Esercizi

Meccanica

ESERCIZI a.a. 2021-2022

ARGOMENTI :

- exe 1,2,3 — CINEMATICA
- exe 4,5,6 — LEGGI DELLA DINAMICA
- exe 7 — LAVORO ED ENERGIA
- exe 8,9 — SISTEMI DI RIFERIMENTO
- exe 10,11,12,13,14 — CORPO RIGIDO
- exe 15 — GRAVITAZIONE

Annotazioni

Numerazione e soluzioni fanno riferimento al materiale fornito dal professor Massaro.

In alcune esercitazioni sono stati fatti argomenti utili per la teoria.

- exe 4 : molle (in serie e parallelo)
  piano inclinato
- exe 5 : macchinario di Atwood
- exe 10 : il problema del razzo
- es 2 exe 11 : Pendolo balistico
- es 5 exe 11 : richiami di urti
- inizio exe 12 : elementi infinitesimi di elementi di superficie e volume nelle varie coordinate

Esercizi che ho saltato:

- exe 5 : 5.2

Esercizi da controllare:

- exe 9 : 9.1
- exe 11 : 11.2 , 11.5
- exe 12 : 12.1 (punto a)
- exe 13 : 13.5

exe 1

1.1

$v_a = 120 \ km/h = 120 \frac{10^3}{3,6 \cdot 10^3} \frac{m}{s} = 33,3 \frac{m}{s}$

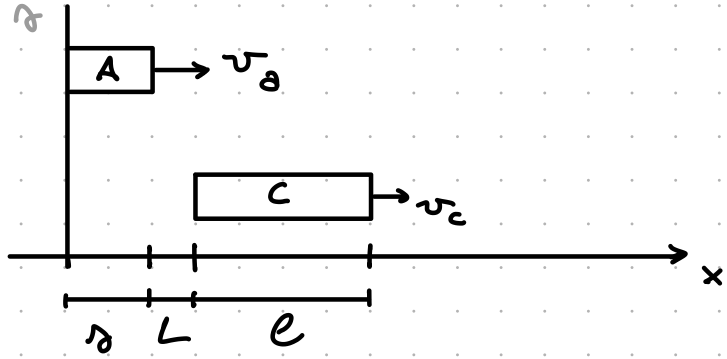
$v_c = 90 \ km/h = 25 \ m/s$

$L = 2 \ m$

$\ell = 11 \ m$

$s = 4 \ m$

τ = TEMPO SORPASSO = ?



FISSO IL SR. t_c $x_{oa} = s$ $x_{oc} = s + L + \ell$ (CONSIDERO LE PARTI FRONTALI DEI VEICOLI)

LEGGI ORARIE : $x(t) = x_0 + v(t - t_0)$, FISSO $t_0 = 0$

$$\begin{cases} x_a(t) = s + v_a t \\ x_c(t) = s + L + \ell + v_c t \end{cases}$$

IL SORPASSO È COMPLETO QUANDO LA PARTE POSTERIORE DI a SUPERA LA PARTE ANTERIORE DI c, QUINDI QUANDO

$$x_a(\tau) = x_c(\tau) + s \Rightarrow s + v_a \tau = s + L + \ell + v_c \tau + s$$

$$\Rightarrow (v_a - v_c)\tau = L + \ell + s \Rightarrow \tau = \frac{L + \ell + s}{v_a - v_c} = \boxed{2,05 \ s}$$

1.2 PISTA : L

VELOCITÀ : v_1 , v_2

TROVA : t_{01} ; x_{02} ; t_{02}

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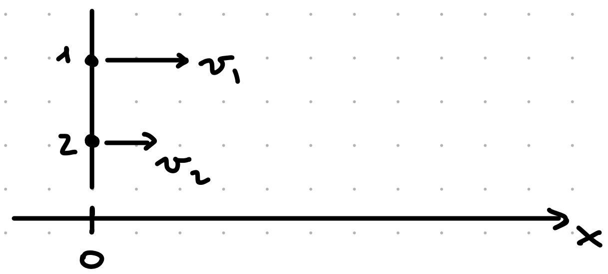
i)



ASSUMO $v_1 > v_2$

$t_{02} = 0$ $\quad t_{01} = ?$

MA VOGLIO 1 CHE VINCE.

LEGGI ORARIE:
$$\begin{cases} x_1(t) = v_1 (t - t_{01}) \\ x_2(t) = v_2 t \end{cases}$$

2 PER FINIRE IL PERCORSO CI METTE: $x_2(\tau_2) = v_2 \tau_2 \equiv L \Rightarrow \tau_2 = \dfrac{L}{v_2}$

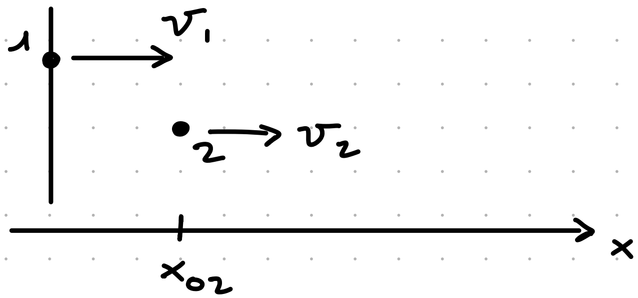
MENTRE 1 : $x_1(\tau_1) = v_1 \tau_1 - v_1 t_{01} \equiv L \Rightarrow \tau_1 = \dfrac{L}{v_1} + t_{01}$

PER VINCERE 1 VOGLIO $\tau_1 < \tau_2$

PER CUI

$$\frac{L}{v_1} + t_{01} < \frac{L}{v_2} \Rightarrow \boxed{t_{01} < \frac{v_1 - v_2}{v_1 v_2} L}$$

ii) ORA VOGLIO CHE VINCA 2.



PRENDO $x_{01} = 0 \qquad t_{01} = 0$

$x_{02} = ? \qquad t_{02} = 0$

I TEMPI PER COMPLETARE IL PERCORSO SONO:

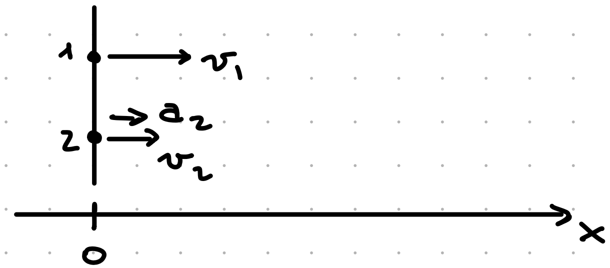
$$x_1(\tau) = v_1 \tau_1 \equiv L \Rightarrow \tau_1 = \frac{L}{v_1}$$

$$x_2(\tau) = x_{02} + v_2 \tau_2 \equiv L \Rightarrow \tau_2 = \frac{L}{v_2} - \frac{x_{02}}{v_2}$$

PER VINCERE 2: $\tau_2 < \tau_1$

$$\Rightarrow \frac{L}{v_2} - \frac{x_{02}}{v_2} < \frac{L}{v_1} \Rightarrow \frac{x_{02}}{v_2} > \frac{v_1 - v_2}{v_1 v_2} L \Rightarrow \boxed{x_{02} > \frac{v_1 - v_2}{v_1} L}$$

iii) VOGLIO CHE VINCA 2.



HO a_2 ; $v_{02} = 0$; $t_{01} = 0$; $t_{02} = ?$; $x_{0,1,2} = 0$

LEGGI ORARIE: $\begin{cases} x_1(t) = v_1 t \\ x_2(t) = \frac{1}{2} a_2 (t - t_{02})^2 \end{cases}$

GLI ARRIVI AL TRAGUARDO SONO:

$$\begin{cases} x_1(\tau_1) \equiv L \\ x_2(\tau_2) \equiv L \end{cases} \Rightarrow \begin{cases} \tau_1 = L/v_1 \\ (\tau_2 - t_{02})^2 = \dfrac{2L}{a_2} \end{cases} \Rightarrow \begin{cases} \tau_1 = L/v_1 \\ \tau_2 = \sqrt{\dfrac{2L}{a_2}} + t_{02} \end{cases}$$

VINCE 2 SE $\tau_2 < \tau_1$

$$\Rightarrow \sqrt{\frac{2L}{a_2}} + t_{02} < \frac{L}{v_1} \Rightarrow \boxed{t_{02} < \frac{L}{v_1} - \sqrt{\frac{2L}{a_2}}}$$

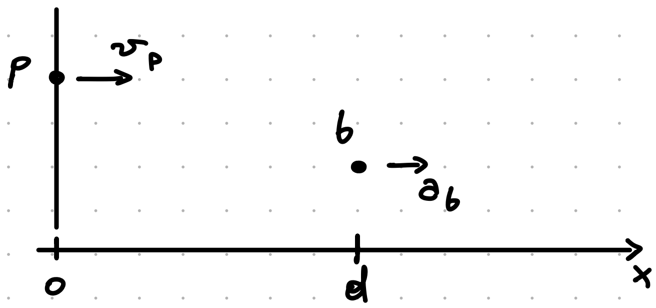
1.3

$v_p = 3\ m/s$

$x_{0b} = d$

$v_{0b} = 0$

$a_b = 0{,}3\ m/s^2$



DETERMINA: i) $d = 12\ m$, p SALE SU b?

ii) d_c VALORE CRITICO

iii) $d = d_c$, TROVA v_b IN QUEL ISTANTE E v_{mb}

LEGGI ORARIE: $x_p(t) = v_p t$ (FISSO $t_{0p} = t_{0b} = 0$)

$x_b(t) = d + \frac{1}{2} a_b t^2$

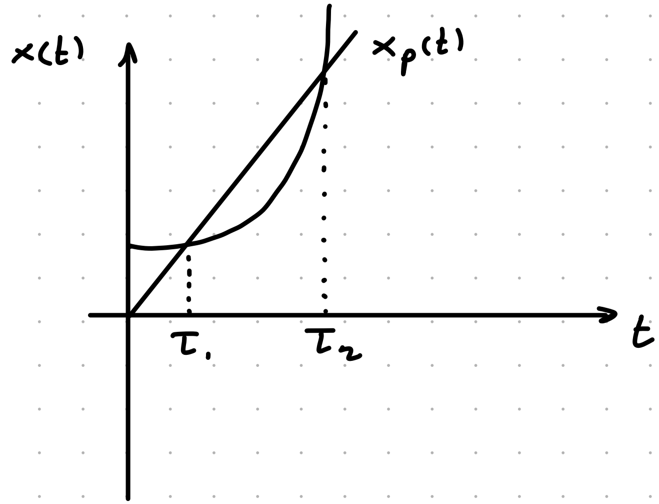
QUANDO p SALE?

$$x_p(\tau) = x_b(\tau) \Rightarrow \frac{1}{2} a_b \tau^2 - v_p \tau + d = 0$$

$$\frac{v_p \pm \sqrt{v_p^2 - 2 a_b d}}{a_b} = \tau_{1,2}$$

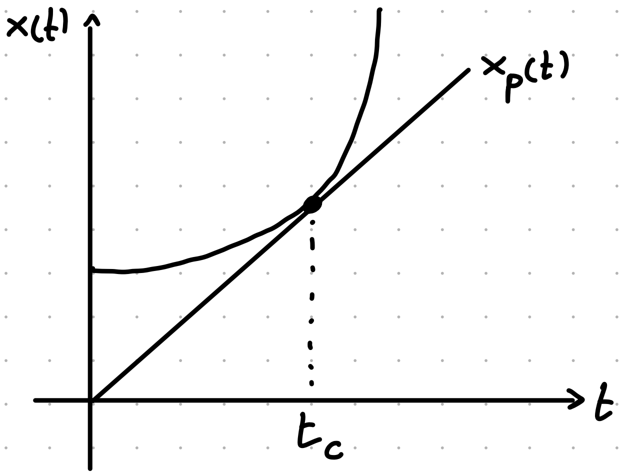
i) $d = 12\ m$ $\quad \tau_{1,2} = \dfrac{3\frac{m}{s} \pm \sqrt{1{,}8}\ \frac{m}{s}}{0{,}3\ \frac{m}{s^2}} = \begin{cases} 14{,}47\ s \\ 5{,}52\ s \end{cases}$

SI p RAGGIUNGE b DOPO $\tau_1 = 5{,}52\ s$ (E SE NON SALISSE LO RIINCONTREREBBE DOPO $\tau_2 = 14.47\ s$)



ii) d_c È T.C. $\tau_{1,2}$ COINCIDONO $\Rightarrow \sqrt{v_p^2 - 2a_b d_c} = 0$

$$\Rightarrow 2a_b d_c = v_p^2 \Rightarrow \boxed{d_c = \frac{v_p^2}{2a_b}}$$



iii) SE $d = d_c$ ALLORA $\tau_c = \frac{v_p}{a_b}$ $\Rightarrow$ P RAGGIUNGE B IN τ_c

$$x_b(t) = d_c + \frac{1}{2} a_b t^2 \rightsquigarrow v_b(t) = \frac{dx}{dt} = a_b t$$

$$\Rightarrow \boxed{v_b(\tau_c) = a_b \cdot \frac{v_p}{a_b} = v_p}$$

E LA VELOCITÀ MEDIA SARÀ:

$$x_b(\tau_c) = d_c + \frac{1}{2} a_b \frac{v_p^2}{a_b^2} = d_c + \frac{v_p^2}{2a_b}$$

$$v_{mb} = \frac{x_b(\tau_c) - x_{0b}}{\tau_c - t_{0b}} = \frac{d_c + \frac{v_p^2}{2a_b}}{v_p/a_b} = \frac{a_b d_c}{v_p} + \frac{v_p^2}{2a_b} \frac{a_b}{v_p} = \frac{a_b}{v_p} \frac{v_p^2}{2a_b} + \frac{v_p}{2}$$

$$= \frac{v_p}{2} + \frac{v_p}{2} = \boxed{v_p \equiv v_{bm}}$$

1.4 t_0 INIZIA A NEVICARE

NEVE HA INTENSITÀ COSTANTE ρ

$v \propto 1/h(t)$

$\Delta t_1 = 2h$ PULISCE $L_1 = 4\ km$

$\Delta t_2 = 2h$ PULISCE $L_2 = 2\ km$

t_{01} = ORE 12.00

DETERMINA t_0.

LEGGE ORARIA DELLO SPAZZANEVE:

$$v(t) = \frac{c}{h(t)}, c \in \mathbb{R} \quad ; \quad v(t) = \frac{dx}{dt} = \frac{c}{h(t)} \Rightarrow dx = \frac{c}{h(t)} dt$$

$$\Rightarrow x(t) = c \int_{t_0}^{t} \frac{1}{h(t)} dt$$

MA IL TESTO MI DICE CHE LA NEVE CADE IN MODO COSTANTE NEL TEMPO

$$\Rightarrow h(t) \propto t - t_0$$

$$\Rightarrow v = \frac{c}{t - t_0}$$

$$\Rightarrow \int_a^b v(t)dt = c \log(t-t_0)\Big|_a^b$$

SEMPRE IL TESTO MI DICE (FISSO $t_{01} = 0$)

$$\begin{cases} x(t_1) = 4\,km = c\log(t-t_0)\Big|_{t_{01}=0}^{t_1=2h} = c\log\left(\frac{t_1-t_0}{-t_0}\right) = c\log\left(\frac{t_0-t_1}{t_0}\right) \\ x(t_2) = 2\,km = c\log(t-t_0)\Big|_{t_1=2h}^{t_2=4h} = c\log\left(\frac{t_2-t_0}{t_1-t_0}\right) \end{cases}$$

POSSO FARNE IL RAPPORTO:

$$\frac{x(t_1)}{x(t_2)} = 2 = \frac{\log\left(\frac{t_0-t_1}{t_0}\right)}{\log\left(\frac{t_2-t_0}{t_1-t_0}\right)} \Rightarrow 2\log\left(\frac{t_2-t_0}{t_1-t_0}\right) = \log\left(\frac{t_0-t_1}{t_0}\right)$$

$$\Rightarrow \left(\frac{t_2-t_0}{t_1-t_0}\right)^2 = \left(\frac{t_0-t_1}{t_0}\right) \Rightarrow t_0(t_2-t_0)^2 = (t_0-t_1)(t_1-t_0)^2$$

$$\Rightarrow t_0(t_2^2+t_0^2-2t_0t_2) = (t_0-t_1)^3$$

$$\Rightarrow t_0 t_2^2 + t_0^3 - 2t_0^2 t_2 = t_0^3 - t_1^3 - 3t_0^2 t_1 + 3t_0 t_1^2$$

$$\Rightarrow 3t_0^2 t_1 - 2t_0^2 t_2 + t_0 t_2^2 - 3t_0 t_1^2 + t_1^3 = 0$$

$$\Rightarrow t_0^2 (3t_1 - 2t_2) + t_0 (t_2^2 - 3t_1^2) + t_1^3 = 0$$

$$\tau = 2h \quad : \quad t_1 = \tau \ ; \ t_2 = 2\tau$$

$$\Rightarrow t_0^2 (3\tau - 4\tau) + t_0 (4\tau^2 - 3\tau^2) + \tau^3 = 0$$

$$\Rightarrow -\tau t_0^2 + \tau^2 t_0 + \tau^3 = 0$$

$$\Rightarrow t_0^{(1,2)} = \frac{-\tau^2 \pm \sqrt{\tau^4 + 4\tau^4}}{-2\tau} = \frac{-\tau^2 \pm \sqrt{5\tau^4}}{-2\tau} = \frac{\tau \pm \tau\sqrt{5}}{2} \underset{(\tau = 2h)}{=} 1 \pm \sqrt{5} = \begin{cases} 3,236\ h \\ -1,236\ h \end{cases}$$

IO HO FISSATO LE 12:00 COME $t_{01} = 0$ PER CUI IL TEMPO CHE MI INTERESSA È QUELLO NEGATIVO.

HA INIZIATO A NEVICARE ALLE 12:00 − 1.236 h = 12:00 − 1 h − 14 min − 9 s

$$\boxed{= 10:45:51}$$

$$1.236\ h = 74.16\ \text{min}$$
$$= 1\ h,\ 14.16\ \text{min}$$
$$= 1\ h,\ 849\ s$$
$$= 1\ h,\ 14\ \text{min},\ 9\ s$$

exe 2

2.1 v_0 , $x_0 \neq 0$

$a = -k\sqrt{v}$ $\quad k \in \mathbb{R}$

DETERMINA : τ = TEMPO PER FERMARSI

ℓ = SPAZIO PERCORSO

$x(t)$

$$a = \frac{dv}{dt} = -k\sqrt{v} \Rightarrow v^{-1/2}\,dv = -k\,dt \Rightarrow -kt = \int_{v_0}^{v} v^{-1/2}\,dv$$

$$\Rightarrow -kt = \frac{v^{1/2}}{-\frac{1}{2}+1}\Bigg|_{v_0}^{v} = 2v^{1/2}\Big|_{v_0}^{v} = 2v^{1/2}(t) - 2v_0^{1/2}$$

$$\Rightarrow v^{1/2}(t) = v_0^{1/2} - \frac{k}{2}t \Rightarrow v(t) = \left(v_0^{1/2} - \frac{k}{2}t\right)^2$$

VOGLIO τ T.C. $v(\tau) = 0 \Rightarrow \sqrt{v_0} = \frac{k}{2}\tau \Rightarrow \boxed{\tau = \frac{2\sqrt{v_0}}{k}}$

CERCO $x(t)$, SO $v(t) = \frac{dx}{dt} \Rightarrow x(t) = \int_{t_0}^{t} v(t)\,dt$

$$\Rightarrow x(t) = \int_{t_0}^{t} dt \left(v_0 + \frac{k^2}{4} t^2 - k v_0^{1/2} t \right) + x_0$$

$$\Rightarrow \boxed{x(t) = x_0 + v_0 (t - t_0) + \frac{k^2}{12}(t^3 - t_0^3) - \frac{k v_0^{1/2}}{2}(t^2 - t_0^2)}$$

ASSUMO $t_0 = 0$

$$x(\tau) = \ell = v_0 \frac{2\sqrt{v_0}}{k} + \frac{k^2}{12} \frac{8 v_0^{3/2}}{k^3} - \frac{k v_0^{1/2}}{2} \frac{4 v_0}{k^2} + x_0$$

$$= \frac{2}{k} v_0^{3/2} + \frac{2}{3k} v_0^{3/2} - \frac{2}{k} v_0^{3/2} + x_0$$

$$\Rightarrow \boxed{\ell = x_0 + \frac{2}{3k} v_0^{3/2}}$$

ANALISI DIMENSIONALE: $a = -k\sqrt{v} \Rightarrow \frac{m}{s^2} = [k] \frac{m^{1/2}}{s^{1/2}} \Rightarrow [k] = \frac{m^{1/2}}{s^{3/2}}$

$$\Rightarrow [\tau] = \frac{m^{1/2}}{s^{1/2}} \frac{s^{3/2}}{m^{1/2}} = s \qquad [\ell] = \frac{s^{3/2}}{m^{1/2}} \frac{m^{3/2}}{s^{3/2}} = m$$

2.2 $x(t) = A\log\left(1+\frac{t^2}{\tau^2}\right)$ $A = 1\ m$ $\tau = 1\ s$

DETERMINA
- $v(t)$, $a(t)$ E SE TORNA AL RIFUGIO DISTANTE L.
- v_m IN $[0,\tau]$ E $[\tau, 2\tau]$
- v_{MAX} E t_{MAX}

$$v(t) = \frac{dx}{dt} = A \cdot \frac{1}{1+\frac{t^2}{\tau^2}} \cdot \frac{2t}{\tau^2} = A\frac{2t}{\tau^2}\frac{\tau^2}{\tau^2+t^2} \Rightarrow \boxed{v(t) = \frac{2At}{\tau^2+t^2}}$$

$$a(t) = \frac{dv}{dt} = \frac{2A}{\tau^2+t^2} - \frac{2At}{(\tau^2+t^2)^2}2t \qquad \boxed{a(t) = \frac{2A}{\tau^2+t^2} - \frac{4At^2}{(\tau^2+t^2)^2}}$$

$$x(t_1) = L = A\log\left(1+\frac{t_1^2}{\tau^2}\right) \Rightarrow \log\left(1+\frac{t_1^2}{\tau^2}\right) = \frac{L}{A}$$

$$\Rightarrow 1+\frac{t_1^2}{\tau^2} = e^{L/A} \Rightarrow \frac{t_1^2}{\tau^2} = e^{L/A}-1 \Rightarrow \boxed{t_1 = \tau\left(e^{L/a}-1\right)^{1/2}}$$

VEDO $x(0) = 0$ $x(\tau) = A\log 2$

$x(2\tau) = A\log 5$

QUINDI

$$v_m(0,\tau) = \frac{x(\tau)-x(0)}{\tau} = \boxed{A\frac{\log 2}{\tau}}$$

$$v_m(\tau,2\tau) = \frac{x(2\tau)-x(\tau)}{2\tau-\tau} = \frac{\log(5)-\log(2)}{\tau}A$$

$$\Rightarrow \boxed{v_m(\tau,2\tau) = A\frac{\log(5/2)}{\tau}}$$

$$v_{MAX} \Rightarrow \frac{dv}{dt}=0 \Rightarrow a(t_{MAX})=0 \Rightarrow \frac{2A}{\tau^2+t^2} - \frac{4At^2}{(\tau^2+t^2)^2} = 0$$

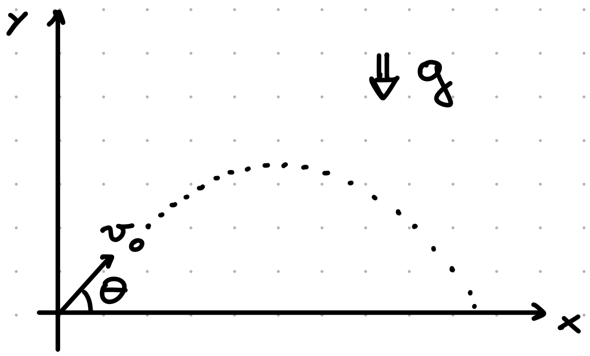
$$\Rightarrow \frac{2A}{\tau^2+t^2} = \frac{4At^2}{(\tau^2+t^2)^2} \Rightarrow 1 = \frac{2t^2}{\tau^2+t^2}$$

$$\Rightarrow \tau^2+t^2 = 2t^2 \Rightarrow \boxed{t_{MAX}=\tau}$$

$$v_{MAX} = v(t_{MAX}) = \frac{2A\,t_{MAX}}{\tau^2+t_{MAX}^2} = \frac{2A\tau}{2\tau^2} = \frac{A}{\tau} \Rightarrow \boxed{v_{MAX} = \frac{A}{\tau}}$$

2.3 v_0, θ

DETERMINA:
- t_{volo} ; x_G (GITTATA) ; θ_{max}
- TRAIETTORIA ; $\vec{a}$; R (RAGGIO DI CURVATURA)



IN UN MOTO PARABOLICO SO DI AVERE:

$$\begin{cases} x(t) = v_{0x} t \\ y(t) = v_{0y} t + \frac{1}{2} a t^2 \overset{!}{=} v_{0y} t - \frac{1}{2} g t^2 \end{cases}$$

$$v_{0x} = v_0 \cos\theta$$
$$v_{0y} = v_0 \sin\theta$$

CERCO LA TRAIETTORIA:

$$t = \frac{x(t)}{v_{0x}} \Rightarrow y(t) = v_{0y} \frac{x}{v_{0x}} - \frac{1}{2} g \frac{x^2}{v_{0x}^2}$$

$$\Rightarrow y(x) = x \frac{v_0 \sin\theta}{v_0 \cos\theta} - \frac{1}{2} g \frac{x^2}{v_0^2 \cos^2\theta}$$

$$\Rightarrow \boxed{y(x) = x \tan\theta - \frac{g}{2v_0^2} \frac{x^2}{\cos^2\theta}}$$

CERCO LA GITTATA:

$$\Rightarrow y(x_G) = 0 \Rightarrow x_G \operatorname{tg}\theta - \frac{g}{2v_0^2} \frac{x_G^2}{\cos^2\theta} = 0$$

$$\Rightarrow x_G\left(tg\theta - \frac{g}{2v_0^2}\frac{x_G}{\cos^2\theta}\right)=0$$

$$\Rightarrow x_G = 0 \quad \vee \quad \frac{g}{2v_0^2}\frac{x_G}{\cos^2\theta} = tg\,\theta$$

$$\Rightarrow x_G = 0 \quad \vee \quad x_G = \frac{2v_0^2}{g}\cos^2\theta\,\frac{\sin\theta}{\cos\theta}$$

QUINDI IL RISULTATO INTERESSANTE È:

$$\boxed{x_G = \frac{2v_0^2}{g}\cos\theta\,\sin\theta}$$

PER IL TEMPO DI VOLO POSSO USARE ANCHE SOLO IL MOTO SU X ESSENDO I MOTI INDIPENDENTI.

$$x(t_v) = x_G = v_{0x}\,t_{volo} \Rightarrow t_{volo} = \frac{x_G}{v_{0x}}$$

$$\Rightarrow t_{volo} = \frac{2v_0^2}{g}\sin\theta\cos\theta\,\frac{1}{v_0\cos\theta}$$

$$\Rightarrow \boxed{t_{volo} = \frac{2v_0}{g}\sin\theta}$$

L'ANGOLO θ_{MAX} LO PRENDO IMPONENDO:

$$\frac{dx_G}{d\theta} = 0 \Rightarrow \frac{2v_0^2}{g}\left(\cos^2\theta - \sin^2\theta\right) = 0$$

$$\Rightarrow \cos^2\theta = \sin^2\theta \Rightarrow \theta_{MAX} = \frac{\pi}{4} + k\frac{\pi}{2}$$

QUELLO CHE CI INTERESSA È:

$$\boxed{\theta_{MAX} = \frac{\pi}{4}}$$

POSSO TROVARE PER COMPLETEZZA IL PUNTO IN CUI IL PALLONE RAGGIUNGE LA QUOTA MASSIMA:

$$\frac{dy(x)}{dx} = 0 \Rightarrow \frac{d}{dx}\left\{x\,tg\theta - \frac{g}{2v_0^2}\frac{x^2}{\cos^2\theta}\right\} = 0$$

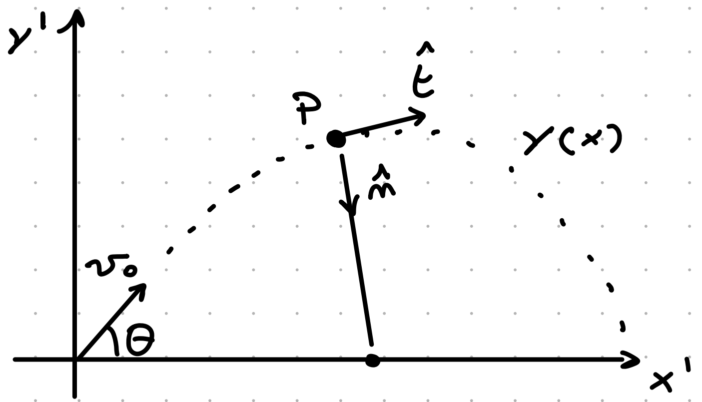
$$\Rightarrow tg\theta - \frac{g}{v_0^2}\frac{x}{\cos^2\theta} = 0 \Rightarrow \frac{\sin\theta}{\cos\theta}\frac{v_0^2}{g}\cos^2\theta = x_{MAX}$$

$$\Rightarrow x_{MAX} = \frac{v_0^2}{g}\sin\theta\cos\theta$$

$$y(x_{MAX}) = y_{MAX} = \frac{v_0^2}{g}\sin^2\theta - \frac{g}{2v_0^2}\frac{1}{\cos^2\theta}\frac{v_0^4}{g^2}\sin^2\theta\cos^2\theta$$

$$\Rightarrow y_{MAX} = \frac{v_0^2}{g}\sin^2\theta - \frac{v_0^2}{2g}\sin^2\theta = \frac{v_0^2}{2g}\sin^2\theta$$

PER TROVARE LE COMPONENTI DI $\vec{a}$ MI METTO NEL S.R. DEL PALLONE:



VOGLIO CALCOLARE LE COMPONENTI DI

$$\vec{a} = a_n\hat{n} + a_t\hat{t}$$

LO TRATTO COME MOTO CIRCOLARE

$$\begin{cases} v_\theta = v_0 \\ v_r = -gt \end{cases} \Rightarrow v = |\vec{v}| = \sqrt{v_\theta^2 + v_r^2} = \sqrt{v_0^2 + g^2t^2}$$

$$a_\theta \equiv a_t = \frac{dv}{dt} = \frac{1}{2}\frac{1}{\sqrt{v_0^2+g^2t^2}}\,2g^2t = \frac{g^2t}{\sqrt{v_0^2+g^2t^2}}$$

DUNQUE

$$\boxed{a_\theta = \frac{g^2}{\sqrt{v_0^2+g^2t^2}}\,t}$$

$$a_r \equiv a_n = \frac{v^2}{R} \qquad \text{E} \qquad \text{SO} \qquad a = g = \sqrt{a_n^2 + a_t^2}$$

$$\Longrightarrow a_n = \sqrt{g^2 - a_t^2}$$

$$a_n = \left(g^2 - \frac{g^4}{v_0^2+g^2t^2}\,t^2\right)^{1/2} = \left(\frac{v_0^2g^2 + g^4t^2 - g^4t^2}{v_0^2+g^2t^2}\right)^{1/2}$$

$$= \left(\frac{v_0^2g^2}{v_0^2+g^2t^2}\right)^{1/2}$$

DUNQUE

$$\boxed{a_n = \frac{v_0 g}{(v_0^2+g^2t^2)^{1/2}}}$$

IL RAGGIO DI CURVATURA LO PRENDO DA a_n:

$$a_n = \frac{v^2}{R} \Rightarrow R = \frac{v^2}{a_n} = \frac{(v_0^2 + g^2t^2)^{1/2}}{v_0 g}(v_0^2 + g^2t^2)$$

$$\Rightarrow \boxed{R = \frac{(v_0^2 + g^2t^2)^{3/2}}{v_0 g}}$$

2.4

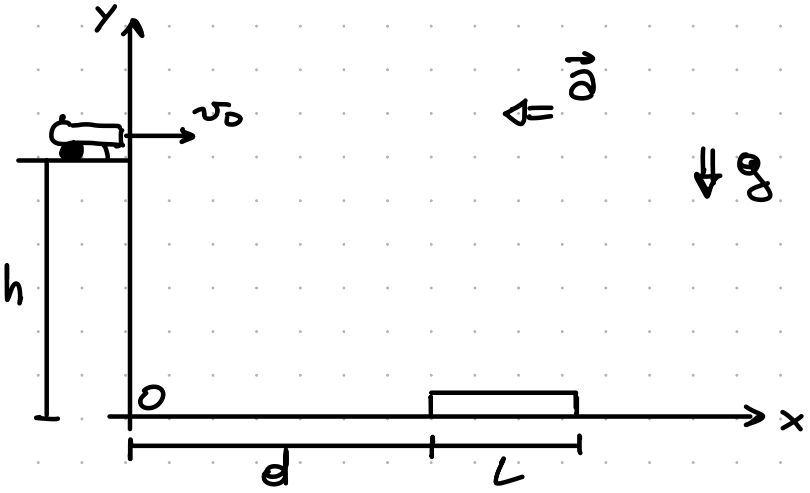
$h = 2\ km$

$L = 50\ m$

$d = 4\ km$

$a_x = -kv_x$

$k = \dfrac{\log 10}{20}\ \dfrac{1}{s}$



DETERMINA

- GITTATA
- INTERVALLO DI v_0 tc COLPISCI L'ARMERIA
- $v^{PERF}_{MIN} = 205\ \frac{m}{s}$, IL PROIETTILE PERFORA IL TETTO DELL'ARMERIA?

MI VIENE DATO:

$$\begin{cases} a_x = -kv_x \\ a_y = -g \end{cases} \quad ; \quad a_i = \frac{dv_i}{dt} \Rightarrow \begin{cases} \dfrac{dv_x}{v_x} = -k\,dt \\ dv_y = -g\,dt \end{cases}$$

$$\Rightarrow \begin{cases} \log\left(\dfrac{v_x}{v_0}\right) = -kt \\ v_y = -gt \end{cases} \Rightarrow \boxed{\begin{cases} v_x = v_0\, e^{-kt} \\ v_y = -gt \end{cases}}$$

$$v_i = \frac{dx_i}{dt} \Rightarrow \begin{cases} \frac{dx}{dt} = v_0 e^{-kt} \\ \frac{dy}{dt} = -gt \end{cases} \Rightarrow \begin{cases} x(t) = v_0 \int_0^t e^{-kt}\, dt \\ y(t) - h = -\frac{1}{2} g t^2 \end{cases}$$

$$\Rightarrow \begin{cases} x(t) = -\frac{v_0}{k} e^{-kt} \Big|_0^t \\ y(t) = h - \frac{1}{2} g t^2 \end{cases} \Rightarrow \boxed{\begin{cases} x(t) = \frac{v_0}{k}\left(1 - e^{-kt}\right) \\ y(t) = h - \frac{1}{2} g t^2 \end{cases}}$$

CALCOLO LA GITTATA :

$$y(t_G) = 0 \Rightarrow h - \frac{g}{2} t_G^2 = 0 \Rightarrow t_G = \sqrt{\frac{2h}{g}}$$

$$\Rightarrow x(t_G) = \boxed{x_G = \frac{v_0}{k}\left(1 - e^{-k\sqrt{\frac{2h}{g}}}\right)}$$

COLPISCO L'ARMERIA SE :

$$d \leq x_G \leq d + L$$

$$\Rightarrow d \leq \frac{v_0}{k}\left(1 - e^{-k\sqrt{\frac{2h}{g}}}\right) \leq d + L$$

$$\Rightarrow \boxed{\frac{kd}{1-e^{-k\sqrt{\frac{2h}{g}}}} \leq v_0 \leq \frac{k(d+L)}{1-e^{-k\sqrt{2h/g}}}}$$

HO TROVATO: $t_G = \sqrt{\frac{2h}{g}}$; $\begin{cases} v_x(t) = v_0 e^{-kt} \\ v_y(t) = -gt \end{cases}$

QUINDI QUANDO TOCCA TERRA HO:

$$\begin{cases} v_x(t_G) = v_0\, e^{-k\sqrt{\frac{2h}{g}}} \\ v_y(t_G) = -g\sqrt{\frac{2h}{g}} \end{cases} \Rightarrow \begin{cases} v_x(t_G) = v_0\, e^{-k\sqrt{\frac{2h}{g}}} \\ v_y(t_G) = -\sqrt{2hg} \end{cases}$$

DEVO CONTROLLARE CHE AGLI ESTREMI DELL'ARTIGLIERIA

$$v_{0,1} = \frac{kd}{1-e^{-k\sqrt{\frac{2h}{g}}}} \qquad v_{0,2} = \frac{k(d+L)}{1-e^{-k\sqrt{2h/g}}}$$

IO ABBIA $v = |\vec{v}| = \sqrt{v_x^2(t_G) + v_y^2(t_G)} > v_{MIN}^{PERF}$

(CHIAMO $\alpha = k\sqrt{\frac{2h}{g}}$)

$$v\Big|_{v_{01}} = \left(\frac{k^2 d^2}{(1-e^{-\alpha})^2} e^{-2\alpha} + 2hg\right)^{1/2} = \left(\frac{k^2 d^2}{(e^{\alpha}-1)^2} + 2hg\right)^{1/2}$$

$$v\Big|_{v_{02}} = \left(\frac{k^2 (d+L)^2}{(1-e^{-\alpha})^2} e^{-2\alpha} + 2hg\right)^{1/2} = \left(\frac{k^2 (d+L)^2}{(e^{\alpha}-1)^2} + 2hg\right)^{1/2}$$

METTO I NUMERI E VEDO SE $v < v_{MIN}^{PERF}$

$$v\Big|_{v_{01}} = 204{,}285 \ m/s$$

$$v\Big|_{v_{02}} = 204{,}438 \ m/s$$

exe3

3.1 PISTA CIRCOLARE R

$v(t) = v_0 e^{-t/\tau}$ $\qquad v_0, \tau \in \mathbb{R}$

TROVA: $s(t)$, a_c, a_t, t_1, t_m

MOTO CIRCOLARE VARIO HO:

$$v = \frac{ds}{dt} \Rightarrow ds = v\,dt \Rightarrow s(t) - s_0 = v_0 \int_0^t e^{-t'/\tau}\,dt'$$

$$\Rightarrow s(t) = s_0 - v_0 \tau \, e^{-t'/\tau}\Big|_0^t \underset{(s_0=0)}{\Rightarrow} s(t) = -v_0\tau\left(e^{-t/\tau} - 1\right)$$

$$\Rightarrow \boxed{s(t) = v_0\tau\left(1 - e^{-t/\tau}\right)}$$

$$a_\theta = \frac{dv}{dt} = \boxed{-\frac{v_0}{\tau} e^{-t/\tau}} \qquad \boxed{a_c = \frac{v^2}{R} = \frac{v_0^2}{R} e^{-2t/\tau}}$$

$$v_\theta = v(t) = r\frac{d\theta}{dt} \Rightarrow \frac{v_0}{r} e^{-t'/\tau}\,dt' = d\theta$$

$$\Rightarrow \theta - \theta_0 = \frac{v_0}{r}(-\tau)\left(e^{-t/\tau} - 1\right)$$

$$\underset{(\theta_0 = 0)}{\Rightarrow} \boxed{\theta(t) = \frac{v_0}{r}\tau\left(1 - e^{-t/\tau}\right)}$$

(NON MI SERVIVA)

DOPO n GIRI $\quad s(t_n) = 2\pi R n = v_0 \tau \left(1 - e^{-t_n/\tau}\right)$

$$\Rightarrow e^{-t_n/\tau} = \left(1 - \frac{2\pi n R}{v_0 \tau}\right) \quad \Rightarrow -\frac{t_n}{\tau} = \log\left(1 - \frac{2\pi R n}{v_0\tau}\right)$$

$$\Rightarrow \boxed{t_n = -\tau \log\left(1 - \frac{2\pi R}{v_0 \tau} n\right)}$$

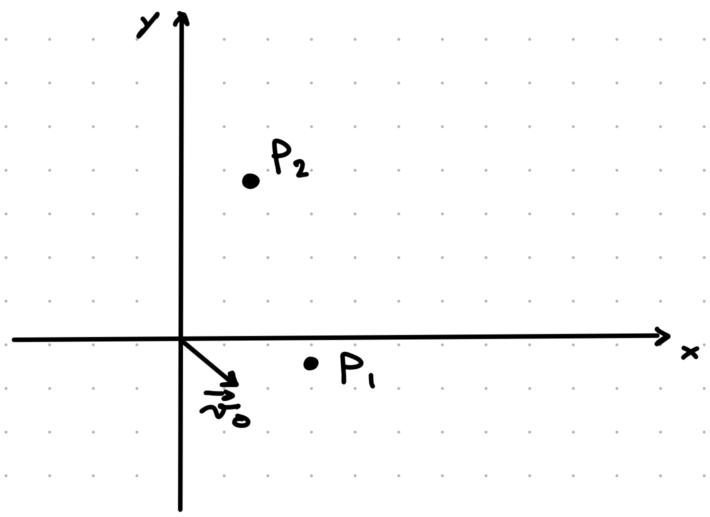
$$\underset{(n=1)}{\Rightarrow} \boxed{t_1 = -\tau \log\left(1 - \frac{2\pi R}{v_0\tau}\right)}$$

3.2 $\vec{v}_0 = (v_{0x}, v_{0y}) = (9, -4)\ m/s$

$a(t) = ct^2$, $c = 4\ m/s^4$

TROVA $y(x)$ E LA LEGGE ORARIA

BOE : $P_1 = (7, -1)\ m$ $P_2 = (5, 3)\ m$



LUNGO x HO MRU

$$x(t) = x_0 + v_{0x} t$$

$$\Rightarrow \boxed{x(t) = v_{0x} t}$$

LUNGO y HO MOTO ACCELERATO

$$a(t) = ct^2 = \frac{dv}{dt} \Rightarrow dv = ct^2 dt \Rightarrow \boxed{v_y(t) = v_{0y} + \frac{c}{3} t^3}$$

$$v_y(t) = \frac{dy}{dt} \Rightarrow dy = v_y dt \Rightarrow \boxed{y(t) = v_{0y} t + \frac{c}{12} t^4}$$

DUNQUE LA LEGGE ORARIA È :

$$\begin{cases} x(t) = v_{0x} t \\ y(t) = v_{0y} t + \frac{c}{12} t^4 \end{cases}$$

$$\Rightarrow t = \frac{x}{v_{0x}} \quad \Rightarrow \quad \boxed{y(x) = \frac{v_{0y}}{v_{0x}} x + \frac{c}{12 v_{0x}^4} x^4}$$

TRAIETTORIA

METTO P_1 E P_2 IN $y(x)$

$$P_1 : \quad -1 = -\frac{4}{9} 7 + \frac{4}{12} \frac{1}{9^4} 7^4 \quad \Rightarrow \quad -1 = -\frac{28}{9} + \frac{1}{3}\left(\frac{7}{9}\right)^4$$

$$\Rightarrow -1 = -3.11 + 0.121 \qquad \underline{\underline{NO}}$$

$$P_2 : \quad 3 = -\frac{4}{9} 5 + \frac{4}{12} \frac{5^4}{9^4} \quad \Rightarrow \quad 3 = -2.22 + 0.03 \qquad \underline{\underline{NO}}$$

<u>NON COLPISCE LE BOE</u>.

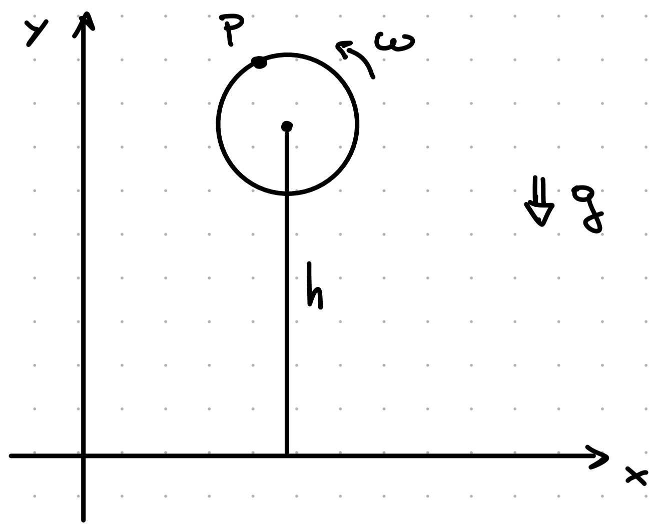
3.3 MONETA DI RAGGIO R

$\omega = \text{cost}$

SCRIVI : • COMPONENTI v

A $t=0$ $\quad \theta_0 = 0 \quad y_c = h$

TROVA : • # GIRI PRIMA DI TOCCARE TERRA



MOTO CENTRO MONETA : MRUA

$$y_c(t) = h - \frac{1}{2} g t^2$$

TEMPO CHE CI METTE A CADERE :

$$y_c(\tau) = 0 = h - \frac{1}{2} g \tau^2 \Rightarrow \boxed{\tau = \sqrt{\frac{2h}{g}}}$$

$\omega = \text{cost} \Rightarrow$ MOTO CIRCOLARE UNIFORME

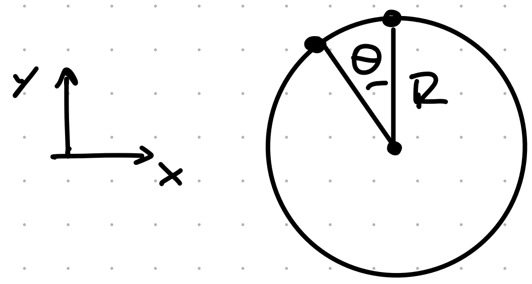
$$\Rightarrow \omega = \frac{d\theta}{dt} \Rightarrow \theta = \theta_0 + \omega t$$

MA SCELGO $\theta(t)$ ANGOLO RISPETTO LA VERTICALE $\Rightarrow \theta_0 = 0$

$$\Rightarrow \boxed{\theta(t) = \omega t}$$

CERCO LE COMPONENTI DI v

AVRÒ COME GENERICA POSIZIONE:



$$\begin{cases} x(t) = -R\sin(\theta(t)) \\ y(t) = h + R\cos(\theta(t)) - \frac{1}{2}gt^2 \end{cases}$$

$$v_i(t) = \frac{dx_i}{dt} \Rightarrow \begin{cases} v_x(t) = -\frac{d\theta}{dt} R\cos(\theta(t)) = -\omega R\cos(\omega t) \\ v_y(t) = -\frac{d\theta}{dt} R\sin(\theta(t)) - gt = -\omega R\sin(\omega t) - gt \end{cases}$$

IL MODULO È

$$v = \sqrt{\omega^2 R^2 \cos^2(\omega t) + \omega^2 R^2 \sin^2(\omega t) + g^2t^2 + 2gt\omega R\sin(\omega t)}$$

$$= \sqrt{\omega^2 R^2 + g^2 t^2 + 2gt\omega R\sin(\omega t)}$$

QUANTO HA GIRATO IN $t = \tau$

$$\theta(\tau) = \omega\tau \Rightarrow \theta(\tau) = \omega\sqrt{\frac{2h}{g}}$$

IL # DI GIRI È

$$\frac{\theta(\tau)}{2\pi} = \frac{\omega}{2\pi}\sqrt{\frac{2h}{g}} = \sqrt{\frac{h\omega^2}{2\pi^2 g}}$$

OPPURE UN ALTRO MODO:

HO $T = \frac{2\pi}{\omega}$ TEMPO PER FARE 1 GIRO

$$\# \text{ GIRI} = \frac{\tau}{T} = \frac{\omega}{2\pi}\sqrt{\frac{2h}{g}} = \sqrt{\frac{h\omega^2}{2\pi^2 g}}$$

3.4

$\omega = \text{cost}$

$r(t) = r_0 e^{-\omega t/2}$

DETERMINA:
- LE COMPONENTI DELLA VELOCITÀ
- L'ACCELERAZIONE
- POSIZIONE E v DOPO UN GIRO
- RAGGIO DI CURVATURA

HO IN COORDINATE (r, θ)

$$\vec{v}(t) = v_\theta \hat{\theta} + v_r \hat{r} = r \frac{d\theta}{dt} \hat{\theta} + \frac{dr}{dt} \hat{r}$$

HO $\omega = \text{cost} \Rightarrow \boxed{\theta(t) = \omega t}$

DUNQUE

$$\boxed{\begin{cases} v_r = -\frac{\omega r_0}{2} e^{-\omega t/2} \\ v_\theta = \omega r(t) = \omega r_0 e^{-\frac{\omega t}{2}} \end{cases}}$$

E POI

$$\vec{a}(t) = a_\theta \hat{\theta} + a_r \hat{r} = \left[r \frac{d^2\theta}{dt^2} + 2 \frac{dr}{dt} \frac{d\theta}{dt} \right] \hat{\theta} + \left[\frac{d^2 r}{dt^2} - r \left(\frac{d\theta}{dt} \right)^2 \right] \hat{r}$$

DUNQUE

$$\begin{cases} a_\theta = 2\left(-\frac{\omega r_0}{2} e^{-\frac{\omega t}{2}}\right)\omega \\ a_r = \frac{\omega^2 r_0}{4} e^{-\omega t/2} - \omega^2 r(t) \end{cases} \Rightarrow \boxed{\begin{cases} a_\theta = -\omega r_0 e^{-\frac{\omega t}{2}} \\ a_r = \frac{\omega^2 r_0}{4} e^{-\omega t/2} - \omega^2 r_0 e^{-\frac{\omega t}{2}} \end{cases}}$$

DOPO UN GIRO HO

$$\theta(\tau) = 2\pi = \omega\tau \Rightarrow \tau \equiv T = \frac{2\pi}{\omega}$$

DUNQUE

$$r(\tau) = r_0 e^{-\frac{\omega}{2}\frac{2\pi}{\omega}} = \boxed{r_0 e^{-\pi}}$$

$$\boxed{\begin{cases} v_r(\tau) = -\frac{\omega r_0}{2} e^{-\pi} \\ v_\theta(\tau) = \omega r_0 e^{-\pi} \end{cases}} \Rightarrow v^2 = \frac{\omega^2 r_0^2}{4} e^{-2\pi} + \omega^2 r_0^2 e^{-2\pi} = \frac{5\omega^2 r_0^2}{4} e^{-2\pi}$$

$$a_c(\tau) = \frac{\omega^2 r_0}{4} e^{-\pi} - \omega^2 r_0 e^{-\pi} = -\frac{3}{4}\omega^2 r_0 e^{-\pi}$$

$$|a_c| \equiv |a_r| = \frac{v^2}{\rho} \Rightarrow \rho = \frac{v^2}{|a_c|} = \left(\frac{5}{4}\omega^2 r_0^2 e^{-2\pi}\right)\left(+\frac{4}{3}\frac{1}{\omega^2 r_0} e^{\pi}\right) = \boxed{+\frac{5}{3} e^{-\pi} r_0}$$