Esercizi

Meccanica

TUTORAGGI 2023 - 2024

ARGOMENTI :

- 1, 2 CINEMATICA
- 2, 3, 4 DINAMICA, LAVORO ED ENERGIA
- 5 SISTEMI DI RIFERIMENTO
- 6 URTI
- 7, 8 CORPO RIGIDO

ANNOTAZIONI

- ES 5.3 DA RIVEDERE PUNTO 6
- ES 5.4

TUTORAGGIO 1

1.1 GRAFICO NEL TESTO.

$$v_{m_1} = \frac{\Delta s}{\Delta t} = \frac{2}{20}\,\frac{m}{s} = 0,1\ m/s$$

IN $t = 0s$ VEDO CHE HO PENDENZA $\sim 0\ m/s$

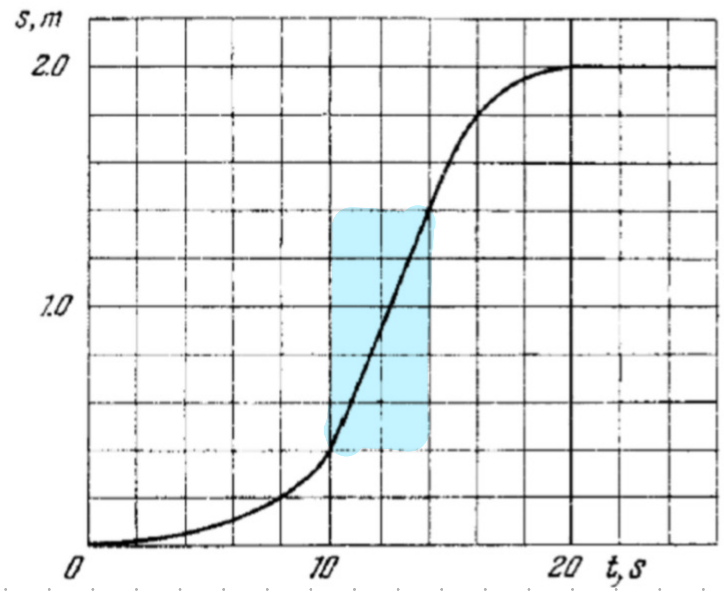
IN $t = 12s$ INDIVIDUO UNA REGIONE IN CUI LA LEGGE ORARIA SIA APPROSSIMABILE AD UNA RETTA E CALCOLO IL COEFFICIENTE ANGOLARE

$$v = \frac{1,4 - 0,4}{14 - 10}\,\frac{m}{s} = \frac{1}{4}\,\frac{m}{s} = 0,25\ m/s$$

$$v_{m,2} = \frac{\Delta s}{\Delta t} = \frac{1,8 - 0,2}{16 - 8}\,\frac{m}{s} = \frac{1,6}{8}\,\frac{m}{s} = 0,2\,\frac{m}{s}$$

v_{MAX} È $v = 0,25\ m/s$

IL CORPO È FERMO IN $t = 0$ E $t = 20s$



1.2

$\ell = 10$ km $\qquad t_1 = 5$ min

$v_L = 4$ m/s $\qquad \Delta t = 135$ min

$v_T = 1$ m/s

HO LE LEGGI $\qquad x_L(t) = v_L t$

$x_T(t) = v_T t$

A t_1 SI TROVANO A $\qquad x_L(t_1) \equiv \tilde{x}_L = v_L t_1 = 1200$ m

$x_T(t_1) \equiv \tilde{x}_T = v_T t_1 = 300$ m

LA LEPRE DORME A $\tilde{x}_L$ E LA TARTARUGA ARRIVA A $\tilde{x}_L$ IN

$$\tilde{x}_L = v_T \tilde{t} \Rightarrow \tilde{t} = \frac{\tilde{x}_L}{v_T} = 1200 \text{ s} = 20 \text{ min}$$

E SUPERA LA LEPRE. POI QUANDO SI SVEGLIA LA LEPRE A $t_1 + \Delta t$
LA TARTARUGA È IN

$$x_T(t_1 + \Delta t) \equiv x_T' = v_T(t_1 + \Delta t) = 8400 \text{ m}$$

LA TARTARUGA ARRIVA AL TRAGUARDO IN

$$t' = \frac{\ell - x_T}{v_T} = 1600 \text{ s}$$

TEMPO IN CUI LA LEPRE È ARRIVATA A $x(t_1) = \tilde{x}_L + v_L t' = 7600$ m
QUINDI T TAGLIA IL TRAGUARDO CON UN DISTACCO DI 2400 m

LA TARTARUGA PERCORRE ℓ IN

$$t^T_{TOT} = \frac{\ell}{v_T} = 10000\ s$$

LA LEPRE CI METTEREBBE SENZA DORMIRE

$$t^L_{TOT} = \frac{\ell}{v_L} = 2500\ s$$

INDICO CON t_D IL TEMPO DEL PISOLINO E IMPONGO CHE T ED L TAGLINO INSIEME IL TRAGUARDO

$$t^T_{TOT} = t^L_{TOT} + t_D \Rightarrow t_D = t^T_{TOT} - t^L_{TOT} = 7500\ s = 125\ min$$

1.3

HO
$$\begin{cases} x_1(t) = d + vt - \frac{1}{2}a_0t^2 \\ x_2(t) = vt \end{cases}$$

IMPONGO

$x_1(\tau) = x_2(\tau)$ IN CUI τ = ISTANTE D'IMPATTO

$$\Rightarrow d + v\tau - \frac{1}{2}a_0\tau^2 = v\tau \quad \Rightarrow \quad \tau = \sqrt{\frac{2d}{a_0}}$$

E IMPATTANO IN

$$\tilde{x} = x_2(\tau) = v\tau = v\sqrt{\frac{2d}{a_0}}$$

ORA HO $x_1(t) = d + vt - \frac{1}{2}a_0t^2$ $\quad v_1(t) = v - a_0t$

$x_2(t) = vt - \frac{1}{2}a_1t^2$ $\quad v_2(t) = v - a_1t$

IMPONGO ANCORA $x_1(\tau) = x_2(\tau)$ PER LA CONDIZIONE D'IMPATTO, INSIEME ALLA CONDIZIONE DI FRENATA $v(\tau) = 0$

$$\begin{cases} x_1(\tau) = d + v\tau - \frac{1}{2}a_0\tau^2 \\ 0 = v - a_0\tau \end{cases} \vee \begin{cases} x_2(\tau) = v\tau - \frac{1}{2}a_1\tau^2 \\ 0 = v - a_1\tau \end{cases}$$

$$\begin{cases} x_1(\tau) = d + \frac{v^2}{a_0} - \frac{1}{2}\frac{v^2}{a_0} \\ \tau = v/a_0 \end{cases} \vee \begin{cases} x_2(\tau) = \frac{v^2}{a_1} - \frac{1}{2}\frac{v^2}{a_1} \\ \tau = v/a_1 \end{cases}$$

$$\Rightarrow \quad d + \frac{v^2}{2a_0} = \frac{v^2}{2a_1} \quad \Rightarrow \quad \frac{2a_1}{v^2} = \frac{1}{\left(d + \frac{v^2}{2a_0}\right)}$$

$$\Rightarrow \quad a_1 = \frac{v^2}{2\left(d + \frac{v^2}{2a_0}\right)} \quad , \quad \tau = \frac{v}{a_1}$$

1.4 $\quad x = A\sin(\omega t) \qquad A = 20\ \text{cm} \qquad T = \frac{\pi}{2}\ s \quad \Rightarrow \quad \omega = \frac{2\pi}{T} = 4\ \frac{\text{rad}}{s}$

$$v(t) = \frac{dx}{dt} = A\omega\cos(\omega t) \quad \Rightarrow \quad v_{MAX} = A\omega = 80\ \text{cm}/s = 0{,}8\ m/s$$

$$x(\tau) = \frac{A}{2} \equiv A\sin(\omega\tau) \quad \Rightarrow \quad \tau = \frac{1}{\omega}\sin^{-1}\left(\frac{1}{2}\right) = 0{,}13\ s$$

$$a(t) = -\omega A\sin(\omega t) \quad \Rightarrow \quad |a(\tau)| = 1{,}59\ m/s^2$$

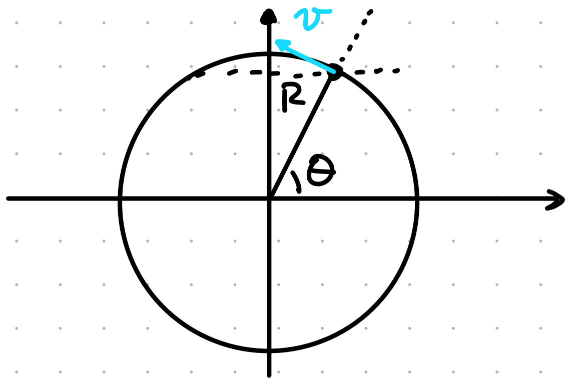
1.5 $v_0 = 5 \ cm/s \qquad R = 20 \ cm$

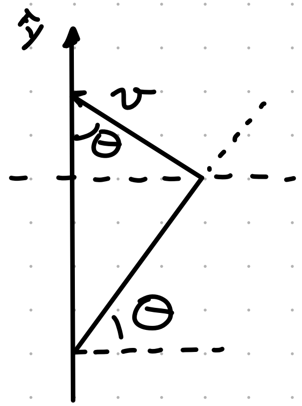
MI POSSO SCRIVERE $\omega = \dfrac{v_0}{R} = \dfrac{1}{4} \dfrac{rad}{s}$

$\Rightarrow \omega = cost \quad \Rightarrow \theta(t) = \omega t \quad \Rightarrow \theta\left(t_1 = \frac{4\pi}{3}\right) = \frac{\pi}{3} \ rad$

IN COORDINATE CARTESIANE $x = R\cos\theta = 10 \ cm, \ y = R\sin\theta = 17{,}3 \ cm$

HO

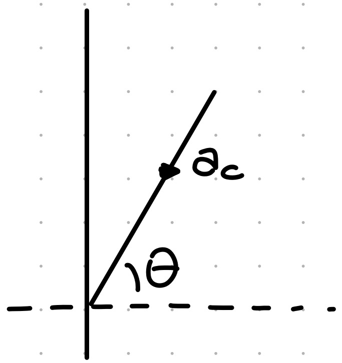




$$\begin{cases} v_x = -v\sin\theta = -4.33 \ cm/s \\ v_y = v\cos\theta = 2{,}5 \ cm/s \end{cases}$$

$$a_c = \frac{v^2}{R} = 1{,}25 \ m/s^2$$

$$\begin{cases} a_{cx} = -a_c\cos\theta = -0{,}625 \ m/s^2 \\ a_{cy} = -a_c\sin\theta = 1{,}083 \ m/s^2 \end{cases}$$



TUTORAGGIO 2

2.1

$$\begin{cases} x(t) = \frac{1}{2}\alpha t^2 - 3\beta t \\ y(t) = At^2 - 4Bt + 2C \end{cases} \Rightarrow \begin{cases} v_x = \alpha t - 3\beta \\ v_y = 2At - 4B \end{cases} \Rightarrow \begin{cases} a_x = \alpha \\ a_y = 2A \end{cases}$$

HO $|a|^2 = a_x^2 + a_y^2 = \alpha^2 + 4A^2 \Rightarrow |a|^2 \equiv 4A^2 \Leftrightarrow \boxed{\alpha = 0}$

$$|v|^2\Big|_{\alpha=0} = v_x^2 + v_y^2 = 9\beta^2 + (2At - 4B)^2 = 9\beta^2 + 4A^2t^2 + 16B^2 - 16ABt$$

2.2

HO $\begin{cases} x(t) = v_{ox} t \\ y(t) = v_{oy} t - \frac{1}{2} g t^2 \end{cases}$

SO $\begin{cases} x(t_1) = L = v_{ox} t_1 \\ y(t_1) = h = v_{oy} t_1 - \frac{1}{2} g t_1^2 \end{cases} \Rightarrow \begin{cases} v_{ox} = L/t_1 \\ v_{oy} = h/t_1 + \frac{g t_1}{2} \end{cases}$

QUINDI

$$|v| = \sqrt{\frac{L^2}{t_1^2} + \left(\frac{h}{t_1} + g\frac{t_1}{2}\right)^2}$$

ALLA QUOTA MASSIMA HO $v_y(t_{MAX}) = 0 = v_{oy} - g t_{MAX} \Rightarrow t_{MAX} = \frac{v_{oy}}{g}$

$$\Rightarrow h_{MAX} = y(t_{MAX}) = \frac{v_{oy}^2}{g} - \frac{v_{oy}^2}{2g} = \frac{1}{2}\frac{v_{oy}^2}{g}$$

HO $v_y(t_1) = v_{oy} - g t_1$ $\qquad v_x = v_{oy} = \text{COST}$

POSSO FARE $\frac{v_y(t_1)}{v_x} = \frac{v_{0y} - g t_1}{v_{0x}} = tg\theta \Rightarrow \theta = tg^{-1}\left(\frac{v_{0y} - g t_1}{v_{0x}}\right)$

2.3 $\begin{cases} r(t) = A - \mu t \\ \theta(t) = \omega t \end{cases} \Rightarrow \begin{cases} x(t) = r(t)\cos\theta(t) = (A-\mu t)\cos(\omega t) \\ y(t) = r(t)\sin\theta(t) = (A-\mu t)\sin(\omega t) \end{cases}$

$$\Rightarrow \begin{cases} v_x(t) = -\mu\cos(\omega t) - \omega(A-\mu t)\sin(\omega t) \\ v_y(t) = -\mu\sin(\omega t) + \omega(A-\mu t)\cos(\omega t) \end{cases}$$

$$\Rightarrow \begin{cases} a_x(t) = \mu\omega\sin(\omega t) + \mu\omega\sin(\omega t) - \omega^2(A-\mu t)\cos(\omega t) = 2\mu\omega\sin(\omega t) - \omega^2 x(t) \\ a_y(t) = -2\mu\omega\cos(\omega t) - \omega^2 y(t) \end{cases}$$

$t = 0$ $\begin{cases} x(0) = A \\ y(0) = 0 \end{cases} \Rightarrow \begin{cases} v_x = -\mu \\ v_y = \omega A \end{cases} \Rightarrow \begin{cases} a_x = -\omega^2 A \\ a_y = -2\mu\omega \end{cases}$

DEVO CALCOLARE t_F ED ω: IN $t=0$ $(x = A, y = 0)$ QUINDI SONO SU x E POI IN $t = t_F$ $(x = 0, y = A/2)$ SONO SU y
$\Rightarrow$ HO FATTO UNA ROTAZIONE DI $\pi/2$

$$r(t_F) = A - \mu t_F = A/2 \qquad \Rightarrow \begin{cases} t_F = -\frac{A}{2\mu} \\ \omega = \frac{\pi\mu}{A} \end{cases}$$
$$\theta(t_F) = \omega t_F = \pi/2$$

USANTO QUINDI t_F, ω E $\omega t_F = \pi/2$ HO

$$\begin{cases} v_x(t_F) = -\omega(A - \mu t_F) = -\frac{\pi\mu}{A}\frac{A}{2} = -\frac{\pi\mu}{2} \\ v_y(t_F) = -\mu \end{cases}$$

$$\begin{cases} a_x(t_F) = 2\mu\omega = 2\mu\frac{\pi\mu}{A} = \frac{2\pi\mu^2}{A} \\ a_y(t_F) = -\omega^2\frac{A}{2} = -\frac{\pi^2\mu^2}{A^2}\frac{A}{2} = -\frac{\pi^2\mu^2}{2A} \end{cases}$$

IL RAGGIO DI CURVATURA SARÀ:

$$R = \frac{v^3}{|\vec{a} \times \vec{v}|} = \frac{\left(\frac{\pi^2\mu^2}{4} + \mu^2\right)^{3/2}}{|\vec{a} \times \vec{v}|}$$

$$\vec{a} \times \vec{v} = \begin{vmatrix} i & j & z \\ a_x & a_y & 0 \\ v_x & v_y & 0 \end{vmatrix} = \begin{pmatrix} 0 \\ 0 \\ a_x v_y - a_y v_x \end{pmatrix}$$

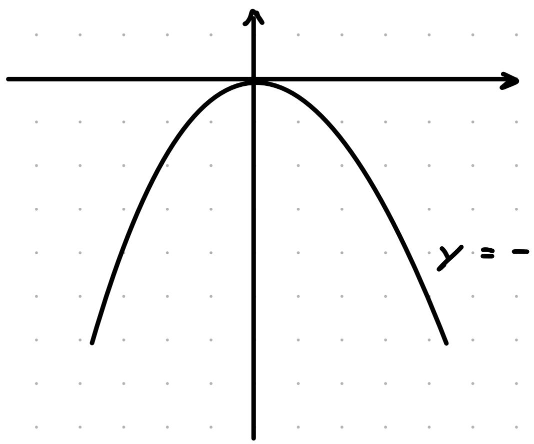
$$|\vec{a} \times \vec{v}| = \left| \frac{2\pi\mu^2}{A}(-\mu) - \left(-\frac{\pi^2\mu^2}{2A}\right)\left(-\frac{\pi\mu}{2}\right) \right| = \left| -\frac{2\pi\mu^3}{A} - \frac{\pi^3\mu^3}{4A} \right| = \frac{\pi\mu^3}{4A}(8 + \pi^2)$$

$$R = \frac{\mu^3}{8}(\pi^2 + 4)^{3/2} \frac{4A}{\pi\mu^3} \frac{1}{(8 + \pi^2)} = \frac{A(\pi^2 + 4)^{3/2}}{2\pi(8 + \pi^2)}$$

24

$y = -x^2$ $\qquad v_x = v_0$

$x(0) = 0$



$[\alpha] = 1/m$

PER LE DIMENSIONI

$$r(t) = \sqrt{x^2(t) + y^2(t)}$$

$$\theta(t) = tg^{-1}\left(\frac{y(t)}{x(t)}\right)$$

IO HO $x(t) = v_0 t \Rightarrow y(t) = -v_0^2 t^2 \alpha$

DUNQUE

$$r(t) = \sqrt{v_0^2 t^2 + v_0^4 t^4} = v_0 t \sqrt{1 + v_0^2 t^2 \alpha^2}$$

VETTORIALMENTE

$$\vec{r}(t) = (x(t), y(t)) = (v_0 t, -v_0^2 t^2 \alpha)$$

POSSO CALCOLARE

$$\begin{cases} v_x = v_0 \\ v_y = -2v_0^2 t \alpha \end{cases} \qquad \begin{cases} a_x = 0 \\ a_y = -2v_0^2 \alpha \end{cases}$$

$$R = \frac{v^3}{|\vec{a} \times \vec{v}|} = \left(v_0^2 + 4v_0^4 t^2 \alpha^2\right)^{3/2} \left(\left\| \begin{matrix} i & j & k \\ v_0 & -2v_0 t\alpha & 0 \\ 0 & -2v_0^2\alpha & 0 \end{matrix} \right\| \right)^{-1} = \frac{v_0^3 (1 + 4v_0^2 t^2 \alpha^2)^{3/2}}{2v_0^3 \alpha}$$

$$\Rightarrow R = \frac{1}{2\alpha}\left(1 + 4v_0^2 t^2 \alpha^2\right)^{3/2}$$

$$v = \frac{dx}{dt} \Rightarrow dx = v\,dt \Rightarrow \Delta x = \int_0^T v\,dt = \int_0^T v_0 \sqrt{1 + 4v_0^2 t^2 \alpha^2}\,dt$$

$$\Rightarrow \begin{array}{l} y = 2v_0 t\alpha \\ dy = 2v_0\alpha\,dt \end{array} \;;\; \Delta x = v_0 \int_0^{2v_0T\alpha} \sqrt{1+y^2}\,\frac{dy}{2v_0\alpha} \Rightarrow \begin{array}{l} y = \cosh\theta \\ dy = \sinh\theta\,d\theta \end{array}$$

$$\Rightarrow \Delta x = \frac{1}{2\alpha}\int_0^{\cosh^{-1}(2v_0T\alpha)} \sinh^2\theta\,d\theta = \frac{1}{2\alpha}\left[-\frac{\theta}{2} + \frac{1}{4}\sinh(2\theta)\right]_0^{\cosh^{-1}(2v_0T\alpha)}$$

$$= \frac{1}{4\alpha}\left[-\theta + \frac{1}{2}\sinh(2\theta)\right]_0^{\cosh^{-1}(2v_0T\alpha)}$$

2.5 DISEGNO NEL TESTO. SCRIVO LE LEGGI DEL MOTO PER OGNUNO

$$\begin{cases} m_1 a = -T_2 + T_3 \\ m_2 a = -m_2 g + T_2 \\ m_3 a = m_3 g - T_3 \end{cases} \qquad \begin{cases} m_1 a = -m_2(g+a) + m_3(g-a) \\ T_2 = m_2(a+g) \\ T_3 = m_3(g-a) \end{cases}$$

$$\begin{cases} a(m_1 + m_2 + m_3) = g(m_3 - m_2) \\ // \end{cases}$$

$$\Rightarrow a = \frac{m_3 - m_2}{m_1 + m_2 + m_3} g$$

$$\Rightarrow T_2 = \frac{m_1 + 2m_3}{m_1 + m_2 + m_3} m_2 g$$

$$\Rightarrow T_3 = \frac{m_1 + 2m_2}{m_1 + m_2 + m_3} m_3 g$$

con $\quad m_1 = m_2 \quad , \quad m_3 = 2m_1$

$$\Rightarrow T_2 = \frac{5}{4} m_2 g \quad , \quad T_3 = \frac{3}{4} m_3 g$$

Tutoraggio 3

3.1 Mi scrivo il sistema delle equazioni del moto

$$\begin{cases} m_1 a = -m_1 g + T_1 \\ m_2 a = -m_2 g + T_2 \\ m_3 a = -T_1 - T_2 + m_3 g \end{cases} \xrightarrow{R_3 \to R_3+R_1+R_2} (m_1+m_2+m_3)a = (m_3-m_1-m_2)g$$

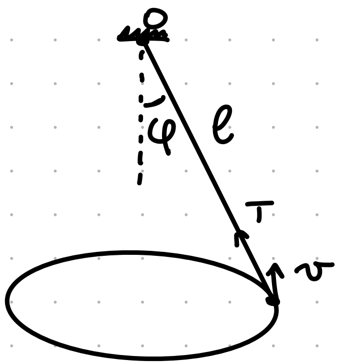
$$\Rightarrow a = \frac{m_3-m_1-m_2}{m_1+m_2+m_3}\, g = \frac{-m_1}{3m_1} = -\frac{1}{3} g = -3,27 \ m/s^2$$

verso l'alto

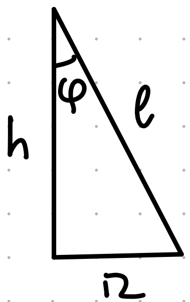
$$R_1 \Rightarrow T_1 = m_1(a+g) = m_1 \frac{2}{3} g = 6,54 \ N$$

$$R_2 \Rightarrow T_2 = m_2(a+g) = m_2 \frac{2}{3} g = 6,54 \ N$$

3.2



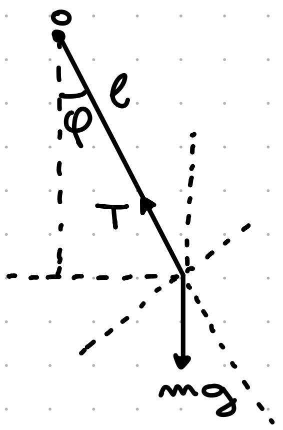
Ho le forze bilanciate



$$\Rightarrow h = \sqrt{\ell^2 - R^2} = 0,86 \ m$$

$$\Rightarrow \varphi = tg^{-1}\left(\frac{R}{h}\right) = \pi/6$$

IN UN GENERICO ISTANTE



SCOMPONGO $T\cos\varphi - mg = 0$

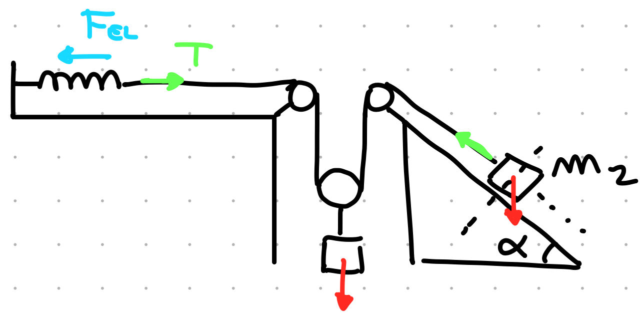
$$\Rightarrow T = \frac{mg}{\cos\varphi} = 11,32 \ N$$

E LA COMPONENTE SUL PIANO DELLA CIRCONFERENZA
È LA FORZA CENTRIPETA (O PUOI VEDERLA COME
LA FORZA CHE BILANCIA LA CENTRIFUGA)

$$m a_c = T\sin\varphi \Rightarrow a_c = \frac{T\sin\varphi}{m} \quad \text{CON} \quad a_c = \omega^2 R \ , \quad \omega = \frac{2\pi}{P}$$

$$\Rightarrow \frac{4\pi^2}{P^2} R = \frac{T\sin\varphi}{m} \Rightarrow P = \sqrt{\frac{4m\pi^2}{T\sin\varphi} R} = 1,87 \ s$$

3.3 SCRIVO LE EQUAZIONI DEL MOTO



$$\begin{cases} -k\Delta\ell + T = 0 \\ m_1 g - 2T = 0 \\ m_2 g\sin\alpha - T = 0 \end{cases}$$

$$\Rightarrow \begin{cases} T = k\Delta\ell \\ T = \frac{m_1}{2} g \\ T = m_2 g \sin\alpha \end{cases} \qquad \begin{cases} \Delta\ell = \frac{m_1 g}{2k} = 0,03 \ m = 3 \ cm \\ T = \frac{m_1 g}{2} = 29,62 \ N \\ \alpha = \sin^{-1}\left(\frac{T}{m_2 g}\right) = 49,01° \end{cases}$$

MOTO PARABOLICO

$$\begin{cases} v_x(t) = v_{0x} \\ v_y(t) = v_{0y} - gt \end{cases}$$

3.4

HO $p_0 = (v_{0x}, v_{0y})\, m$

$p_1 = (v_{0x}, v_{0y} - gt)\, m$

$$\Rightarrow \Delta\vec{p} = (0, -mgt)$$

IL TEMPO TOTALE DEL MOTO È

$$y(t) = v_{0y}t - \frac{1}{2}gt^2 \Rightarrow y(t_{TOT}) \equiv 0 = v_{0y}t_{TOT} - \frac{1}{2}gt_{TOT}^2$$

$$\Rightarrow t_{TOT} = \left\{0; \frac{2v_{0y}}{g}\right\}$$

DUNQUE

$$\Delta\vec{p}_{TOT} = (0, -2mv_{0y})$$

$$\Rightarrow |\Delta\vec{p}_{TOT}| = \sqrt{0 + (-2mv_{0y})^2} = 2mv_0 \sin\theta$$

3.5 $\Delta v = v_1 - v_0$, $F = -\alpha v^2$

HO $dp = F\,dt \Rightarrow m\,dv = -\alpha v^2\,dt \Rightarrow \dfrac{dv}{v^2} = -\dfrac{\alpha}{m}dt$

$$\Rightarrow -\frac{1}{v}\Bigg|_{v_0}^{v_1} = -\frac{\alpha}{m}t \Rightarrow \frac{1}{v_1} - \frac{1}{v_0} = \frac{\alpha}{m}t \Rightarrow t = \frac{m}{\alpha}\left(\frac{1}{v_1} - \frac{1}{v_0}\right)$$

DEVO TROVARE α. POSSO USARE IL TEOREMA DELLE FORZE VIVE

$$dW = F \cdot ds \Rightarrow dW = F\,dx \quad \text{ma} \quad dW = dK = d\left(\frac{m}{2}v^2\right) = mv\,dv$$

$$\Rightarrow F\,dx = mv\,dv \Rightarrow -\alpha v^2 dx = mv\,dv$$

$$\Rightarrow \frac{dv}{v} = -\frac{\alpha}{m}dx \Rightarrow \ln\left(\frac{v_1}{v_0}\right) = -\frac{\alpha}{m}h$$

$$\Rightarrow \alpha = -\frac{m}{h}\ln\left(\frac{v_1}{v_0}\right)$$

DUNQUE HO

$$t = \frac{h}{\ln\left(\frac{v_1}{v_0}\right)}\left(\frac{1}{v_0} - \frac{1}{v_1}\right)$$

TUTORAGGIO 4

4.1 FIGURA NEL TESTO.

UTILIZZO IL TEOREMA DELLE FORZE VIVE VISTO CHE C'È F_{ATT}:

$$W = \Delta K \Rightarrow mgh_1 - mgh_2 - mg\cos\alpha\,\mu L_1 - mg\cos\beta\,\mu L_2 - mg\mu s = 0$$

$$\Rightarrow mg(h_1 - h_2) - mg\mu\left(\cos\alpha \frac{h_1}{\sin\alpha} + \cos\beta \frac{h_2}{\sin\beta}\right) - mg\mu s = 0$$

$$\Rightarrow h_1 - h_2 = -\mu\left(h_1 \cot\alpha + h_2 \cot\beta + s\right)$$

4.2 USO LA CONSERVAZIONE DELL'ENERGIA TRA A E B

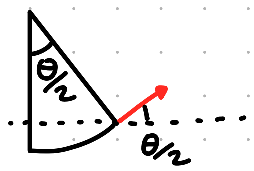
$$\frac{1}{2}Mv^2 + MgH = \frac{1}{2}Mv_B^2 + Mg\left(R - R\cos\left(\frac{\theta}{2}\right)\right)$$

$$\Rightarrow v_B = \left(v^2 + 2gH - 2g\left(R - R\cos(\theta/2)\right)\right)^{1/2}$$

$v = 0$

$$\Rightarrow v_B = \sqrt{2g\left(H - R\left(1 - \cos(\theta/2)\right)\right)} = 7{,}58\ m/s$$

L'ANGOLO DI INCLINAZIONE È $\theta/2$
E HO MOTO PARABOLICO



DUNQUE

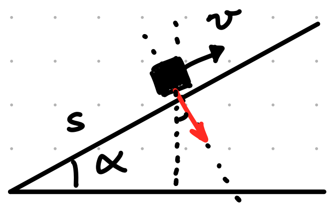
$$\begin{cases} y = R(1-\cos(\theta/2)) + v_B \sin\frac{\theta}{2}\, t - \frac{1}{2} g t^2 \\ x = v_B \cos\frac{\theta}{2}\, t \end{cases}$$

E $\quad v_y = v_B \sin\frac{\theta}{2} - gt \quad , \quad t_{MAX} \Rightarrow v_y = 0 \Rightarrow t_{MAX} = \frac{v_B}{g}\sin\frac{\theta}{2}$

DUNQUE

$$y_{MAX} = R\left(1-\cos\frac{\theta}{2}\right) + \frac{v_B^2}{g}\sin\frac{\theta}{2}\sin\frac{\theta}{2} - \frac{v_B^2}{2g}\sin^2\frac{\theta}{2} = 0{,}79 \text{ m}$$

4.3

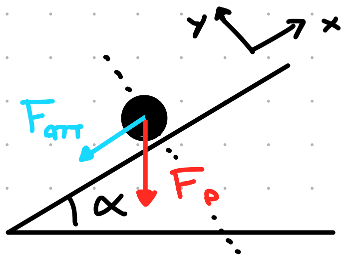


USO IL TEOREMA DELLE FORZE VIVE

$$W = \Delta K \Rightarrow -mg\mu\cos\alpha \cdot s - mg \cdot s\sin\alpha = -\frac{m}{2}v_0^2$$

$$\Rightarrow \quad sg(\mu\cos\alpha + \sin\alpha) = \frac{v_0^2}{2}$$

$$\Rightarrow \quad s = \frac{v_0^2}{2g(\mu\cos\alpha + \sin\alpha)} = 0{,}68$$



$$\begin{cases} ma_x = -N\mu - mg\sin\alpha \\ ma_y \equiv 0 = N - F_P\cos\alpha \end{cases}$$

$$\Rightarrow a_x = -g(\mu\cos\alpha + \sin\alpha)$$

$$x(t) = v_0 t - \frac{1}{2} g(\mu\cos\alpha + \sin\alpha)t^2 \quad ; \quad v(t) = v_0 - g(\mu\cos\alpha + \sin\alpha)t$$

IN $\quad x(\tau) \equiv s \quad ; \quad v(\tau) = 0 \quad \Rightarrow \quad \tau = \dfrac{v_0}{g(\mu\cos\alpha + \sin\alpha)} = 0{,}45\ s$

POI TORNA GIÙ E L'a_x CAMBIA

$$\begin{cases} m a_x = F_{ATT} - mg\sin\alpha \\ 0 = N - mg\cos\alpha \end{cases} \Rightarrow \quad a_x = g(\mu\cos\alpha - \sin\alpha)$$

$$x(t) = s + \frac{1}{2} g(\mu\cos\alpha - \sin\alpha)t^2 \quad , \quad v(t) = g(\mu\cos\alpha - \sin\alpha)t$$

SE $\quad x(\tilde{\tau}) \equiv 0 = s + \frac{1}{2} g(\mu\cos\alpha - \sin\alpha)\tilde{\tau}^2$

$$\Rightarrow \quad \tilde{\tau} = \left(\frac{-2s}{g(\mu\cos\alpha - \sin\alpha)} \right)^{1/2} = 0{,}65\ s$$

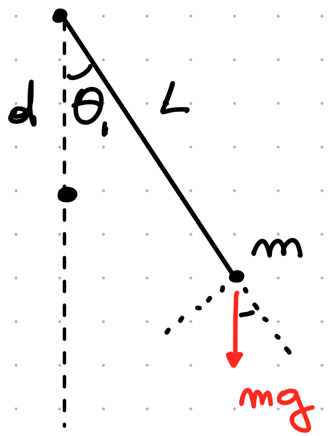
QUINDI

$$t_{TOT} = \tau + \tilde{\tau} = 1{,}10\ s$$

HO

$$W_{ATT} = F_{ATT} \cdot (2s) = \mu mg\cos\alpha \cdot 2s = 2{,}31\ J$$

4.4



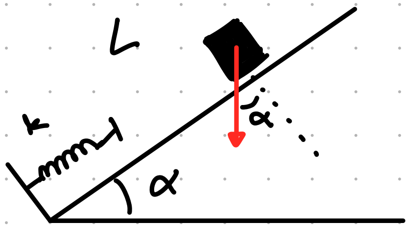
USO LA CONSERVAZIONE DELL'ENERGIA (ORIGINE NEL PUNTO PIÙ BASSO)
$(d = L/2)$

$$mg(L - L\cos\theta_1) = mg\left(\frac{L}{2} - \frac{L}{2}\cos\theta_2\right)$$

$$\Rightarrow \quad 1 - \cos\theta_1 = \frac{1}{2}(1 - \cos\theta_2)$$

$$\Rightarrow \quad \theta_2 = \cos^{-1}\left(1 - 2(1-\cos\theta)\right) = 42{,}94°$$

4.5



USO LA CONSERVAZIONE DELL'ENERGIA

$$mgL\sin\alpha = \frac{1}{2}k\Delta\ell^2 - mg\Delta\ell\sin\alpha$$

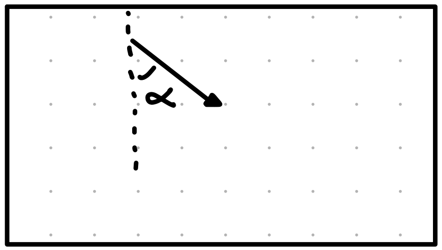
$$\Rightarrow \quad \Delta\ell^2 - \frac{2mg\sin\alpha}{k}\Delta\ell - \frac{2mgL\sin\alpha}{k} = 0$$

$$\Delta\ell_{1,2} = \frac{\frac{2mg\sin\alpha}{k} \pm \sqrt{\frac{4m^2g^2\sin^2\alpha}{k^2} + 4\frac{2mgL\sin\alpha}{k}}}{2}$$

$$= \frac{mg\sin\alpha \pm \sqrt{m^2g^2\sin^2 + 2mgkL\sin\alpha}}{k} = \begin{cases} 1{,}46 \text{ m} \\ -1{,}07 \text{ m} \end{cases}$$

TUTORAGGIO 5

5.1



HO SUL FINESTRINO

$$\begin{cases} v'_{px} = v'_p \sin\alpha \\ v'_{py} = v'_p \cos\alpha \end{cases}$$

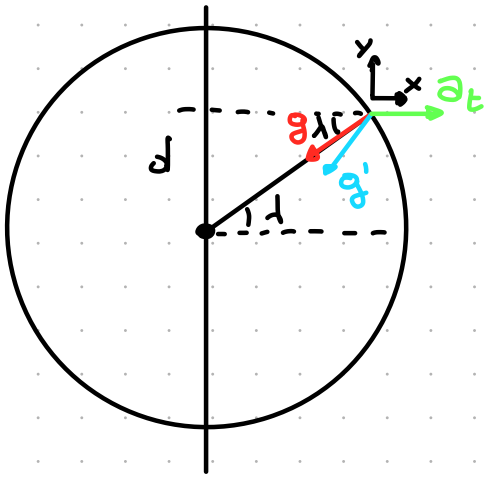
CON IL CAMBIO DI SR

$$\begin{cases} v_{px} = 0 = v'_{px} - v_A \\ v_{py} \equiv v_p = v'_{py} \end{cases} \Rightarrow \quad v'_p = \frac{v_A}{\sin\alpha}$$

$$v_p = \frac{v_A}{\sin\alpha}\cos\alpha = 46,20 \ km/h$$

5.2

$g' = 9.79 \ m/s^2 \qquad \lambda = 30°$



$$d = R\sin\lambda$$

HO LA TRASFORMAZIONE DI SR

$$\vec{a} = \vec{a}' + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

$$\Rightarrow \vec{a}' = \vec{a} - \underbrace{\vec{\omega} \times (\vec{\omega} \times \vec{r})}_{\omega^2 \vec{r} \ = \ \vec{a}_t}$$

$\omega^2 \vec{r} = \vec{a}_t$ → CENTRIFUGA (VISTA DA SR INERZIALE)

SCELGO $(\hat{x}, \hat{y})$ SULLA SUPERFICIE IN CUI HO

$$\vec{g}' = g'_x \hat{x} + g'_y \hat{y}$$ PER $g'_{x,y}$ USO LA TRASFORMAZIONE

$$\vec{g}' = -(g\cos\lambda - a_t)\hat{x} - (g\sin\lambda)\hat{y}$$
$$= (-g\cos\lambda + a_t)\hat{x} - g\sin\lambda\,\hat{y}$$

IL MODULO SARÀ

$$g'^2 = g^2\cos^2\lambda + a_t^2 - 2ga_t\cos\lambda + g^2\sin^2\lambda$$

$$\Rightarrow g^2 - 2ga_t\cos\lambda + a_t^2 - g'^2 = 0 \qquad (a_t = \omega^2 R\cos\lambda)$$

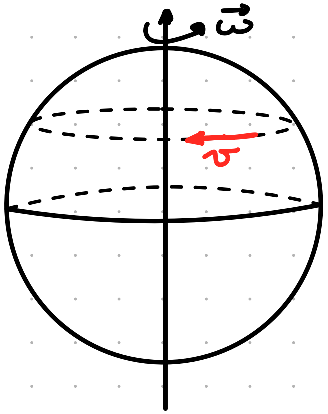
$$\Rightarrow g_{1,2} = \frac{2a_t\cos\lambda \pm \sqrt{4a_t^2\cos^2\lambda - 4a_t^2 + 4g'^2}}{2} = a_t\cos\lambda \pm \sqrt{-a_t^2\sin^2\lambda + g'^2}$$

$$\Rightarrow g_{1,2} = 0{,}029\cdot\cos\lambda \pm \sqrt{-8.511\cdot 10^{-4}\cdot 0{,}25 + 95{,}84} = 9{,}8148\ m/s$$

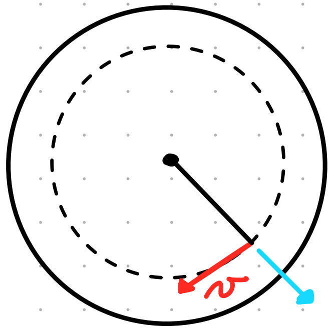
PER TROVARE L'ANGOLO

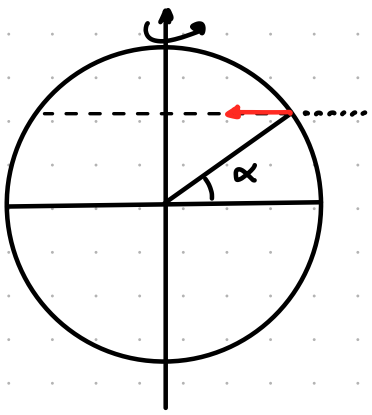
$$\lambda' = tg^{-1}\left(\frac{g\sin\lambda}{g\cos\lambda - a_t}\right) = 30{,}085° \Rightarrow \Delta\lambda = 0{,}085°$$

5.3 L'ACC. DI CORIOLIS È $\vec{a}_c = 2\vec{\omega} \times \vec{v}$



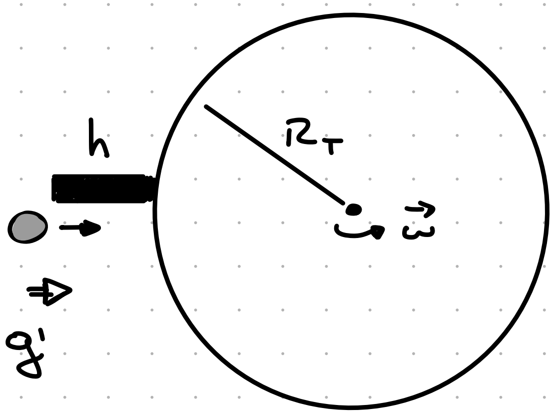
$|a_c| = 0,14544 \; m/s$

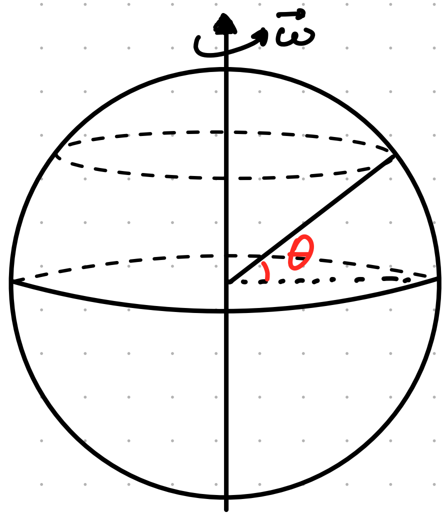




NON CAPISCO $\vec{a}_c$

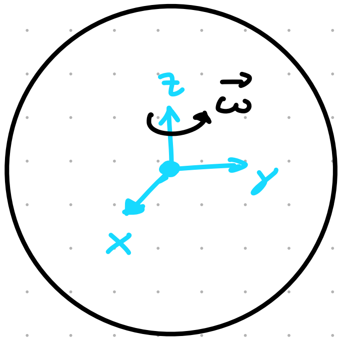
5.4 DALL'ALTO

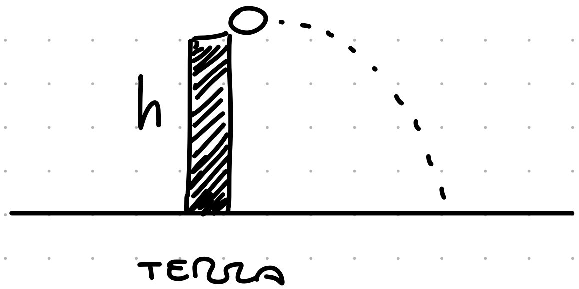




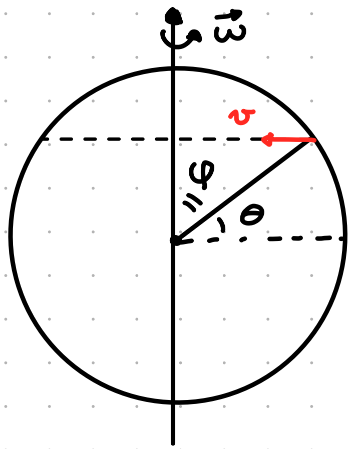
(VEDI TEORIA P. 102)

PRENDIAMO UN SR AD ASSI FISSI IN CUI LA TERRA RUOTA





S.5



CONOSCO LA RELAZIONE $\vec{a} = \vec{a}' + \vec{\omega} \times (\vec{\omega} \times \vec{r}) + 2\vec{\omega} \times \vec{v}'$

MA IN CUI

$$\omega = \frac{2\pi}{T_T} \quad , \quad \vec{v}' = \vec{v}_0 \quad , \quad \vec{a} = \vec{g}$$

E VOGLIO CHE $a' = g$

VEDO

$$\vec{\omega} \times \vec{r} = \omega r \sin\varphi \, \hat{u}_p = \omega r \cos\theta \, \hat{u}_p$$

VERSORE DEL PARALLELO

$$\vec{\omega} \times (\vec{\omega} \times \vec{r}) = \omega^2 r \cos\theta \, \hat{u}_m$$

VERSO $\hat{z}$

HO

$$0 = \omega^2 r \sin\varphi \, \hat{u}_m + 2\omega v_0 \, \hat{u}_m$$

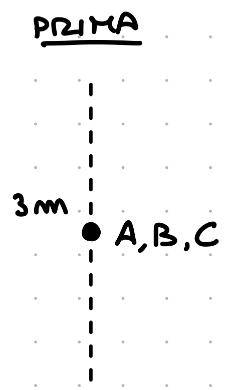
IN COMPONENTI $\omega^2 r \cos\theta + 2\omega v_0 = 0$

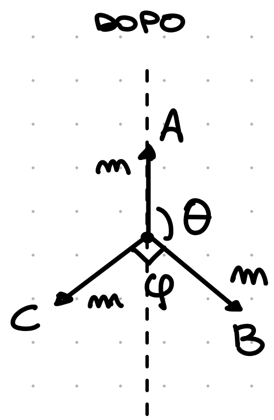
$$\Rightarrow \quad v_0 = -\frac{\omega r \cos\theta}{2}$$

HO $\quad r = R_T \quad \Rightarrow \quad v_0 = -\frac{\omega R_T \cos\theta}{2} = -163{,}78 \ m/s$

TUTORAGGIO 6

6.1 a) I FRAMMENTI GIACCIONO TUTTI SU UN PIANO PERCHÉ SI DEVE CONSERVARE LA QUANTITÀ DI MOTO TOTALE





$\theta = \frac{3\pi}{4}$

$\varphi = \pi/2$

K NOTA

CONSERVO L'IMPULSO NELLE DUE DIREZIONI NEL PIANO DEL MOTO

$$\begin{cases} 0 = m v_A - m v_B \cos\left(\frac{\varphi}{2}\right) - m v_c \cos\left(\frac{\varphi}{2}\right) \\ 0 = m v_B \sin\left(\frac{\varphi}{2}\right) - m v_c \sin\left(\frac{\varphi}{2}\right) \end{cases}$$

$$\Rightarrow \begin{cases} v_A = (v_B + v_C)\cos(\varphi/2) \\ v_B = v_C \end{cases}$$

$$\Rightarrow \begin{cases} v_A = 2 v_B \cos\left(\frac{\varphi}{2}\right) \\ v_B = v_C \end{cases}$$

POSSO USARE LA CONOSCENZA DI K

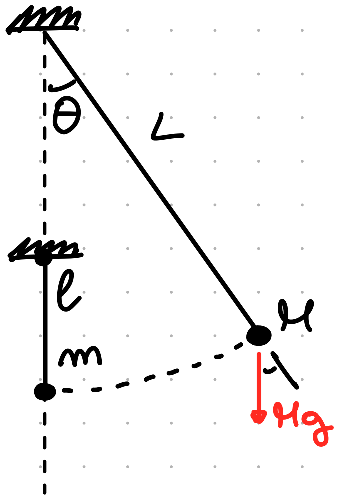
$$K = \frac{m}{2}\left(v_A^2 + v_B^2 + v_C^2\right)$$

$$\Rightarrow 4 v_B^2 \cos^2\left(\frac{\varphi}{2}\right) + v_B^2 + v_B^2 = \frac{2K}{m}$$

$$\Rightarrow 4 v_B^2 = \frac{2K}{m} \qquad \Rightarrow v_B = \sqrt{\frac{K}{2m}} \equiv v_C$$

DUNQUE $\quad v_A = 2 v_B \cdot \frac{\sqrt{2}}{2} = \sqrt{K/m}$

6.2



HO UN URTO ELASTICO, QUINDI CONSERVO ENERGIA E QUANTITÀ DI MOTO.

METTO L'ORIGINE DI $\hat{y}$ SULLA MASSA m (PUNTO PIÙ BASSO) E CONSERVO L'ENERGIA

$$Mg(L - L\cos\theta) = \frac{1}{2} M v_M^2$$

$$\Rightarrow v_M = \sqrt{2gL(1-\cos\theta)} = 1{,}15 \ m/s$$

CONSERVO L'IMPULSO PRIMA E DOPO L'URTO INSIEME ALL'ENERGIA

$$\begin{cases} M v_M = M v_M' + m v_m' \\ \frac{M}{2} v_M^2 = \frac{M}{2} v_M'^2 + \frac{m}{2} v_m'^2 \end{cases}$$

$$\begin{cases} v_m' = \frac{M}{m}(v_M - v_M') \\ M v_M^2 = M v_M'^2 + m \cdot \frac{M^2}{m^2}(v_M^2 + v_M'^2 - 2 v_M v_M') \end{cases}$$

$$\Rightarrow M v_M'^2 - M v_M^2 + \frac{M^2}{m}(v_M^2 + v_M'^2 - 2 v_M v_M') = 0$$

$$v_M'^2\left(1 + \frac{M}{m}\right) - 2\frac{M}{m} v_M v_M' + v_M^2\left(\frac{M}{m} - 1\right) = 0$$

$$v_{M\pm}' = \frac{2\frac{M}{m}v_M \pm \sqrt{4\frac{M^2}{m^2}v_M^2 - 4v_M^2\left(\frac{M^2}{m^2} - 1\right)}}{2\left(\frac{M}{m} + 1\right)} = \frac{2\frac{M}{m}v_M \pm 2v_M}{2\left(\frac{M}{m}+1\right)}$$

$$\Rightarrow v_{M\pm}' = \frac{\frac{M}{m} \pm 1}{\frac{M}{m} + 1} v_M$$

IL SEGNO + MI DA $v_M' = v_M$ $\Rightarrow$ NON C'È L'URTO

$$\Rightarrow v_M' = \frac{\frac{M}{m} - 1}{\frac{M}{m} + 1} v_M = \frac{M - m}{m} \frac{m}{M + m} v_M$$

$$\Rightarrow v_M' = \frac{M-m}{M+m} v_M = 0{,}38 \ m/s$$

PER CUI

$$v_m' = \frac{M}{m}(v_M - v_M')$$

$$= \frac{M}{m} v_M \left(1 - \frac{M-m}{M+m} \right) = \frac{M}{m} v_M \left(\frac{M+m-M+m}{M+m} \right)$$

$$= \frac{2M}{M+m} v_M = 1{,}53 \ m/s$$

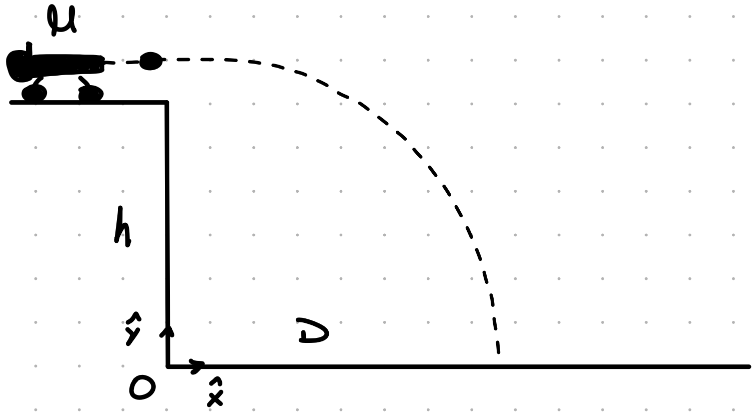
USO LA CONSERVAZIONE DELL'ENERGIA SU m PER TROVARE L'ANGOLO FINALE

$$\frac{m}{2} v_m'^2 = mg(\ell - \ell\cos\varphi)$$

$$\Rightarrow 1 - \cos\varphi = \frac{v_m'^2}{2g\ell} \qquad \Rightarrow \cos\varphi = 1 - \frac{v_m'^2}{2g\ell}$$

$$\Rightarrow \varphi = \cos^{-1}\left(1 - \frac{v_m'^2}{2g\ell} \right) = 66{,}2^\circ$$

6.3



STUDIO IL MOTO PARABOLICO PER TROVARE v_0 CON CUI PARTE m

$$\begin{cases} x(t) = v_0 t \\ y(t) = h - \frac{1}{2} g t^2 \end{cases}$$

ARRIVO IN $(D, 0)$:

$$\begin{cases} D = v_0 \tau \\ 0 = h - \frac{1}{2} g \tau^2 \end{cases} ; \quad \begin{cases} v_0 = D/\tau \\ \tau = \sqrt{\frac{2h}{g}} \end{cases} \Rightarrow \quad v_0 = D\sqrt{\frac{g}{2h}} = 110{,}74 \ m/s$$

USO LA CONSERVAZIONE DELL'IMPULSO DURANTE L'ESPULSIONE

$$0 = -M v_c + m v_0 \quad \Rightarrow \quad v_c = \frac{m}{M} v_0 = 0{,}111 \ m/s$$

USO IL TEOREMA DELLE FORZE VIVE

$$F_{ATT} \cdot d \geq \frac{M}{2} v_c^2 \quad \Rightarrow \quad F_{ATT} \geq \frac{M v_c^2}{2d} = 30{,}80 \ N$$

6.4 W , $F_{ATT} = -b v$

HO $W = \frac{dL}{dt} = \frac{F_M\, dx}{dt} = F_M v$

EQUAZIONE DEL MOTO : $m \frac{dv}{dt} = F_M + F_{ATT}$

A REGIME HO $ma = 0$

$$\Rightarrow F_M + F_{ATT} = 0 \Rightarrow F_M = -F_{ATT} \Rightarrow \frac{W}{v} = bv$$

$$\Rightarrow v = \sqrt{W/b}$$

SE HO $\alpha = \frac{dm}{dt}$ VALE

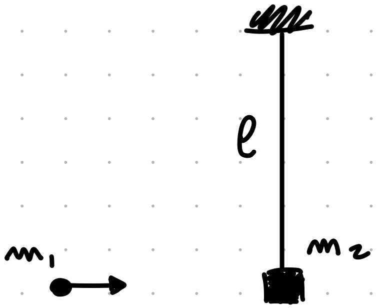
$$\frac{dp}{dt} = \frac{dm}{dt} v + m \frac{dv}{dt}$$, HO $v = v_{ORIZZ}$

E CHIEDO CHE $\frac{dv}{dt} \equiv 0 \Rightarrow \frac{dp}{dt} = \frac{dm}{dt} v$

DUNQUE $\Delta F = \frac{dp}{dt} = \alpha v$ FORZA NECESSARIA

TROVO LA POTENZA $\Delta F \cdot v = \Delta W = \alpha v^2 = \alpha \frac{W}{b}$

6.5



URTO TOTALMENTE ANELASTICO, CONSERVO L'IMPULSO

$$m_1 u = (m_1 + m_2) u'$$

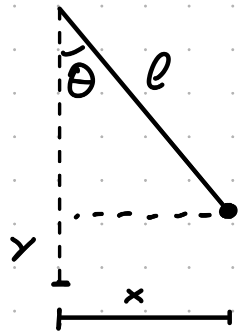
$$\Rightarrow u = \frac{m_1 + m_2}{m_1} u'$$

POSSO USARE ANCHE LA CONSERVAZIONE DEL ENERGIA (NON DELL'URTO) SUL SISTEMA $m_1 + m_2$

$$\frac{m_1 + m_2}{2} u'^2 = (m_1 + m_2) g \ell \cos\theta$$

$$\Rightarrow \begin{cases} u = \dfrac{m_1 + m_2}{m_1} u' \\ \dfrac{1}{2} u'^2 = g \ell \cos\theta \end{cases} \Rightarrow u = \frac{m_1 + m_2}{m_1} \sqrt{2 g \ell \cos\theta}$$

$\ell \cos\theta$ È LO SPOSTAMENTO VERTICALE y



PITAGORA $\quad \ell^2 = x^2 + (\ell - y)^2$

$$\Rightarrow \ell^2 = x^2 + \ell^2 + y^2 - 2\ell y$$

$$\Rightarrow y^2 - 2\ell y + x^2 = 0$$

$$y_{1,2} = \frac{2\ell \pm \sqrt{4\ell^2 - 4x^2}}{2} = \ell \pm \sqrt{\ell^2 - x^2}$$

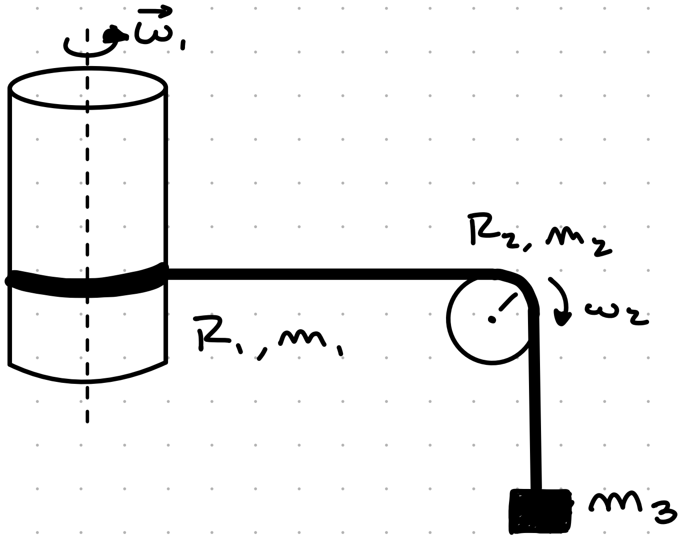
$\Rightarrow$ SOL "$-$" : $y = \ell\left(1 - \sqrt{1 - \frac{x^2}{\ell^2}}\right) \underset{x \ll \ell}{\sim} \ell\left(1 - 1 + \frac{1}{2}\frac{x^2}{\ell^2}\right) = \frac{1}{2}\frac{x^2}{\ell}$

COSÌ HO

$$\mu = \frac{m_1 + m_2}{m_1}\sqrt{2gy} \sim \frac{m_1 + m_2}{m_1}\sqrt{\frac{g}{\ell}}\, x$$

TUTORAGGIO 7

7.1

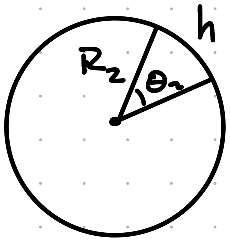


USO LA CONSERVAZIONE DELL'ENERGIA (TUTTO FERMO = MOTO)

$$m_3 g h = \frac{m_3}{2} v^2 + \frac{1}{2} I_{CIL}\,\omega_1^2 + \frac{1}{2} I_{CAR}\,\omega_2^2$$

CON $I_{CIL} = \frac{1}{2} m_1 R_1^2$ $\qquad I_{CAR} = \frac{1}{2} m_2 R_2^2$

E CONOSCO



$R_2\theta_2 = h$ MA ANCHE $h = R_1\theta_1$

E SO $\omega_1 = \frac{v}{R_1}$ $\qquad \omega_2 = \frac{v}{R_2}$

$$m_3 g R_2 \theta_2 = \frac{m_3}{2} v^2 + \frac{m_1}{4} v^2 + \frac{m_2}{4} v^2$$

POSSO DERIVARE RISPETTO t

$$m_3 g R_2 \omega_2 = m_3 v a + \frac{m_1}{2} v a + \frac{m_2}{2} v a$$

$$m_3 g v = m_3 v a + \frac{m_1}{2} v a + \frac{m_2}{2} v a$$

$$a \left(m_3 + \frac{m_1}{2} + \frac{m_2}{2} \right) = m_3 g$$

$$\Rightarrow a = \frac{2 m_3}{2 m_3 + m_1 + m_2} g = \frac{4}{15} g$$

ORA USO LE EQUAZIONI CARDINALI

$$\begin{cases} I_{CIL} \dot{\omega}_1 = T_{12} R_1 \\ I_{CAR} \dot{\omega}_2 = -T_{12} R_2 + T_{23} R_2 \\ m_3 a = m_3 g - T_{23} \end{cases} \Rightarrow T_{23} = m_3 (g - a)$$

$$\Rightarrow T_{23} = \frac{m_3 (m_1 + m_2)}{2 m_3 + m_1 + m_2} g$$

$$\omega_2 = \frac{v}{R_2} \Rightarrow \dot{\omega}_2 = \frac{a}{R_2}$$

2° RIGA $\Rightarrow$ $$\frac{m_2}{2} R_2^2 \frac{a}{R_2} = -T_{12} R_2 + T_{23} R_2$$

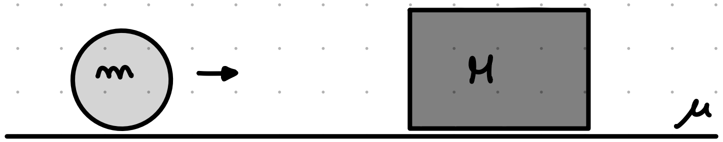
$$\Rightarrow \frac{m_2}{2} a = -T_{12} + T_{23} \Rightarrow T_{12} = -\frac{m_2}{2} a + T_{23}$$

$$\Rightarrow T_{12} = \frac{-m_2 m_3}{2m_3 + m_1 + m_2} g + \frac{m_1 m_3 + m_2 m_3}{2m_3 + m_1 + m_2} g$$

$$= \frac{m_1 m_3}{2m_3 + m_1 + m_2} g$$

$$= \frac{20}{15} g = \frac{4}{3} g$$

7.2



URTO COMPLETAMENTE ANELASTICO

HO SICURAMENTE IL TEOREMA DELLE FORZE VIVE

$$W = -F_{ATT} L = \Delta K = 0 - \frac{1}{2}(M+m){v'_0}^2$$

CON $F_{ATT} = (M+m)g\mu$

URTO COMPLETAMENTE ANELASTICO POSSO CONSERVARE SOLO LA QUANTITÀ DI MOTO

$$m v_0 = (m+M) v_0' \Rightarrow v_0' = \frac{m}{M+m} v_0$$

DUNQUE

$$L(M+m)g\mu = \frac{M+m}{2}\left(\frac{m}{m+M}\right)^2 v_0^2 \Rightarrow v_0^2 = \frac{2(M+m)^2}{m^2} g\mu L$$

$$\Rightarrow v_0 = \sqrt{\frac{2(M+m)^2}{m^2} g\mu L} = 36{,}35 \ m/s$$

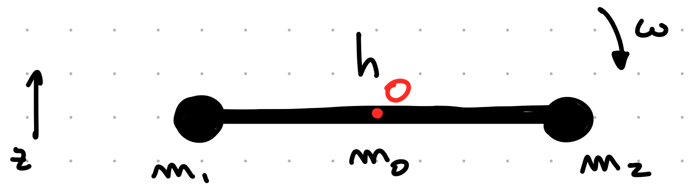
7.3 ℓ , $\lambda = \lambda_0\left(1 + \frac{x^2}{\ell^2}\right)$

PER DEFINIZIONE $x_{cm} = \frac{\int x \, dm}{\int dm}$ CON $\frac{dm}{dx} = \lambda$

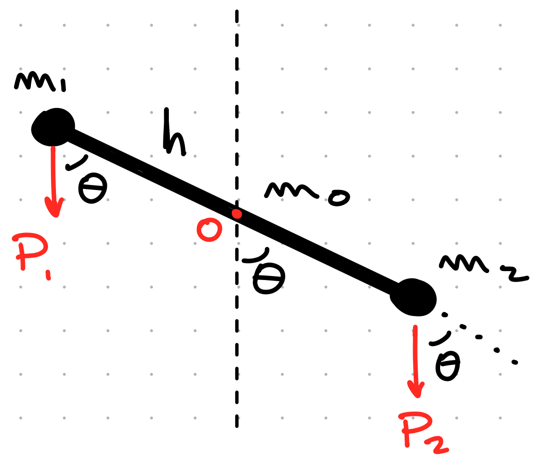
$$\Rightarrow x_{cm} = \frac{\int x \lambda \, dx}{\int \lambda \, dx} = \frac{\lambda_0 \int_0^\ell \left(x + \frac{x^3}{\ell^2}\right) dx}{\lambda_0 \int_0^\ell \left(1 + \frac{x^2}{\ell^2}\right) dx} = \frac{\frac{\ell^2}{2} + \frac{1}{\ell^2}\frac{\ell^4}{4}}{\ell + \frac{1}{\ell^2}\frac{\ell^3}{3}} = \frac{\ell^2\left(\frac{1}{2} + \frac{1}{4}\right)}{\ell\left(1 + \frac{1}{3}\right)}$$

$$\Rightarrow x_{cm} = \frac{3}{4} \cdot \frac{3}{4} \ell = \frac{9}{16} \ell$$

7.4



POSIZIONE INIZIALE



MI CALCOLO IL MOMENTO D'INERZIA

$$I = I_{ASTA} + m_1\left(\frac{h}{2}\right)^2 + m_2\left(\frac{h}{2}\right)^2$$

$$= \frac{1}{12} m_0 h^2 + \frac{1}{4} m_1 h^2 + \frac{1}{4} m_2 h^2$$

IN UN ISTANTE GENERICO HO IN GIOCO

$$P_{1_t} = m_1 g \sin\theta \qquad P_{2t} = m_2 g \sin\theta$$

E MI SCRIVO LA LEGGE CARDINALE

$$-I\ddot{\theta} = -P_1\frac{h}{2} + P_2\frac{h}{2}$$

$$\Rightarrow -\left(\frac{1}{12} m_0 h^2 + \frac{1}{4} m_1 h^2 + \frac{1}{4} m_2 h^2\right)\ddot{\theta} = -m_1 g \sin\theta \frac{h}{2} + m_2 g \sin\theta \frac{h}{2}$$

$$\Rightarrow -\frac{h^2}{12}\left(m_0 + 3m_1 + 3m_2\right)\ddot{\theta} = \left(m_2 g - m_1 g\right)\frac{h}{2}\sin\theta$$

$$\Rightarrow -\ddot{\theta} = \frac{6(m_2 - m_1)g}{(m_0+3m_1+3m_2)h}\sin\theta$$

PICCOLE OSCILLAZIONI $\Rightarrow \sin\theta \sim \theta$

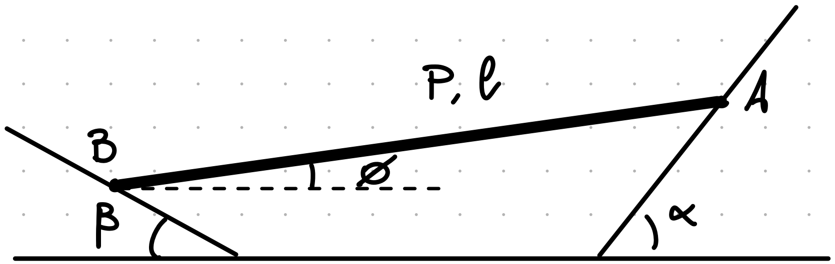
$$\Rightarrow \ddot{\theta} = -\frac{6(m_2-m_1)g}{(m_0+3m_1+3m_2)h}\theta$$

$$\Rightarrow \omega^2 = \frac{6(m_2-m_1)g}{(m_0+3m_1+3m_2)h}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{(m_0+3m_1+3m_2)h}{6(m_2-m_1)g}} = 2\pi\sqrt{\frac{12m_1 h}{6m_1 g}} = 2\pi\sqrt{\frac{2h}{g}}$$

m_1, $m_2 = 2m_1$
$m_0 = 3m_1$

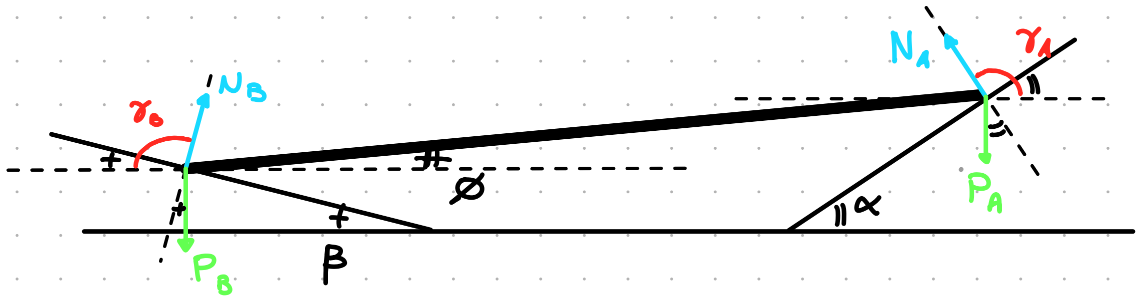
7.5



EQUILIBRIO $\Rightarrow$ $\vec{a}_{cm} = \vec{0}$ E $\vec{\alpha} = 0$ $\Rightarrow$ $\vec{F}_{RIS} = \vec{0}$, $\vec{M}_{RIS} = 0$

EQUAZIONE CARDINALE IN COMPONENTI

$$\begin{cases} \gamma_B = \frac{\pi}{2} + \beta \\ \gamma_A = \frac{\pi}{2} + \alpha \end{cases}$$



SCOMPONENDO $P_{A,B}$ TROVA LE REAZIONI VINCOLARI (USO LA I EQUAZIONE CARDINALE)

$$\begin{cases} N_A = P \cos\alpha \\ N_B = P \cos\beta \end{cases}$$

CON LA II EQUAZIONE CARDINALE, SCEGLIENDO IL BARICENTRO COME POLO E IMPONENDO $\vec{M}_{RIS} = \vec{0}$ TROVO (DEVO CALCOLARE I MOMENTI DI P_A E P_B):

$\ell/2$

$$0 = -\frac{\ell}{2} N_B \sin(\pi - \gamma_B - \phi) + \frac{\ell}{2} N_A \sin(\pi - \gamma_A + \phi)$$

$$\cos\beta \sin\left(\pi - \frac{\pi}{2} - \beta - \phi\right) = \cos\alpha \sin\left(\pi - \frac{\pi}{2} - \alpha + \phi\right)$$

$$\Rightarrow \sin\left(\frac{\pi}{2} - \beta - \phi\right) = \sin\left(\frac{\pi}{2} + \phi - \alpha\right)\frac{\cos\alpha}{\cos\beta} \qquad \left(\cos\theta = \sin\left(\frac{\pi}{2} + \theta\right)\right)$$

$$\Rightarrow \cos(\beta + \phi) = \cos(\alpha - \phi)\frac{\cos\alpha}{\cos\beta} \qquad \left(\cos(\theta \pm \varphi) = \cos\theta\cos\varphi \mp \sin\theta\sin\varphi\right)$$

$$\Rightarrow \cos\beta\cos\phi - \sin\beta\sin\phi = (\cos\alpha\cos\phi + \sin\alpha\sin\phi)\left(\frac{\cos\alpha}{\cos\beta}\right)$$

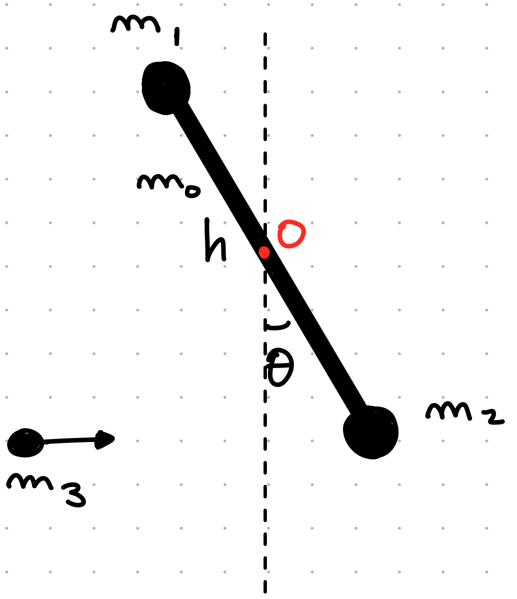
$$\Rightarrow \cancel{\cos\phi}\left(\cos\beta - \frac{\cos^2\alpha}{\cos\beta}\right) = \cancel{\sin\phi}\left(\sin\beta + \frac{\sin\alpha\cos\alpha}{\cos\beta}\right)$$

$$\tan\phi = \frac{\cos\beta - \frac{\cos^2\alpha}{\cos\beta}}{\sin\beta + \frac{\sin\alpha\cos\alpha}{\cos\beta}} \sim 0{,}27$$

NO

TUTORAGGIO 8

8.1



URTO COMPLETAMENTE ANELASTICO, POSSO CONSERVARE LA QUANTITÀ DI MOTO NELL'URTO, MA USO INVECE IL MOMENTO ANGOLARE

$$m_3 v \frac{h}{2}\cos\theta = I\,\omega'$$

CON MOMENTO D'INERZIA

$$I = \frac{1}{12} m_0 h^2 + m_1 \left(\frac{h}{2}\right)^2 + (m_2+m_3)\left(\frac{h}{2}\right)^2$$

$$= \frac{h^2}{12}\left(m_0 + 3(m_1+m_2+m_3)\right)$$

DUNQUE DOPO L'URTO SI HA

$$\omega' = \frac{m_3 v h}{2I}\cos\theta = \frac{6 m_3 v \cos\theta}{h\left(m_0+3(m_1+m_2+m_3)\right)}$$

ORA POSSO CONSERVARE L'ENERGIA TRA LA SITUAZIONE DOPO L'URTO E DOPO UN GIRO (CONDIZIONE MINIMA ⇒ ASTA FERMA → VERTICALE CON m_2+m_3 IN ALTO) (ORIGINE = BARICENTRO)

$$m_1 g \frac{h}{2}\cos\theta - (m_2+m_3) g \frac{h}{2}\cos\theta + \frac{1}{2} I \omega'^2 \geq -m_1 g \frac{h}{2} + (m_2+m_3) g \frac{h}{2}$$

$$\Rightarrow \frac{1}{2} I \left(\frac{m_3 v h \cos\theta}{2I} \right)^2 \geq -m_1 g \frac{h}{2}(1+\cos\theta) + (m_2+m_3) g \frac{h}{2}(1+\cos\theta)$$

$$\Rightarrow \frac{m_3^2 v^2 h^2 \cos^2\theta}{8I} \geq g \frac{h}{2}(1+\cos\theta)(m_2+m_3-m_1)$$

$$\Rightarrow v^2 \geq \frac{4 I g h (1+\cos\theta)}{m_3^2 h^2 \cos^2\theta}(m_2+m_3-m_1)$$

$$\Rightarrow v^2 \geq \frac{gh(m_0+3(m_1+m_2+m_3))}{3 m_3^2 \cos^2\theta}(1+\cos\theta)(m_2+m_3-m_1)$$

m_1, $m_2 = 2m_1$, $m_3 = m_1$, $m_0 = 3m_1$

$$\Rightarrow v^2 \geq \frac{15 m_1 g h}{3 m_1^2 \cos^2\theta}(1+\cos\theta)\, 2m_1$$

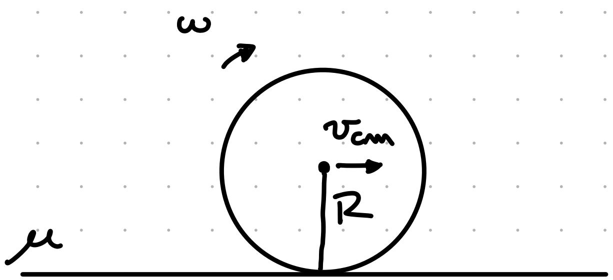
$$\Rightarrow v^2 \geq \frac{10 g h (1+\cos\theta)}{\cos^2\theta}$$

$$\Rightarrow \quad v \geq \sqrt{\frac{10(1+\cos\theta)}{\cos^2\theta}\, gh} = \sqrt{\frac{40\left(1+\frac{\sqrt{3}}{2}\right)}{3}\, gh}$$

$$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad v \geq \sqrt{\frac{20(2+\sqrt{3})}{3}\, gh}$$

8.2



HO $\quad I_{cil} = \frac{1}{2} m R^2$

POSSO USARE IL TEOREMA DELLE FORZE VIVE

$$W = -F_{att}\,\ell = 0 - \frac{1}{2} I \omega^2$$

$$\Rightarrow \quad \omega^2 = \frac{2F_{att}\,\ell}{I} \quad \Rightarrow \quad \omega = \sqrt{\frac{2F_{att}\,\ell}{I}}$$

NON CONOSCO NÉ F_{att} NÉ ℓ! USO LE EQUAZIONI CARDINALI

POLO // CENTRO CILINDRO
$$\begin{cases} \begin{cases} m a_y = -F_{ATT} \\ m a_y = 0 = N - mg \end{cases} \\ I\alpha = -F_{ATT} R \end{cases} \qquad \begin{cases} m a_y = -F_{ATT} \\ N = mg \\ F_{ATT} R = I\alpha \end{cases}$$

$$a_y = a_{cm} = \frac{dv}{dt} \Rightarrow \frac{dv}{dt} = -\frac{F_{ATT}}{m} \Rightarrow v(t) = v_{0cm} - \frac{F_{ATT}}{m} t$$

$$\alpha = \frac{d\omega}{dt} \Rightarrow \omega(t) = \omega_0 - \frac{F_{ATT} R}{I} t$$

AD UN CERTO ISTANTE VOGLIO CHE SI FERMI :

$$v(\tau) = 0 \Rightarrow \tau = \frac{m v_{0cm}}{F_{ATT}}$$

$$\Rightarrow \omega(\tau) = 0 = \omega_0 - \frac{F_{ATT} R}{I} \frac{m v_{0cm}}{F_{ATT}}$$

$$\Rightarrow \omega_0 = \frac{m R v_{0cm}}{I} = \frac{2 v_{0cm}}{R}$$

SE INIZIA MOTO DI PURO ROTOLAMENTO ALLORA AL TEMPO $\tilde{\tau}$

$$\begin{cases} v(\tilde{\tau}) = v_{cm} = v_{0cm} - \dfrac{F_{ATT}}{m}\tilde{\tau} \\ \\ \omega(\tilde{\tau}) = \omega_0 - \dfrac{F_{ATT} R}{I}\tilde{\tau} \end{cases}$$

$$\Rightarrow \quad \tilde{\tau} = \frac{m}{F_{ATT}}\left(v_{0cm} - v_{cm}\right)$$

$$\Rightarrow \quad \omega(\tilde{\tau}) = \omega_0 - \frac{F_{ATT} R}{I}\,\frac{m}{F_{ATT}}\left(v_{0cm} - v_{cm}\right)$$

$$\Rightarrow \quad \omega(\tilde{\tau}) = \omega_0 - \frac{mR}{I}\left(v_{0cm} - v_{cm}\right)$$

USO $\quad \omega(\tilde{\tau}) = \dfrac{v_{cm}(\tilde{\tau})}{R}, \quad v_{cm} = -\dfrac{4}{9}v_{0cm}$

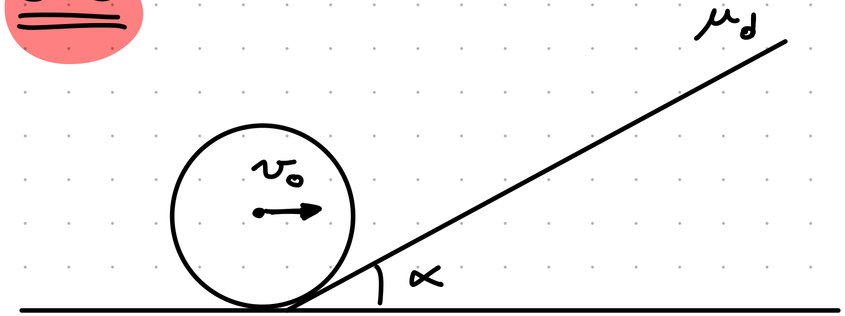
$$\Rightarrow \quad \omega(\tilde{\tau}) \equiv \frac{v_{cm}(\tau)}{R} = \omega_0 - \frac{2}{R}\left(v_{0cm} + \frac{4}{9}v_{0cm}\right)$$

$$\Rightarrow \quad -\frac{4}{gR} v_{0cm} = \omega_0 - \frac{2}{R}\frac{13}{9} v_{0cm}$$

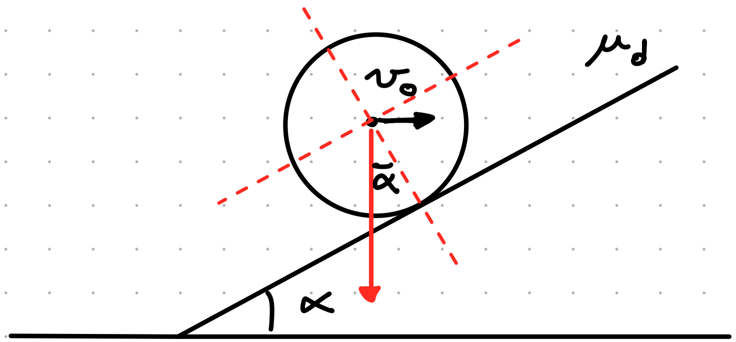
$$\Rightarrow \quad \omega_0 = \frac{v_{0cm}}{gR}\left(-4 + 26\right)$$

$$\Rightarrow \quad \omega_0 = \frac{22}{9}\frac{v_{0cm}}{R}$$

8.3



IN UN GENERICO ISTANTE



SCRIVO LE EQUAZIONI CARDINALI CON POLO NELL'ASSE DI SIMMETRIA DEL CILINDRO ($\hat{x}, \hat{y}$ SUL PIANO INCLINATO)

$$\begin{cases} \begin{cases} m a_x = -F_{ATT} - mg\sin\alpha \\ m a_y \equiv 0 = N - mg\cos\alpha \end{cases} \\ -I\alpha = R F_{ATT} \end{cases} \quad , \quad I = \frac{1}{2} m R^2$$

$$\begin{cases} \begin{cases} N = mg\cos\alpha \\ a_x = -\dfrac{F_{ATT}}{m} - g\sin\alpha \end{cases} \\ I\dot{\omega} = +R F_{ATT} \end{cases} \Rightarrow \begin{cases} v_{cm}(t) = v_{0cm} - \left(\dfrac{F_{ATT}}{m} + g\sin\alpha\right)t \\ \omega(t) = \omega_0 + \dfrac{R F_{ATT}}{I} t \end{cases}$$

HO $$F_{ATT} = \mu_d N = \mu_d mg\cos\alpha$$

$$\Rightarrow \begin{cases} v_{cm}(t) = v_{0cm} - (\mu_d g\cos\alpha + g\sin\alpha)t \\ \omega(t) = \omega_0 + \dfrac{R\mu_0 mg}{I}\cos\alpha\, t \end{cases}$$

ALL'ISTANTE τ ROTOLA E STRISCIA, QUINDI VALE CHE

$$\omega(\tau) = \frac{v_{cm}(\tau)}{R}$$

$$\Rightarrow \omega_0 + \frac{2\mu_d g}{R}\cos\alpha\,\tau = \frac{v_{0cm}}{R} - \frac{\mu_d g}{R}\cos\alpha\,\tau - \frac{g\sin\alpha}{R}\tau$$

$$\Rightarrow \omega_0 - \frac{v_{0cm}}{R} = \left(-\frac{3\mu_d g}{R}\cos\alpha - \frac{g\sin\alpha}{R}\right)\tau$$

$$\Rightarrow \omega_0 R - v_{0cm} = -(3\mu_d g\cos\alpha + g\sin\alpha)\,\tau$$

$$\Rightarrow \tau = \frac{v_{0cm} - \omega_0 R}{3\mu_d g\cos\alpha + g\sin\alpha}$$

MA IL RULLO È SOLO IN MOTO TRASLATORIO PRIMA DI SALIRE SUL PIANO $\Rightarrow \omega_0 = 0$

$$\Rightarrow \tau = \frac{v_{0cm}}{g\sin\alpha + 3\mu_d g\cos\alpha}$$

POSSO TROVARE L'ISTANTE IN CUI SI FERMA

$$v(\tilde{\tau}) \equiv 0 = v_{0cm} - (\mu_d g\cos\alpha + g\sin\alpha)\,\tilde{\tau}$$

$$\Rightarrow \tilde{\tau} = \frac{v_{0cm}}{g\sin\alpha + \mu_d g\cos\alpha} = 1{,}02\ s$$

HO

$$v(t) = v_{0cm} - (\mu_d g\cos\alpha + g\sin\alpha)t$$

$$\Rightarrow \ell(t) = v_{0cm}t - \frac{1}{2}(\mu_d g\cos\alpha + g\sin\alpha)t^2$$

DA 0 A τ IL RULLO PERCORRE ℓ_1 E DOPODICHE INIZIA A ROTOLARE SENZA STRISCIARE

$$\ell_1 = v_{0cm}\tau - \frac{1}{2}(\mu_g g\cos\alpha + g\sin\alpha)\tau^2 = 10,81 \text{ m}$$

E POI COMINCIA A RALLENTARE E FERMARSI. UTILIZZO LA CONSERVAZIONE DELL'ENERGIA

$$\frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2 = mg\Delta\ell\sin\alpha$$

VALE

$$\omega(\tau) = \frac{v(\tau)}{R} \quad , \quad I = \frac{1}{2}mR^2$$

$$\Rightarrow \frac{m}{2}v_{cm}^2(\tau) + \frac{m}{4}v_{cm}^2 = mg\Delta\ell\sin\alpha$$

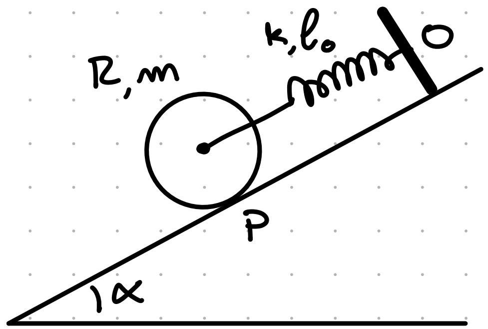
$$\Rightarrow \frac{3}{4}v_{cm}^2(\tau) = g\Delta\ell\sin\alpha$$

$$\Rightarrow \Delta \ell = \frac{3 v_{cm}^2(\tau)}{4 g \sin\alpha} = \frac{3}{4 g \sin\alpha}\left(v_{0cm} - \frac{(\mu_d g\cos\alpha + g\sin\alpha)}{(\mu_d 3 g\cos\alpha + g\sin\alpha)} v_{0cm}\right)^2$$

$$\Rightarrow \Delta \ell = 3{,}45 \text{ m}$$

$$\Rightarrow \ell = \ell_1 + \Delta\ell = 14{,}26 \text{ m}$$

8.4



TROVA $x(t)$.

EQUAZIONI CARDINALI (POLO P)

$$\begin{cases} \begin{cases} m a_x = k\Delta\ell - mg\sin\alpha \\ m a_y = 0 = N - mg\cos\alpha \end{cases} \\ I\dot{\omega} = mg\sin\alpha R - k\Delta\ell R \end{cases} \quad , \quad \Delta\ell = x - \ell_0$$

SE MOTO DI PURO ROTOLAMENTO $\omega = \dfrac{v_{cm}}{R}$

$$\Rightarrow \dot{\omega} = \frac{\dot{v}}{R}$$

$$\Rightarrow I \frac{\dot{v}}{R} = \left(mg \sin\alpha - k(x - l_0)\right) R \quad , \quad I = I_{cm} + mR^2 = \frac{3}{2} m R^2$$

$$\Rightarrow 3\frac{m}{2} R \dot{v} = \left(mg \sin\alpha - k(x - l_0)\right) R$$

$$\Rightarrow \ddot{x} = \frac{2}{3} g \sin\alpha - \frac{2}{3}\frac{k}{m} x + \frac{2}{3}\frac{k}{m} l_0$$

$$\Rightarrow \ddot{x} = -\frac{2}{3}\frac{k}{m} x + \frac{2}{3}\frac{k}{m} l_0 + \frac{2}{3} g \sin\alpha$$

OSCILLATORE ARMONICO SOTTOPOSTO AD UNA FORZA COSTANTE :

$$\omega = \sqrt{\frac{2k}{3m}}$$

SOL NON SO !