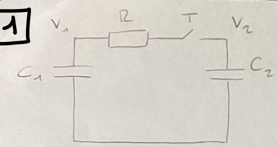
ESERCIZI ELETTRICITÀ E MAGNETISMO FOGLI $\boxed{\text{EM-18, EM-19}}$

EM-18 : 1, 3

(2, 4, 5 ESERCIZI SU SPETTROMETRO DI MASSA, CICLOTRONE E SELETTORE DI VELOCITÀ)

$\boxed{1}$



$V_{10} = 500\ V$ $\quad V_{20} = 200V$

$C_1 = 2\mu F$ $\quad C_2 = 4\mu F$

$R = 3M\Omega$

SUPPONGO $V_1 > V_2$ (FISSO IL VERSO DI i)

$q_{01} = C_1 V_{10}$ $\quad q_{02} = C_2 V_{01}$ $\quad$ VALE $q_{01} + q_{02} = q_1 + q_2$

SI CHIUDE T $\quad V_1 + V_2 = R\,i(t) \rightarrow \frac{q_1}{C_1} - \frac{q_2}{C_2} = -R\frac{dq}{dt}$

$$\frac{C_2 q_1 - C_1(q_{01}+q_{02}-q_1)}{C_1 C_2} = -R\frac{dq}{dt} \rightarrow \frac{dq}{C_2 q_1 - C_1(q_0 - q_1)} = -\frac{dt}{RC_1C_2} \quad , \ q_0 = q_{01} + q_{02}$$

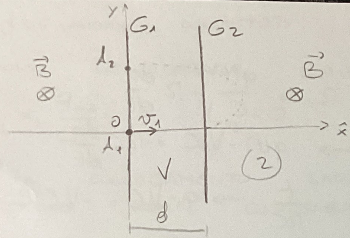
$$\rightarrow \frac{dq}{q_1(C_2+C_1) - (C_1 q_0)} = -\frac{dt}{RC_1C_2} \rightarrow \frac{1}{C_2+C_1}\ln\left(\frac{q_1(C_2+C_1) - C_1(q_0)}{-C_1 q_0}\right) = -\frac{t}{RC_1C_2}$$

$$q_1(C_2+C_1) - C_1 q_0 = -C_1 q_0\, e^{-\frac{t}{RC_1C_2}(C_2+C_1)} \quad , \ \frac{1}{C_s} = \frac{C_1+C_2}{C_1C_2} \quad , \ \tau = RC_s$$

$$q_1(t) = \frac{C_1 q_0}{C_2+C_1} - \frac{C_1}{C_2+C_1} q_0\, e^{-\frac{t}{\tau}}$$

$\boxed{q_2 = q_0 - q_1(t)}$ $\quad \boxed{V_1(t) = \frac{q_1(t)}{C_1}}$ $\quad \boxed{V_2(t) = \frac{q_2(t)}{C_2}}$

3



$A_2: h = 9,2$ cm , $t = 1,22\cdot 10^{-7}$ s

IL PROTONE PROCEDERÀ IN MOTO RETTILINEO FINO G_2, POI DI MOTO CIRCOLARE UNIFORME IN $|x| > d$, E POI DI NUOVO RETTILINEO FINO A_2

TRA G_1 E G_2 PRENDE UNA EN. CINETICA

$$E_k = \frac{1}{2} m v^2 = qV \Rightarrow v = \sqrt{\frac{2eV}{m}}$$

ENTRA IN 2, SENTE $\vec F = e\vec v \times \vec B \Rightarrow |F| = evB = ma = m\frac{v^2}{r}$

$\Rightarrow r = \frac{mv}{eB}$, QUINDI $\omega = \frac{v}{r} = \frac{eB}{m}$

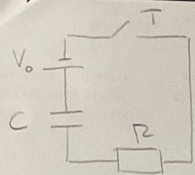
COSÌ $T = \frac{2\pi}{\omega} = \frac{2\pi m}{eB}$ CHE POSSO CALCOLARE

IN (2) COMPIE UNA SEMICIRCONFERENZA, QUINDI CI IMPIEGA UN TEMPO

$t_2 = T/2 = \frac{\pi m}{eB} = 4,09\cdot 10^{-8}$ s

$v = 1,99\cdot 10^{6}$

6



$q_0 = 0$

IN UN GENERICO ISTANTE

$$V_0 = V_C + V_R = \frac{q(t)}{C} + Ri = \frac{q(t)}{C} + R\frac{dq}{dt}$$

$$\Rightarrow V_0 C = q(t) + RC\frac{dq}{dt} \rightarrow q(t) - V_0 C = -RC\frac{dq}{dt}$$

$$\rightarrow \frac{dq}{q - V_0 C} = -\frac{dt}{RC}, \quad \tau = RC \rightarrow \ln\left(\frac{q - V_0 C}{-V_0 C}\right) = -\frac{t}{RC} \rightarrow q - V_0 C = -V_0 C e^{-t/\tau}$$

$$\Rightarrow \boxed{q(t) = V_0 C\left(1 - e^{-t/\tau}\right)}$$

QUINDI

$$\boxed{V_C = \frac{q(t)}{C} = V_0\left(1 - e^{-t/\tau}\right)} \quad ; \quad \boxed{V_R = V_0 - V_C = V_0 e^{-t/\tau}}$$

$$I(t) = \frac{dq}{dt} = \frac{V_0}{RC} C e^{-t/\tau} = \boxed{\frac{V_0}{R} e^{-t/\tau}}$$

$$\boxed{W_R(t) = V_R(t)\, I(t) = \frac{V_0^2}{R} e^{-2t/\tau}}$$

$\boxed{7}$ $d >>$ DIMENSIONI CONDUTTORI . C_A, $C_B = 3C_A$ $q_{iA} = 1\,\mu C$ $q_{iB} = 0$

LI POSSO CONSIDERARE COMPLETAMENTE ISOLATI

$$V_{iA} = \frac{q_{iA}}{C_A} \qquad V_{iB} = 0$$

DOPO IL COLLEGAMENTO SARANNO ALLO STESSO POTENZIALE (SI COMPORTANO COME CONDENSATORI IN PARALLELO)

$$\frac{q_{fA}}{C_A} = \frac{q_{fB}}{C_B} \Rightarrow q_{fA} = \frac{q_{fB}}{3} \qquad (\text{POICHÉ } C_B = 3C_A)$$

VALE $q_{iA} + q_{iB} = q_{iA} = q_{fA} + q_{fB} \Rightarrow q_{fA} = q_{iA} - q_{fB}$

$$\Rightarrow q_{iA} - q_{fB} = \frac{q_{fB}}{3} \rightarrow q_{iA} = \frac{4}{3} q_{fB} \rightarrow q_{fB} = \frac{3}{4} q_{iA} = 0{,}75\,\mu C$$

$$\Rightarrow q_{fA} = 0{,}25\,\mu C$$

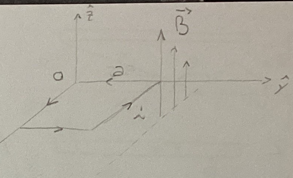
$\boxed{1}$

$a = 20\ cm$

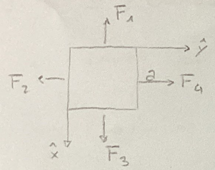
$i = 5\ A$

$B = \alpha x \hat{u}_z \quad , \alpha = 0{,}2\ T/m$

$F_{TOT} = ? \qquad U_p = ?$



VEDO



VISTO CHE $B = B(x)$ E NON DA y VEDO

$F_1 = i a B(0) = 0\ N$

$\vec{F}_3 = i a B(a) = i a^2 \alpha = 0{,}04\ N\ \hat{x}$

PERÒ

$$\vec{F}_2 = i(-\hat{y}) \int_0^a B(x)dx = -i\hat{y}\int_0^a \alpha x\,dx = -i\alpha \frac{a^2}{2}\hat{y} = -0{,}02\ \hat{y}\ N$$

$$\vec{F}_4 = i\hat{y}\int_0^a B(x)dx = i\alpha\frac{a^2}{2}\hat{y} = 0{,}02\ \hat{y}\ N$$

QUINDI

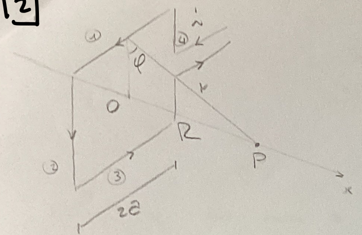
$$\vec{F}_{TOT} = \vec{F}_3 + \vec{F}_2 + \vec{F}_4 = \vec{F}_3 = \boxed{0{,}04\ N\ \hat{x}}$$

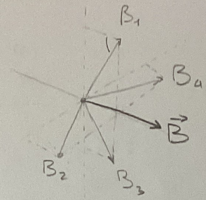
CALCOLO $\vec{m} = i\Sigma\hat{z} = i a^2 \hat{z} = 0{,}2\ \hat{z}\ A m^2 \qquad dm = i\, d\Sigma\, \hat{n}$

$$dU = -d\vec{m}\cdot\vec{B} = -i\vec{B}\cdot\hat{z}\, d\Sigma$$

$$U = -i\int_\Sigma \vec{B}\cdot\hat{z}\, d\Sigma = -i\alpha\int_0^a dy\int_0^a x\,dx = -i\alpha a \left.\frac{x^2}{2}\right|_0^a = -i\alpha\frac{a^3}{2} = -4\cdot 10^{-3}\ J$$

$\boxed{2}$



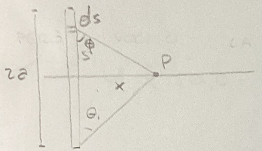


IL CAMPO DI UN FILO

$$\vec{B} = \frac{\mu_0 i}{4\pi}\int \frac{dl \times \hat{r}}{r^2}$$

IN P HO 4 CONTRIBUTI E SI VEDE CHE LE COMPONENTI $\perp \hat{x}$ SI CANCELLANO 2 A 2

UN SINGOLO CONTRIBUTO (RICAVO BIOT-SAVART)



$$dB = \frac{\mu_0 i}{4\pi}\int \frac{ds \times \hat{r}}{r^2} = \frac{\mu_0 i}{4\pi}\,\frac{\sin\varphi\, ds}{r^2}$$

$$s\,\mathrm{ctg}\varphi = x \rightarrow ds = \frac{x\, d\varphi}{\sin^2\varphi}$$

$$r = \sqrt{x^2+s^2}\,, \quad r = \frac{x}{\sin\varphi} \rightarrow \frac{1}{r^2} = \frac{\sin^2\varphi}{x^2}$$

$$\Rightarrow dB = \frac{\mu_0 i}{4\pi}\sin\varphi\,\frac{\cancel{\sin^2\varphi}}{x^2}\,\frac{x\, d\varphi}{\cancel{\sin^2\varphi}} = \frac{\mu_0 i \sin\varphi\, d\varphi}{4\pi x}$$

$$\Rightarrow \vec{B} = \frac{\mu_0 i}{4\pi x}\int_{-\theta_1}^{\theta_1}\sin\varphi\, d\varphi = \frac{\mu_0 i}{2\pi x}\cos\theta_1\,\hat{B} = \frac{\mu_0 i}{2\pi x}\,\frac{a}{\sqrt{x^2+a^2}}\,\hat{B}$$

(ORA NELLA SPIRA x = DISTANZA = r DAL PUNTO P

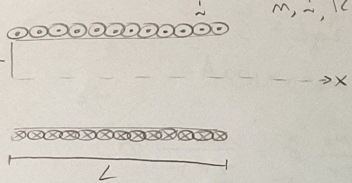
MA VOGLIO SOLO LA COMPONENTE $\hat{x} \rightarrow \cos\varphi = \frac{a}{r}$ E PRENDO 4 CONTRIBUTI

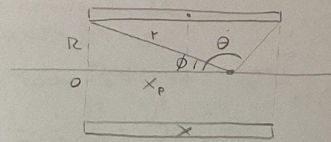
$$\vec{B}_{TOT} = \frac{2\mu_0 i}{\pi}\,\frac{a^2}{r^2}\,\frac{1}{\sqrt{r^2+a^2}} = \boxed{\frac{2\mu_0 i\, a^2}{\pi(R^2+a^2)\sqrt{R^2+2a^2}}\,\hat{x}}$$

$(r^2 = R^2+a^2)$

$\boxed{3}$

m, i, R, R



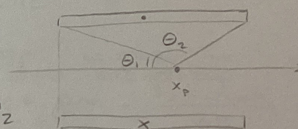


SO IL CAMPO DI UNA SPIRA $\vec{B}_s = \frac{\mu_0 i}{2} \frac{R^2}{\sqrt{(R^2+x^2)^3}} \hat{x}$

PER UN SOLENOIDE AVRÒ $d\vec{B} = m\,dx\,B_s = \frac{\mu_0 m i}{2} \frac{R^2}{(R^2+x^2)^{3/2}} dx\,\hat{x}$

$r \sin\theta = R \Rightarrow \boxed{R^2 = r^2 \sin^2\theta} \Rightarrow \frac{1}{r^2} = \frac{\sin^2\theta}{R^2}$

$x - x_p = -R \operatorname{cotg}\phi \Rightarrow dx = \frac{R\,d\theta}{\sin^2\theta}$



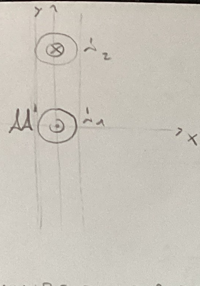
$$d\vec{B} = \frac{\mu_0 m i}{2} R^2 \frac{\sin^3\theta}{R^3} \frac{R\,d\theta}{\sin^2\theta} \hat{x} = \frac{\mu_0 m i}{2} \sin\theta\,d\theta\,\hat{x}$$

$$\vec{B} = \frac{\mu_0 m i}{2} (\cos\theta_1 - \cos\theta_2) \hat{x}, \quad \cos(\pi - \theta_2) = \cos\theta_2' = -\cos\theta_2$$

$$= \frac{\mu_0 m i}{2} (\cos\theta_1 + \cos\theta_2) \hat{x}, \quad \cos\theta_1 = \frac{x_p}{\sqrt{R^2 + x_p^2}}, \quad \cos\theta_2 = \frac{L - x_p}{\sqrt{R^2 + (L - x_p)^2}}$$

$$\Rightarrow \boxed{\vec{B} = \frac{\mu_0 m i}{2} \left(\frac{L - x_p}{\sqrt{R^2 + (L - x_p)^2}} + \frac{x_p}{\sqrt{R^2 + x_p^2}} \right) \hat{x}}$$

4



AA' INDEFINITO

BB', $l = 20\ cm$ $m_2 = 2g$

$i_1 = \text{cost}$ $i_1 = 25\,A$ $i_2 = 100\,A$

CALCOLO $\vec{B}_{AA'} = \dfrac{\mu_0 i_1}{2\pi r}\hat{\phi} = \dfrac{\mu_0 i_1}{2\pi y}\hat{\phi}$

$\vec{F}_g = m_2 g(-\hat{y})$

HO EQUILIBRIO QUANDO LE DUE FORZE m, g SI BILANCERANNO

$$\vec{F}_m = i_2 \int d\vec{l} \times \vec{B}_{AA'} = \frac{\mu_0 i_1 i_2}{2\pi}\int \frac{dl}{y}\hat{y} = \frac{\mu_0 i_1 i_2}{2\pi y} l\,\hat{y}$$

VOGLIO $|\vec{F}_g| = |\vec{F}_m| \Rightarrow \dfrac{\mu_0 i_1 i_2}{2\pi y_0} l = m_2 g \Rightarrow \boxed{y_0 = \dfrac{\mu_0 i_1 i_2 l}{2\pi m_2 g} = 5 \cdot 10^{-3}\,m = 0{,}005\,m}$

IN UN GENERICO ISTANTE $\vec{F}_m + \vec{F}_g = m_2 a = m_2 \dfrac{d^2 y}{dt^2}$

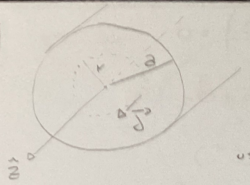
$\dfrac{\mu_0 i_1 i_2 l}{2\pi y} - mg = m\dfrac{d^2 y}{dt^2}$ VALE $\dfrac{\mu_0 i_1 i_2 l}{2\pi} = y_0 m g$

$\left(\dfrac{y_0}{y} - 1\right) mg = m\dfrac{d^2 y}{dt^2} \rightsquigarrow mg\dfrac{y_0 - y}{y} = m\ddot{y}$ SE $y \sim y_0 \Rightarrow$

$\Rightarrow g\dfrac{y_0 - y}{y_0} = \ddot{y}$ SE $y - y_0 = \varepsilon \equiv$ SCOSTAMENTO DA POS. EQUILIBRIO

$\Rightarrow \ddot{\varepsilon} = -\dfrac{g}{y_0}\varepsilon \Rightarrow \omega = \sqrt{g/y_0} \Rightarrow \boxed{T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{y_0}{g}} = 0{,}14\,s}$

$\boxed{5}$



$$\vec{J} = J_0 r \hat{z}$$

NEL GAUGE DI COULOMB SO $\nabla^2 \vec{A} = -\mu_0 \vec{J}$

USO $\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$

MA $\vec{J} = \vec{J}(r) \Rightarrow \vec{A} = \vec{A}(r) \Rightarrow \nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)$

$\Rightarrow$ HO $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A}{\partial r}\right) = -\mu_0 J_0 r \rightarrow \frac{\partial}{\partial r}\left(r\frac{\partial A}{\partial r}\right) = -\mu_0 J_0 r^2$

$\boxed{r<a}$ $\frac{\partial}{\partial r}\left(r\frac{\partial A}{\partial r}\right) = -\mu_0 J_0 r^2 \rightarrow r\frac{\partial A}{\partial r} = -\mu_0 J_0 \frac{r^3}{3} + C \rightarrow \frac{\partial A}{\partial r} = -\mu_0 J_0 \frac{r^2}{3} + \frac{C}{r}$

$\Rightarrow \boxed{\vec{A}(r) = \left(-\mu_0 J_0 \frac{r^3}{9} + c \log r + d\right)\hat{z}}$, $c, d \in \mathbb{R}$

VOGLIO $A(r \to 0) = 0 \Rightarrow C = 0$ E SCELGO $d = 0$ ARBITRARIA

$\Rightarrow \boxed{\vec{A}(r) = -\mu_0 J_0 \frac{r^3}{9}\hat{z}}$, $\vec{A} \equiv \left(0, 0, -\mu_0 J_0 \frac{r^3}{9}\hat{z}\right)$

$\text{rot}\, \vec{v} = \left(\frac{1}{r}\frac{\partial v_z}{\partial \theta} - \frac{\partial v_\theta}{\partial z}, \frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r}, \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial \theta}\right]\right)$ IO HO $\text{rot} = \left(0, -\frac{\partial v_z}{\partial r}, 0\right)$

$\Rightarrow \boxed{\vec{B}(r) = \text{rot}\, A = \mu_0 J_0 \frac{r^2}{3}\hat{\theta}}$

$\boxed{r > a}$ $\quad \nabla^2 \vec{A} = 0 \quad , \quad \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A}{\partial r}\right) = 0 \Rightarrow \frac{\partial}{\partial r}\left(r\frac{\partial A}{\partial r}\right) = 0$

$$\Rightarrow r\left(\frac{\partial A}{\partial r}\right) = C_2 \rightarrow \frac{\partial A}{\partial r} = \frac{C_2}{r} \Rightarrow \vec{A}(r) = \left(C_2 \log(r) + d_2\right)\hat{z}$$

VOGLIO LA CONTINUITÀ IN $r = a$ (LA VOGLIO C^1)

$$A_z \Big| \quad -\frac{\mu_0 J_0 a^3}{9} = C_2 \log(a) + d_2 \rightarrow d_2 = -\frac{\mu_0 J_0 a^3}{9} - C_2 \log(a)$$

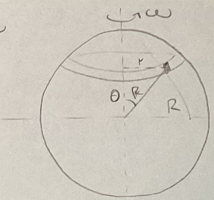
$$\frac{\partial}{\partial r}\Big| \quad -\frac{\mu_0 J a^2}{3} = \frac{C_2}{a} \rightarrow \boxed{C_2 = -\frac{\mu_0 J_0 a^3}{3}} \Rightarrow \boxed{d_2 = -\frac{\mu_0 J_0 a^3}{3}\left(\frac{1}{3} - \log a\right)}$$

QUINDI $\quad \vec{A}_z(r) = -\frac{\mu_0 J_0 a^3}{3}\log r - \frac{\mu_0 J_0 a^3}{3}\left(-\log a + \frac{1}{3}\right)$

$$\boxed{= -\frac{\mu_0 J_0 a^3}{3}\left(\log\left(\frac{r}{a}\right) + \frac{1}{3}\right)\hat{z}}$$

$$\vec{B}(r) = \mathrm{rot}\, A = \left(0, \frac{\mu_0 J_0 a^3}{3r}, 0\right) = \boxed{\frac{\mu_0 J_0 a^3}{3r}\hat{\theta}}$$

6 σ, ω



$v = \omega R$

UNA SPIRA CIRCOLARE

$$dB = \frac{\mu_0 di}{2} \frac{r^2}{\sqrt{(r^2+z^2)^3}} \hat{z} = \frac{\mu_0 di}{2} \frac{R^2 \sin^2\theta}{\sqrt{(R^2\cos^2\theta + R^2\sin^2\theta)^3}}$$

$$= \frac{\mu_0 R^2}{2R^3} di = \frac{\mu_0}{2R} di \sin^2\theta$$

AREA DI UNA SPIRA $\quad d\Sigma = R\, d\theta\, 2\pi R \sin\theta$

$$dq = \sigma\, d\Sigma$$

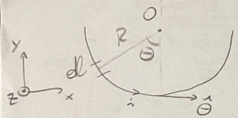
COSÌ CALCOLO $\quad di = \frac{dq}{T} = \sigma R^2 2\pi \sin\theta\, d\theta \frac{\omega}{2\pi}$

$$\Rightarrow dB = \frac{\mu_0}{2R} \sigma R^2 \omega \sin^3\theta\, d\theta = \frac{\mu_0 \sigma R \omega}{2} \sin^3\theta\, d\theta$$

$$\Rightarrow \vec{B} = \frac{4}{3} \frac{\mu_0 \sigma R \omega}{2} = \boxed{\frac{2}{3} \mu_0 \sigma \omega R}$$

$\boxed{7}$

NEL PUNTO O NON AVRÒ CONTRIBUTO DELLA PARTE RETTILINEA DEL FILO



$$\vec{B} = \frac{\mu_0 i}{4\pi}\int \frac{dl \times \hat{r}}{r^2} = \frac{\mu_0 i \hat{z}}{4\pi R^2}\int dl$$

MA $\int dl$ SARÀ LA LUNGHEZZA DI UNA SEMI-CIRCONFERENZA $= \pi R$

$$\Rightarrow \boxed{\vec{B} = \frac{\mu_0 i}{4R}\hat{z}}$$

SE PRENDO UN B ENTRANTE NEL FOGLIO AVRÒ

$$\vec{F} = \int i\, d\vec{l} \times \vec{B} = iB\int (\hat{\theta} \times (-\hat{z}))\, dl$$

$$\hat{\theta} = (\sin\theta, -\cos\theta, 0) \qquad \hat{\theta} \times (-\hat{z}) = \begin{vmatrix} i & j & k \\ \sin\theta & -\cos\theta & 0 \\ 0 & 0 & -1 \end{vmatrix} = \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix}$$

$$\Rightarrow \vec{F} = iB\int_0^\pi (\cos\theta\, \hat{x} + \sin\theta\, \hat{y})\, dl = iB\int_0^\pi (\cos\theta\, \hat{x} + \sin\theta\, \hat{y})\, R\, d\theta$$

$$= iBR\left[\sin\theta\Big|_0^\pi \hat{x} + \cos\theta\Big|_\pi^0 \hat{y}\right] \boxed{= 2iBR\,\hat{y}}$$