMENTE HO

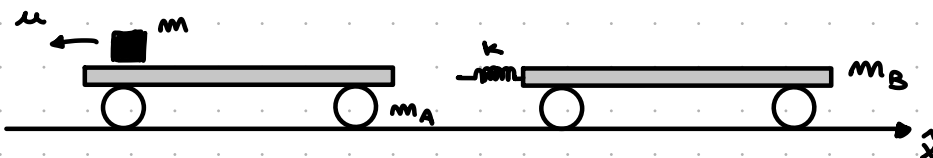
$$\vec{F}_{EL} = - \vec{F}_m \xrightarrow{\text{MODULO}} k \ell_0 = mg$$

QUANDO SI STACCA $m_2 = \frac{1}{3} m$ HO L'EQUAZIONE DEL MOTO

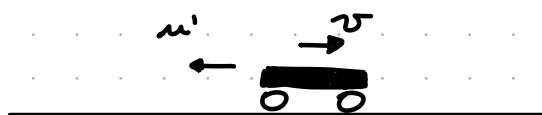
$$m_1 a = F_{EL} - m_1 g = mg - m_1 g = mg - \frac{2}{3} mg$$

$$\Rightarrow \frac{2}{3} m a = \frac{1}{3} mg \Rightarrow a = \frac{1}{2} g$$

2

 m, m_A, m_B 

QUANDO LA PERSONA SALTA HO UN URTO COMPLETAMENTE ANELASTICO
E POSSO CONSERVARE L'IMPULSO (RISPETTO AD UN SIZ ESTERNO)



CON $u' = u - v$ (TRASFORMAZIONE $\vec{v}$ SIZ)

$$0 = -m u' + m_A v = -m(u - v) + m_A v$$

$$\Rightarrow m u = (m_A + m) v \quad \Rightarrow v = \frac{m}{m_A + m} u$$

IL LAVORO LO TROVO CON IL TEOREMA DELLE FORZE VIVE

$$W = \Delta K = \frac{m}{2} (u - v)^2 = \frac{m}{2} u^2 \left(1 - \frac{m}{m_A + m} \right)^2 = \frac{m m_A^2}{2(m_A + m)^2} u^2$$

DA CONTROLLARE

QUANDO A URTA CON B, B SI MUOVE

$$m_A v = (m_A + m_B) \tilde{v}$$

QUANDO LA MOLLA È A $\delta_{\max}$

$$\Rightarrow \tilde{v} = \frac{m_A}{m_A + m_B} v$$

HO LA VELOCITÀ v CON CUI SI MUOVE A E USO
IL LAVORO PER TROVARE $\delta_{\max}$

$$W_{EL} = \Delta K = \frac{m_A + m_B}{2} v^2 - \frac{m_A}{2} v^2 \equiv \Delta U_{EL} = \frac{1}{2} k (\underbrace{\ell^2 - \ell_2^2}_{-\delta_{\max}^2})$$

$$\Rightarrow \frac{m_A + m_B}{2} v^2 - \frac{m_A}{2} v^2 = -\frac{1}{2} k \delta_{\max}^2$$

$$\Rightarrow \frac{m_A^2}{2(m_A + m_B)} v^2 - \frac{m_A}{2} v^2 = -\frac{1}{2} k \delta_{\max}^2$$

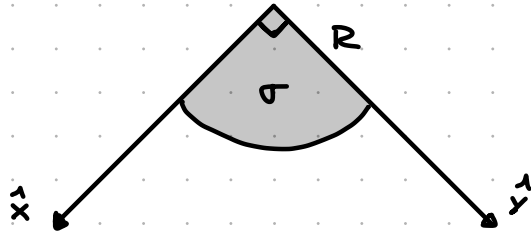
$$\Rightarrow m_A v^2 \left(1 - \frac{m_A}{m_A + m_B} \right) = k \delta_{\max}^2$$

$$\Rightarrow m_A \frac{m^2 \mu^2}{(m + m_A)^2} \frac{m_B}{(m_A + m_B)} = k \delta_{\max}^2$$

$$\Rightarrow \delta_{\max} = \sqrt{\frac{m_A m_B m^2}{(m + m_A)^2 (m_A + m_B)} \frac{\mu^2}{k}}$$

DA CONTROLLARE I
QUADRATI

3



NO $dm = \sigma dS = \sigma r dr d\theta = \sigma dx dy$

$\downarrow$ POLARI $\downarrow$ CARTESIANE

$$r_{cm} = \frac{\int r dm}{\int dm} = \frac{\sigma \int_0^R r^2 dr \int_0^{\pi/2} d\theta}{\sigma \int_0^R r dr \int_0^{\pi/2} d\theta} = \frac{\frac{R^3}{3} \frac{\pi}{2}}{\frac{R^2}{2} \frac{\pi}{2}} = \frac{2}{3} R$$

$$\theta_{cm} = \frac{\int \theta dm}{\int dm} = \frac{\sigma \frac{R^2}{2} \frac{\theta^2}{2} \Big|_0^{\pi/2}}{\sigma \frac{R^2}{2} \frac{\pi}{2}} = \frac{2}{\pi} \frac{\pi^2}{8} = \frac{\pi}{4}$$

IN CARTESIANE

$$\begin{cases} x_{cm} = r_{cm} \cos \theta_{cm} \\ y_{cm} = r_{cm} \sin \theta_{cm} \end{cases}$$

$$\begin{aligned}
 x_{cm} &= \frac{\sigma \int x dx \int dy}{\sigma \frac{\pi R^2}{4}} = \frac{4}{\pi R^2} \int_0^R x dx \int_0^{\sqrt{R^2-x^2}} dy = \frac{4}{\pi R^2} \int_0^R x \sqrt{R^2-x^2} dx \\
 &= \frac{4}{\pi R} \int_0^R x \sqrt{1-\frac{x^2}{R^2}} dx = \frac{4}{\pi} \int_0^1 R y \sqrt{1-y^2} dy = -\frac{4}{\pi} \frac{R}{2} \int_0^1 (-2y) (1-y^2)^{1/2} dy \\
 &= -\frac{2R}{\pi} \frac{(1-y^2)^{3/2}}{3/2} \Big|_0^1 \quad \begin{matrix} y = \frac{x}{R} & dx = R dy \end{matrix} \quad \begin{matrix} f(x) = 1-y^2 \\ f' = -2y \end{matrix}
 \end{aligned}$$

$$\Rightarrow x_{cm} = \frac{4R}{3\pi}$$

$$y_{cm} = \frac{\sigma \int dx \int y dy}{\sigma \frac{\pi R^2}{4}} = \frac{4}{\pi R^2} \int_0^R dx \int_0^{R\sqrt{1-x^2/R^2}} y dy = \frac{4}{\pi R^2} \int_0^R dx \frac{R^2}{2} \left(1 - \frac{x^2}{R^2}\right)$$

$$= \frac{2}{\pi} \int_0^R \left(1 - \frac{x^2}{R^2}\right) dx = \frac{2}{\pi} \left(R - \frac{1}{R^2} \frac{1}{3} R^3\right) = \frac{2R}{\pi} \left(1 - \frac{1}{3}\right) = \frac{4R}{3\pi}$$

$$I_0 = \int b^2 dm = \int \sqrt{x^2 + y^2}^2 \sigma dx dy = \sigma \int_0^R dx \int_0^{R\sqrt{1-x^2/R^2}} (x^2 + y^2) dy$$

$$= \sigma \int_0^R dx \left(x^2 R \sqrt{1 - \frac{x^2}{R^2}} + \frac{1}{3} R^3 \left(1 - \frac{x^2}{R^2}\right)^{3/2} \right)$$

$$= \sigma R \int_0^R dx \left[x^2 \sqrt{1 - \frac{x^2}{R^2}} + \frac{R^2}{3} \left(1 - \frac{x^2}{R^2}\right)^{3/2} \right]$$

$$z = x/R$$

$$= \sigma R^2 \int_0^1 dz \left[R^2 z^2 \sqrt{1 - z^2} + \frac{R^2}{3} (1 - z^2) \sqrt{1 - z^2} \right]$$

$$= \sigma 12^4 \int_0^1 dz \sqrt{1-z^2} \underbrace{\left(z^2 + \frac{1}{3} - \frac{z^2}{3} \right)}_{\frac{2}{3}z^2 + \frac{1}{3}}$$

$$= \frac{\sigma 12^4}{3} \int_0^1 dz (2z^2 + 1) \sqrt{1-z^2}$$

$$\sin \theta = z \quad dz = \cos \theta d\theta$$

$$= \frac{\sigma 12^4}{3} \int_0^{\pi/2} d\theta \cos^2 \theta (2 \sin^2 \theta + 1)$$

$$= \frac{\sigma 12^4}{3} \left[2 \int_0^{\pi/2} d\theta \cos^2 \theta \sin^2 \theta + \int_0^{\pi/2} \cos^2 \theta \right]$$

$$= \frac{\sigma 12^4}{3} \left[2 \int_0^{\pi/2} -\frac{1}{16} \left(e^{i2\theta} + e^{-i2\theta} + 2 \right) \left(e^{i2\theta} + e^{-i2\theta} - 2 \right) + \frac{1}{4} \int_0^{\pi/2} \left(e^{i2\theta} + e^{-i2\theta} + 2 \right) d\theta \right]$$

$$= \frac{\sigma 12^4}{3} \left[2 \int_0^{\pi/2} -\frac{1}{16} \left(e^{i4\theta} + 1 - 2e^{i2\theta} + 1 + e^{-i4\theta} - 2e^{-i2\theta} + 2e^{i2\theta} + 2e^{-i2\theta} - 4 \right) + \frac{1}{4} \left(\frac{1}{2i} e^{i2\theta} - \frac{1}{2i} e^{-i2\theta} + 2\theta \right) \right]_0^{\pi/2}$$

$$= \frac{\sigma 12^4}{3} \left[-2 \frac{1}{16} \left(\frac{1}{4i} e^{i4\theta} - 2\theta - \frac{1}{4i} e^{-i4\theta} \right) \right]_0^{\pi/2} + \frac{1}{4} \left(\sin(2\theta) + 2\theta \right) \Big|_0^{\pi/2}$$

$$= \frac{\sigma R^4}{3} \left[-\frac{1}{8} \left(\frac{1}{2} \sin(4\theta) - 2\theta \right) \right]_0^{\pi/2} + \frac{\pi}{4} \Bigg]$$

$$= \frac{\sigma R^4}{3} \left[+\frac{\pi}{8} + \frac{\pi}{4} \right] = \frac{\sigma R^4}{3} \frac{\pi}{4} \left[1 + \frac{1}{2} \right] = \frac{\sigma R^4 \pi}{3 \cdot 4} \frac{3}{2} = \frac{\sigma R^4 \pi}{4 \cdot 2}$$

MA HO $m = \sigma \frac{\pi R^2}{4}$

$$\Rightarrow I_0 = \frac{\sigma \pi R^2}{4 \cdot 2} R^2 = \frac{m}{2} R^2$$

SCRIVO L'EQUAZIONE CARDINALE

$$M_{\text{EXT}} = r_{\text{cm}} \times (m \vec{g}) = I_0 \ddot{\theta}$$

$$\Rightarrow -I_0 \ddot{\theta} = r_{\text{cm}} m g \sin \theta$$

$$\sin \theta \sim \theta \quad \Rightarrow \quad \ddot{\theta} = -\frac{r_{\text{cm}} m g}{I_0} \theta \quad \Rightarrow \quad \omega = \sqrt{\frac{r_{\text{cm}} m g}{I_0}}$$

$$\text{con } r_{\text{cm}} = \sqrt{x_{\text{cm}}^2 + y_{\text{cm}}^2} = \frac{4}{3\pi} R \sqrt{2}$$

$$\Rightarrow T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{3I_0}{4R\sqrt{2}mg}} = 2\pi \sqrt{\frac{\pi 3mR^2}{8R\sqrt{2}mg}} = 2\pi \sqrt{\frac{3\pi mR}{8\sqrt{2}mg}}$$

uso $I_0 = \frac{1}{2}mR^2$

03/07/2023

1

v_0

USO LA CONSERVAZIONE DELL'ENERGIA E DEL MOMENTO ANGOLARE

$$\begin{cases} \frac{m}{2} v_0^2 - G \frac{M_T m}{R_T} = \frac{m}{2} v_H^2 - G \frac{M_T m}{R_T + r_H} \\ m v_0 R_T = m v_H (R_T + r_H) \end{cases}$$

$$\Rightarrow v_H = \frac{R_T}{R_T + r_H} v_0 \Rightarrow \frac{v_0^2}{2} - \frac{G M_T}{R_T} = \frac{1}{2} \frac{R_T^2}{(R_T + r_H)^2} v_0^2 - \frac{G M_T}{R_T + r_H}$$

$$\Rightarrow \frac{v_0^2}{2} \left(1 - \frac{R_T^2}{(R_T + r_H)^2} \right) = \frac{G M_T}{R_T} \left(1 - \frac{R_T}{R_T + r_H} \right)$$

$$\Rightarrow \frac{v_0^2}{2} \left(1 + \frac{R_T}{R_T + r_H} \right) \left(1 - \frac{R_T}{R_T + r_H} \right) = \frac{G M_T}{R_T} \left(1 - \frac{R_T}{R_T + r_H} \right)$$

$$\Rightarrow 1 + \frac{R_T}{R_T + r_H} = \frac{2 G M_T}{R_T v_0^2} \Rightarrow r_H = R_T \left(\frac{2 G M_T}{R_T v_0^2} - 1 \right)^{-1} - R_T$$

$$\Rightarrow r_H = R_T \left(\frac{2 G M_T - R_T v_0^2}{R_T v_0^2} \right)^{-1} - R_T = R_T \left(\frac{R_T v_0^2}{2 G M_T - R_T v_0^2} - 1 \right)$$

$$\Rightarrow r_H = R_T \frac{2R_T v_0^2 - 2GM_T}{2GM_T - R_T v_0^2}$$

CREDO SIA GIUSTO, SOLO
CALCOLATO DALLA SUPERFICIE
E NON DAL CENTRO DI T

AVEVO VISTO

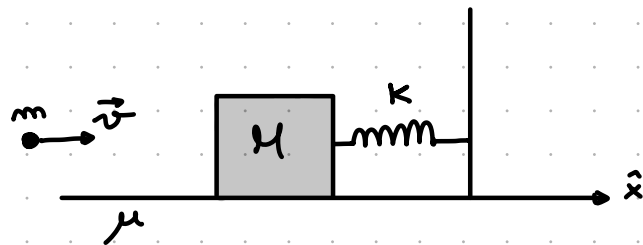
$$v_H = \frac{R_T}{R_T + r_H} v_0 \Rightarrow R_T + r_H = \frac{R_T^2 v_0^2}{2GM_T - R_T v_0^2}$$

$$\Rightarrow v_H = R_T v_0 \frac{2GM_T - R_T v_0^2}{R_T^2 v_0^2} = \frac{2GM_T - R_T v_0^2}{R_T v_0}$$

2

URTO ANELASTICO

→ CONSERVO L'IMPULSO E TROVO LA VELOCITÀ DOPO CHE m SI È CONFICCATO



$$mv = (M+m) \tilde{v} \Rightarrow \tilde{v} = \frac{m}{M+m} v$$

PER TROVARE LA COMPRESSIONE USO IL TEOREMA DELLE FORZE VIVE.

$$W = \Delta K \Rightarrow W = W_c + W_{nc}, \quad W_{nc} = -F_{ATT} \delta_{MAX} = -(M+m)g\mu\delta_{MAX}$$

$$W_c = \Delta U_{EL} = \frac{1}{2}k(\ell_0^2 - \delta_{MAX}^2)$$

METTO ℓ_0 DELLA MOLLA A 0

$$\Rightarrow -\frac{k}{2}\delta_{MAX}^2 - (M+m)g\mu\delta_{MAX} = -\frac{(M+m)}{2}\tilde{v}^2 = -\frac{m^2}{2(M+m)}v^2$$

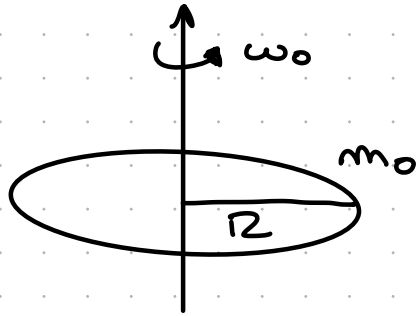
$$\Rightarrow \frac{k}{2}\delta_{MAX}^2 + (M+m)g\mu\delta_{MAX} - \frac{m^2}{2(M+m)}v^2 = 0$$

$$\Rightarrow \delta_{MAX}^{1,2} = -\frac{(M+m)g\mu}{k} \pm \frac{1}{k} \sqrt{(M+m)^2 g^2 \mu^2 + \frac{km^2}{(M+m)} v^2}$$

SCELGO IL +
VISTO CHE
VOGLIO IL
MASSIMO

$$\delta_{MAX} = -\frac{(M+m)g\mu}{k} \pm \sqrt{\frac{\mu^2(M+m)^2 g^2}{k^2} + \frac{m^2}{k(M+m)} v^2}$$

3

ANELLO $\rightarrow m_0, R, \omega_0$ 

so $I_0 = m_0 R^2$

ho poi $\frac{dm}{dt} = k \Rightarrow m(t) = m_0 + kt$

IL MOMENTO ANGOLARE IN TUTTO CIÒ SI CONSERVA

$$I_0 \omega_0 = I(t) \omega(t) \quad \text{con} \quad I(t) = m(t) R^2 = (m_0 + kt) R^2$$

io VOGLIO τ TC $\omega(t) = \omega_0/2$

$$\Rightarrow m_0 R^2 \omega_0 = (m_0 + k\tau) R^2 \frac{\omega_0}{2}$$

$$\Rightarrow 2m_0 = m_0 + k\tau \Rightarrow \tau = \frac{m_0}{k}$$

con $L = \text{cost}$ ho ANCHE

$$I_0 \omega_0 = I(t) \omega(t) \Rightarrow \omega(t) = \frac{m_0 \omega_0}{m_0 + kt}$$

so $\alpha = \frac{d\omega}{dt} = \frac{-m_0 k \omega_0}{(m_0 + kt)^2} = -\frac{k}{m + kt} \omega(t)$

CONTROLLO

$$([K] = K_g / r)$$

$$[\alpha] = \frac{K_g \cdot \frac{K_g}{r} \cdot \frac{\text{rad}}{r}}{K_g^2} = \frac{\text{rad}}{r^2}$$

NOTA

NEUE

SOLUZIONI

HO

$$\alpha = -\frac{k}{m_0 \omega_0} \omega(t)$$

$$[\alpha] = \frac{\frac{K_g}{r}}{K_g \frac{\text{rad}}{r}} \frac{\text{rad}}{r} = \frac{1}{r}$$

CHE

DIMENSIONALMENTE

È

SBAGLIATO

07/09/2023

1

$m, r, R, \Delta t$

PER TROVARE v_r E v_R IMONGO CHE FORZA CENTRIFUGA E GRAVITAZIONALE SI BILANCINO

$$m r \omega^2 = G \frac{Mm}{r^2} \Rightarrow m r \frac{v_r^2}{r^2} = G \frac{Mm}{r^2}$$

$$\Rightarrow v_r = \sqrt{\frac{GM}{r}} \quad \text{E UGUALMENTE} \quad v_R = \sqrt{\frac{GM}{R}}$$

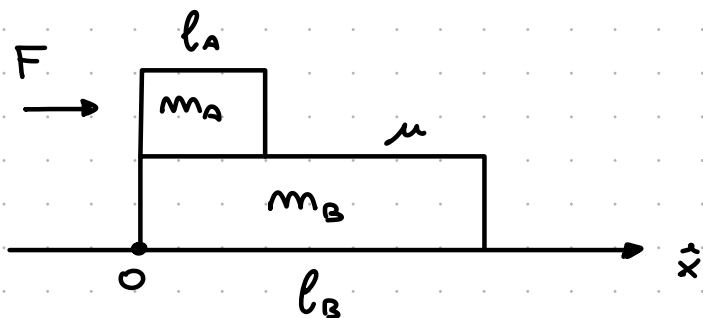
IL LAVORO W CERCO CON IL TEOREMA DELLE FORZE VIVE

$$W_c + W_{nc} = \Delta K \Rightarrow -\Delta U + W = \Delta K \Rightarrow W = \frac{1}{2} m v_R^2 - \frac{1}{2} m v_r^2 + \Delta U$$

$$\Rightarrow W = \frac{m}{2} \left(\frac{GM}{R} - \frac{GM}{r} \right) + \left(-G \frac{Mm}{R} + G \frac{Mm}{r} \right)$$
$$= \frac{1}{2} GMm \left(\frac{1}{R} - \frac{1}{r} \right) - GMm \left(\frac{1}{R} - \frac{1}{r} \right)$$

$$\Rightarrow W = \frac{1}{2} GMm \left(\frac{1}{r} - \frac{1}{R} \right) \longrightarrow \text{LE SOLUZIONI PENSO SIANO SBAGLIATE}$$

LA POTENZA È $P = \frac{W}{\Delta t}$

2 m_A, m_B, l_A, l_B, μ 

SCRIVO LE EQUAZIONI DEL MOTO DEI DUE BLOCCHI

$$\begin{cases} m_A a_{Ax} = F - F_{ATT} \\ m_A a_{Ay} = -m_A g + N_A \equiv 0 \\ m_B a_{Bx} = +F_{ATT} \\ m_B a_{By} = -m_B g + N_B \equiv 0 \end{cases}$$

$$; \begin{cases} a_A = \frac{1}{m_A} (F - F_{ATT}) \\ N_A = m_A g \\ a_B = +F_{ATT} / m_B \\ N_B = m_B g \end{cases}$$

AVENDO

$$F_{ATT} = N_A \mu \Rightarrow F_{ATT} = m_A g \mu$$

TROVO

$$\begin{cases} a_A = \frac{F}{m_A} - g \mu \\ a_B = + \frac{m_A}{m_B} g \mu \end{cases}$$

DA ∂_i TROVO LE VELOCITÀ E LE LEGGI ORARIE

$$\begin{cases} v_A = \left(\frac{F}{m_A} - g\mu \right) t \\ v_B = \frac{m_A}{m_B} g\mu t \end{cases} \quad \begin{cases} x_A(t) = \frac{1}{2} \left(\frac{F}{m_A} - g\mu \right) t^2 \\ x_B(t) = \frac{1}{2} \frac{m_A}{m_B} g\mu t^2 \end{cases}$$

LEGGI ORARIE CHE SI RIFERISCONO AGLI SPIGOLI SINISTRI.
CHIEDO DUNQUE CHE

$$x_A(\tau) + l_A = x_B(\tau) + l_B$$

$$\Rightarrow \frac{1}{2} \left(\frac{F}{m_A} - g\mu \right) \tau^2 + l_A = \frac{1}{2} \frac{m_A}{m_B} g\mu \tau^2 + l_B$$

$$\Rightarrow \frac{1}{2} \tau^2 \left(\frac{F}{m_A} - g\mu - \frac{m_A}{m_B} g\mu \right) = l_B - l_A$$

$$\Rightarrow \tau^2 \left(\frac{m_B F - m_B g\mu - m_A g\mu}{m_A m_B} \right) = 2(l_B - l_A)$$

$$\Rightarrow \tau = \sqrt{\frac{2 m_A m_B (l_B - l_A)}{m_B F - g\mu (m_A + m_B)}}$$

P.I. COMPATTA

$$\frac{1}{2} a_A \tau^2 + l_A = \frac{1}{2} a_B \tau^2 + l_B$$

$$\Rightarrow \frac{1}{2} (a_A - a_B) \tau^2 = l_B - l_A$$

$$\Rightarrow \tau = \sqrt{\frac{2(l_B - l_A)}{a_A - a_B}}$$

PER MUOVERSI ALL'UNISONO L'ATTRITO STATICO DEV'ESSERE
PIÙ GRANDE DELLA FORZA IN GIOCO, O COMUNQUE DEVONO
AVERE STESSA ACCELERAZIONE

$$a_A = a_B \quad \text{con} \quad F_{\text{ATT}} = N \mu_s = F_{\text{ATT}, S}$$

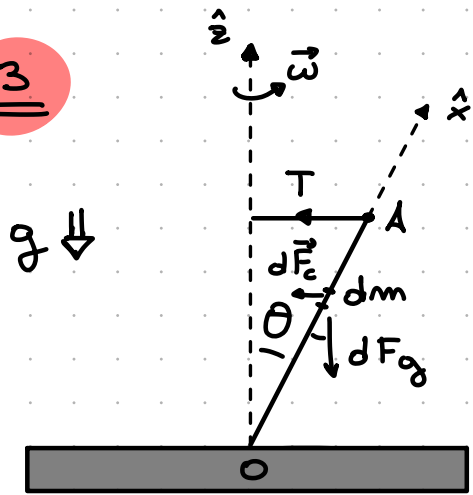
$$\frac{F}{m_A} - g \mu_s = \mu_s \frac{m_A}{m_B} g$$

$$\Rightarrow \mu_s \left(\frac{m_A}{m_B} + 1 \right) g = \frac{F}{m_A}$$

$$\Rightarrow \mu_s = \frac{m_B}{(m_A + m_B) m_A} \frac{F}{g}$$

$$\Rightarrow \mu_s \left(\frac{m_A + m_B}{m_B} \right) g = \frac{F}{m_A}$$

3



$$\lambda = cx$$

$$m = \int \lambda dx = \frac{1}{2} cx^2 \Big|_0^l = \frac{1}{2} cl^2$$

$$x_{cm} = \frac{\int x dm}{m} = \frac{1}{m} \int_0^l cx^2 dx = \frac{c}{m} \frac{1}{3} l^3 = \frac{2}{3} l$$

$$dH_p = \vec{x} \times d\vec{F}_g = dF_g \sin\theta \times = dm g \sin\theta \times$$

$$\Rightarrow H_p = g \sin\theta \int x dm = g \sin\theta \int_0^l cx^2 dx = \frac{1}{3} g \sin\theta cl^3$$

$$\Rightarrow H_p = \frac{2}{3} mgl \sin\theta$$

$$\text{so } dF_c = dm \omega^2 r, \quad r = x \sin\theta$$

$$\Rightarrow d\vec{H}_c = \vec{x} \times d\vec{F}_c = dm \omega^2 x^2 \sin\left(\frac{\pi}{2} + \theta\right) \sin\theta = dm \omega^2 x^2 \cos\theta \sin\theta$$

$$\Rightarrow H_c = \omega^2 \cos\theta \sin\theta \int x^2 dm = \omega^2 c \cos\theta \sin\theta \int_0^l x^3 dx$$

$$\Rightarrow H_c = \frac{1}{4} c \omega^2 l^4 \cos\theta \sin\theta = \frac{1}{2} m \omega^2 l^2 \cos\theta \sin\theta$$

DUNQUE PER AVERE EQUILIBRIO CHIEDO CHE IL MOMENTO
RESULTANTE SIA NULLO

$$0 = \vec{M}_C + \vec{M}_P + \vec{M}_T$$

$$\text{con } M_T = Tl \sin(\theta + \frac{\pi}{2}) = Tl \cos \theta$$

HO M_C USCENTE, M_P ENTRANTE, M_T USCENTE

$$\Rightarrow 0 = M_C - M_P + M_T$$

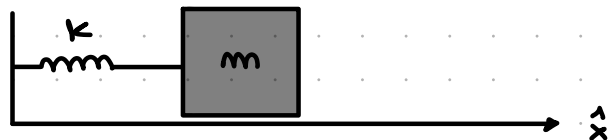
$$\Rightarrow Tl \cos \theta = M_P - M_C \quad \Rightarrow T = \frac{M_P - M_C}{l \cos \theta}$$

$$\Rightarrow T = \frac{2}{3} mg l \sin \theta - \frac{1}{2} m \omega^2 l \sin \theta$$

26/01/2024

11

m, k



$$x(0) = A$$

$$\text{HO} \quad m\ddot{x} = -kx \Rightarrow x(t) = A \sin(\omega t + \varphi), \quad \omega = \sqrt{\frac{k}{m}}$$

$$\text{HA} \quad x(0) = A \Rightarrow A \sin(\varphi) = A \Rightarrow \varphi = \frac{\pi}{2}$$

NOTA $\sin\left(\alpha + \frac{\pi}{2}\right) = \frac{1}{2i} \left(e^{i\alpha} e^{i\frac{\pi}{2}} - e^{-i\alpha} e^{-i\frac{\pi}{2}} \right) = \frac{1}{2i} \left(i e^{i\alpha} - e^{-i\alpha} (-i) \right) = \frac{e^{i\alpha} + e^{-i\alpha}}{2} = \cos \alpha$

DUNQUE SI HA

$$x(t) = A \cos(\omega t)$$

$$\Rightarrow v(t) = \dot{x} = -\omega A \sin(\omega t)$$

PER CUI

$$P(t) = \vec{F} \cdot \vec{v} = (-kx(t) \hat{x}) \cdot (v(t) \hat{x})$$
$$= (-kA \cos(\omega t)) (-\omega A \sin(\omega t)) = k\omega A^2 \sin(\omega t) \cos(\omega t)$$

VORREI $P(t)$ CON $\tilde{t} = T/8$, $T = \text{PERIODO}$

VALE $T = \frac{2\pi}{\omega} \Rightarrow \tilde{t} = \frac{T}{8} = \frac{1}{8} \frac{2\pi}{\omega} = \frac{1}{4} \frac{\pi}{\omega}$

DUNQUE

$$P\left(\frac{T}{8}\right) = \kappa \omega A^2 \sin\left(\omega \frac{1}{4} \frac{\pi}{\omega}\right) \cos\left(\omega \frac{1}{4} \frac{\pi}{\omega}\right) = \kappa \omega A^2 \sin\left(\frac{\pi}{4}\right) \cos\left(\frac{\pi}{4}\right)$$

$$\Rightarrow P\left(\frac{T}{8}\right) = \kappa \omega A^2 \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2} \kappa \omega A^2$$

INVECE $\bar{P}$ MEDIA SU UN T È

$$\bar{P} = \frac{1}{T} \int_0^T P(t) dt = \frac{\omega}{2\pi} \kappa A^2 \int_0^T \sin(\omega t) \cos(\omega t) dt$$

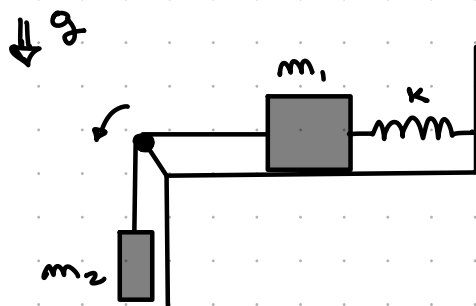
NOTO CHE

$$\begin{aligned} \sin \alpha \cos \beta &= \frac{1}{4i} \left(e^{i\alpha} - e^{-i\alpha} \right) \left(e^{i\beta} + e^{-i\beta} \right) = \frac{1}{4i} \left(e^{i(\alpha+\beta)} + e^{i(\alpha-\beta)} - e^{-i(\alpha-\beta)} - e^{-i(\alpha+\beta)} \right) \\ &= \frac{1}{2} \left(\sin(\alpha+\beta) + \sin(\alpha-\beta) \right) \end{aligned}$$

CON $\alpha = \beta$ $\sin \alpha \cos \alpha = \frac{1}{2} \sin(2\alpha)$

$$\begin{aligned} \bar{P} &= \frac{\kappa \omega^2 A^2}{4\pi} \int_0^T \sin(2\omega t) dt = \frac{\kappa \omega^2 A^2}{4\pi} \left(-\frac{\cos(2\omega t)}{2\omega} \right) \Big|_0^T \\ &= \frac{\kappa \omega A^2}{8\pi} \left(-\cos(2\omega T) + 1 \right) = \frac{\kappa \omega A^2}{8\pi} \left(-\cos\left(2\omega \frac{2\pi}{\omega}\right) + 1 \right) = 0 \end{aligned}$$

2



HO EQUILIBRIO QUINDI

$$\begin{cases} m_1: & 0 = -F_{EL} + T \\ m_2: & 0 = -T + m_2 g \end{cases}$$

$$\Rightarrow \begin{cases} F_{EL} = k\delta = T \\ T = m_2 g \end{cases} \Rightarrow \boxed{\delta = \frac{T}{k} = \frac{m_2 g}{k}}$$

SE ALZO m_2 FINO AD AVERE $\delta_{MOLLA} = 0$ ALLORA UNA VOLTA
FATTA L'OSCILLAZIONE MASSIMA USO IL TEOREMA DELLE FORZE
VIVE (O CONSERVO L'ENERGIA)

$$W = \Delta K$$

$$-\Delta U_{POT} - \Delta U_{EL} = 0 \rightarrow \text{FERMO TUTTO QUANDO PARTO E FERMO ALL'OSCILLAZIONE MASSIMA}$$

$$m_2 g (\Delta z) - \frac{1}{2} k \delta_{MAX}^2 = 0$$

SE LA MOLLA SI ALLUNGA DI $\delta_{MAX} \Rightarrow m_2$ SCENDE DI UNA QUOTA δ_{MAX}

$$m_2 g \delta_{MAX} - \frac{1}{2} k \delta_{MAX}^2 = 0 \Rightarrow \delta_{MAX} = 2 \frac{m_2 g}{k} = 2\delta$$

SCRIVO LE EQUAZIONI DEL MOTO

$$\begin{cases} m_1 \ddot{x}_1 = -kx_1 + T \\ m_2 \ddot{x}_2 = m_2 g - T \end{cases}$$

ESSENDO IL FILO INESTENSIBILE $\Rightarrow \ddot{x}_1 = \ddot{x}_2$ (ACCELERAZIONI UGUALI)

SOMMANDO

$$(m_1 + m_2) \ddot{x}_1 = m_2 g - T - kx_1 + T$$

$$\ddot{x}_1 = -\frac{k}{m_1 + m_2} x_1 + m_2 g$$

→ OSCILLATORE ARMONICO CON FORZA COSTANTE

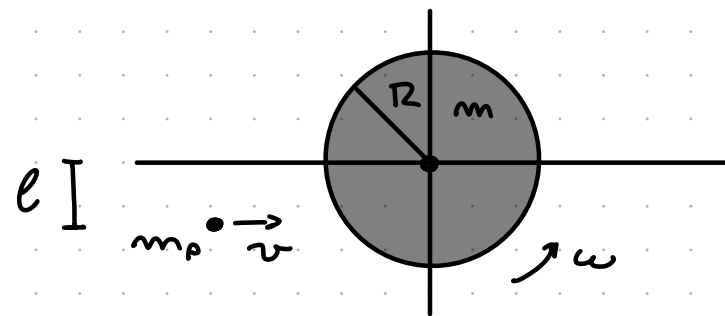
$$\omega = \sqrt{\frac{k}{m_1 + m_2}}$$

$$\Rightarrow T = 2\pi \sqrt{\frac{m_1 + m_2}{k}}$$

3

DISCO , m , R

VERTICALE

 m_p , l , v 

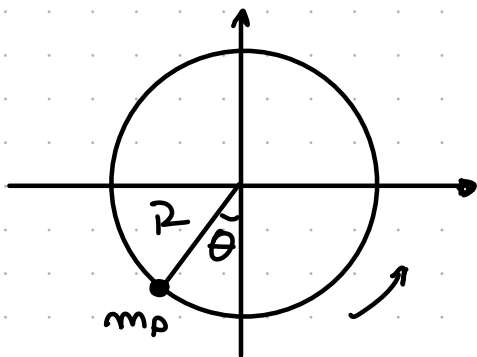
PER TROVARE ω' DOPO L'URTO
 POSSO CONSERVARE IL MOMENTO
 ANGOLARE

$$m_p v l = I \omega' \quad \text{con} \quad I = I_{\text{disco}} + m_p R^2$$

$$m_p v l = (I_{\text{disco}} + m_p R^2) \omega'$$

$$\Rightarrow \omega' = \frac{m_p v l}{\frac{1}{2} m R^2 + m_p R^2} = \frac{2 m_p v l}{(m + 2 m_p) R^2}$$

SE INDICO θ :



m_p DOPO L'URTO SI
 MUOVE CON IL DISCO,
 AD UNA DISTANZA R
 DAL CENTRO E CON
 VELOCITÀ ω'

DUNQUE

$$p_0 = (m_p v, 0)$$

$$p_1 = (m_p v'_x, m_p v'_y)$$

$$= (m_p |\vec{\omega} \times \vec{R}_x|, m_p |\vec{\omega} \times \vec{R}_y|)$$

$$\Rightarrow p_i = (m_p \omega' R \sin \theta, -m_p \omega' R \cos \theta)$$

$$\begin{cases} \Delta p_x = m_p \omega' R \sin \theta - m_p v \\ \Delta p_y = -m_p \omega' R \cos \theta \end{cases}$$

NON PER CI SONO VINCOLI E LUNGO $\hat{x}$ SI CONSERVA L'IMPULSO

$$m_p v = (m_p + m) v_{cm} \quad \Rightarrow \quad v_{cm} = \frac{m_p}{m_p + m} v$$

16/02/2024

1 κ, l, μ, m, A



UTILIZZO IL TEOREMA DELLE FORZE VIVE PER TROVARE $\delta_{\max}$

$$-\Delta U_{EL} + W_{NC} = \Delta K$$

$$\text{HO } v(0) = 0 \quad \text{E} \quad v(\delta_{\max}) = 0$$

$$\frac{1}{2} \kappa (l - x_{\min})^2 - \frac{1}{2} \kappa A^2 + mg\mu (A + l - x_{\min}) = 0$$

$$(l - x_{\min})^2 - A^2 + 2 \frac{mg\mu}{\kappa} (A + l - x_{\min}) = 0$$

$$\downarrow$$
$$l^2 + x_{\min}^2 - 2lx_{\min}$$

$$x_{\min}^2 + 2 \left(\frac{mg\mu}{\kappa} - l \right) x_{\min} - A^2 + l^2 + 2 \frac{mg\mu}{\kappa} (A + l) = 0$$

$$\Rightarrow x_{\min} = \left(\frac{mg\mu}{k} - l \right) \pm \sqrt{\left(\frac{mg\mu}{k} - l \right)^2 - \left(2 \frac{mg\mu}{k} (l+l) - l^2 + l^2 \right)}$$

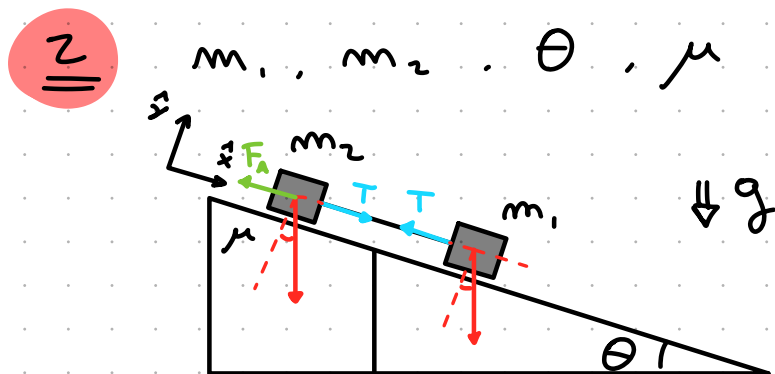
$$\Rightarrow x_{\min} = \left(\frac{mg\mu}{k} - l \right) \pm \sqrt{\left(\frac{mg\mu}{k} - l \right)^2 + \left(l^2 - l^2 - 2 \frac{mg\mu}{k} (l+l) \right)}$$

USO IL TEOREMA ANCHE PER TROVARE v

$$-\Delta U + W_{nc} = \Delta K$$

$$\frac{1}{2} k (l - \underset{x}{x}_{\min})^2 - mg\mu (l - \underset{x}{x}_{\min}) = \frac{m}{2} v^2$$

$$\Rightarrow v = \sqrt{2g\mu(l-x) - \frac{k}{m}(l-x)^2}$$



SCRIVO L'EQUAZIONE DEL MOTO DELLE DUE MASSE

$$\begin{cases} m_1 a_{1x} = m_1 g \sin \theta - T \\ m_1 a_{1y} = -m_1 g \cos \theta + N_1 \equiv 0 \\ m_2 a_{2x} = m_2 g \sin \theta + T - F_{\text{att}} \\ m_2 a_{2y} = -m_2 g \cos \theta + N_2 \equiv 0 \end{cases}$$

$$\begin{cases} m_1 a_{1x} = m_1 g \sin \theta - T \\ N_1 = m_1 g \cos \theta \\ m_2 a_{2x} = m_2 g \sin \theta + T - N_2 \mu \\ N_2 = m_2 g \cos \theta \end{cases}$$

PERO' LA FUNE E' IDEALE $\Rightarrow a_{1x} \equiv a_{2x}$

$$\begin{cases} m_1 a = m_1 g \sin \theta - T \\ m_2 a = m_2 g \sin \theta + T - N_2 \mu \end{cases}$$

SOPRA

$$(m_1 + m_2)a = (m_1 + m_2)g \sin \theta - m_2 g \mu \cos \theta$$

$$\Rightarrow a = g \sin \theta - \frac{m_2 g \mu}{m_1 + m_2} \cos \theta$$

DALLA PRIMA EQUAZIONE

$$\begin{aligned} T &= m_1 g \sin \theta - m_1 a \\ &= \frac{m_1 m_2}{m_1 + m_2} g \mu \cos \theta = 1,91 \text{ N} \end{aligned}$$

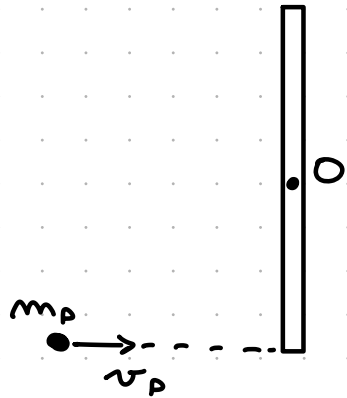
RIPRENDO LE EQUAZIONI DEL MOTO INCLUDENDO F (ATTRITO CAMBIA VERSO)

$$\begin{cases} m_1 a = m_1 g \sin \theta - T \\ N_1 = m_1 g \cos \theta \\ m_2 a = m_2 g \sin \theta + T + N_2 \mu - F \\ N_2 = m_2 g \cos \theta \end{cases}$$

PER AVERE $v \equiv v_0 = \text{cost} \Rightarrow a = 0$

$$\begin{cases} T = m_1 g \sin \Theta \\ N_1 = m_1 g \cos \Theta \\ F = m_2 g \sin \Theta + T + N_2 \mu \\ N_2 = m_2 g \cos \Theta \end{cases}$$

$$\Rightarrow F = (m_1 + m_2) g \sin \Theta + m_2 g \mu \cos \Theta$$

3ASTA, m, l 

URTO TOTALMENTE ANELASTICO $\rightarrow$ CONSERVO $\vec{L}$
IMMEDIATAMENTE PRIMA
E DOPO L'URTO

$$m_p v_p \frac{l}{2} = I \omega$$

$$I = I_{ASTA} + m_p \left(\frac{l}{2} \right)^2 = \frac{1}{12} m l^2 + m_p \frac{l^2}{4}$$

$$\Rightarrow \omega_0 = \frac{l m_p v_p}{2 I} = \frac{\frac{1}{2} m_p v_p l}{\frac{1}{12} m l^2 + \frac{1}{4} m_p l^2} = \frac{2 m_p v_p}{\frac{1}{3} m l + m_p l}$$

HO $\mathcal{M}_{ATT} = -I \frac{d\omega}{dt}$

$$\Rightarrow \omega(t) = \omega_0 - \frac{\mathcal{M}_{ATT}}{I} t$$

$$\Rightarrow \Theta(t) = \omega_0 t - \frac{1}{2} \frac{\mathcal{M}_{ATT}}{I} t^2$$

VOGLIO τ T.C. $\Theta(\tau) = 4\pi$ E $\omega(\tau) = 0$

$$\Rightarrow \omega(\tau) = 0 = \omega_0 - \frac{\mathcal{M}_{\text{ATT}}}{I} \tau$$

$$\Rightarrow \tau = \frac{\omega_0 I}{\mathcal{M}_{\text{ATT}}}$$

$$\Rightarrow \Theta(\tau) = 4\pi = \omega_0 \tau - \frac{1}{2} \frac{\mathcal{M}_{\text{ATT}}}{I} \tau^2$$

$$\Rightarrow 4\pi = \frac{\omega_0^2 I}{\mathcal{M}_{\text{ATT}}} - \frac{1}{2} \frac{\mathcal{M}_{\text{ATT}}}{I} \frac{\omega_0^2 I^2}{\mathcal{M}_{\text{ATT}}^2}$$

$$\Rightarrow 4\pi = \frac{1}{2} \frac{\omega_0^2 I}{\mathcal{M}_{\text{ATT}}}$$

$$\Rightarrow \mathcal{M}_{\text{ATT}} = \frac{\omega_0^2 I}{8\pi}$$

$$\Rightarrow \omega_0 = \frac{e m_p v_p}{2I}$$

$$\mathcal{M}_{\text{ATT}} = \frac{e^2 m_p^2 v_p^2}{4 I^2} \frac{I}{8\pi}$$

$$\Rightarrow \mathcal{M}_{\text{ATT}} = \frac{e^2 m_p^2 v_p^2}{4 \cdot 8\pi} \frac{1}{\frac{1}{12} m \ell^2 + m_p \frac{\ell}{4}} = \frac{m_p^2 v_p^2}{8\pi \left(\frac{1}{3} m + m_p \right)}$$