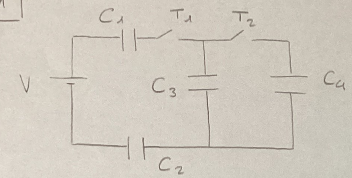
ESERCIZI DI ELETTRICITÀ E MAGNETISMO

$\boxed{\text{FOGLIO EM - 16}}$

5

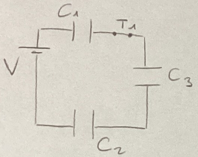
$\boxed{1}$



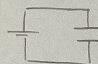
$C_1 = 2\,mF$ $\quad C_2 = 4\,mF$ $\quad C_3 = C_4 = 1\,mF$

$V = 10\,V$

a)



$\Rightarrow$

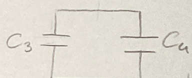


$$\frac{1}{C_{EQ}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = 5{,}7\cdot 10^{-10}\,F = 0{,}57\,mF$$

$$Q_3 = C_{EQ}V = 5{,}7\cdot10^{-9}\,C = \boxed{5{,}7\,mC}$$

$$\boxed{V_3 = \frac{Q_3}{C_3} = 5{,}71\,V}$$

b)



$Q_3 + Q_4 = Q_3' + Q_4' \Rightarrow Q_3' + Q_4' = Q_3$

MA C_3, C_4 IN SERIE $\Rightarrow Q_3' = Q_4' \Rightarrow Q_3' = Q_4' = \frac{Q_3}{2} = \boxed{2{,}85\,mC}$

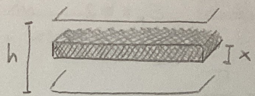
$$\boxed{V_3' = \frac{Q_3'}{C_3} = 2{,}85\,V = V_4'}$$

$\boxed{2}$

$S = 400\ cm^2$ $\qquad V_{IN} = 10^4\ V$

$h = 1\ cm$

$x = 5\ mm$



SENZA LASTRA $\quad E_0 = \frac{\sigma}{\varepsilon_0}\ ,\quad V_0 = \frac{\sigma h}{\varepsilon_0} = Eh\ ,\quad C_0 = \frac{q}{V} = \frac{\sigma S}{\sigma h}\varepsilon_0 = \frac{\varepsilon_0 S}{h} = 35{,}4\ pF$

CON LA LASTRA $\quad V = E(h-x) \Rightarrow C = \frac{\varepsilon_0 S}{h-x}$

$$-\Delta C = C_0 - C = \frac{\varepsilon_0 S}{h} - \frac{\varepsilon_0 S}{h-x} = \varepsilon_0 S\left(\frac{h-x-h}{h(h-x)}\right) = -\frac{\varepsilon_0 S}{h}\frac{x}{h-x} \Rightarrow \boxed{|\Delta C| = 35{,}4 pF}$$

SUPPONIAMO $q = COST$ $\quad (\sigma = COST)$

$q_0 = C_0 V_{IN} = 3{,}54 \cdot 10^{-7}\ C \qquad \sigma = \frac{q_0}{S} = 8{,}85 \cdot 10^{-8}\ C/m^2$

$U_{IN} = \frac{q^2}{2C_0} = 1{,}77 \cdot 10^{-3}\ J \qquad U_F = \frac{q^2}{2C} = 8{,}85 \cdot 10^{-4}\ J$

$\Delta U = -8{,}85 \cdot 10^{-4}\ J \qquad \boxed{W_q = -\Delta U = 8{,}85 \cdot 10^{-4}\ J}$

SUPPONIAMO $V = COST = V_{IN}$

$q_0 = C_0 V_{IN} = 3{,}54 \cdot 10^{-7}\ C \qquad q_F = C V_{IN} = 7{,}08 \cdot 10^{-7}\ C$

$U_{IN} = \frac{q^2}{2C_0} = 1{,}77 \cdot 10^{-3}\ J \qquad U_F = \frac{q_F^2}{2C} = 3{,}54 \cdot 10^{-3}\ J$

$\Delta U = 1{,}77 \cdot 10^{-3}\ J \qquad \boxed{W_V = -\Delta U = -1{,}77 \cdot 10^{-3}\ J}$

MA DEVO CONSIDERARE

$W_{GEN} = -\Delta U_{GEN} = V(q_0 - q)$

$= 3{,}54 \cdot 10^{-3}\ J$

$W_{GEN} = -2W_V$

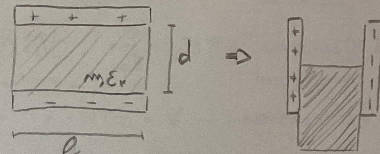
$W = W_V + W_{GEN} = W_V - 2W_2$

$= -W_V = 1{,}77 \cdot 10^{-3}\ J$

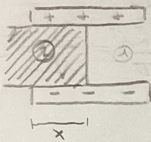
$\boxed{3}$ $l = 20\ cm$, $d = 0{,}32\ cm$, $\varepsilon_r = 2$, $m = 10{,}2\ g$, $Q = 2{,}2 \cdot 10^{-6}\ C$

SO $E = \frac{E_0}{\varepsilon_r}$ QUINDI $E = \frac{\sigma}{\varepsilon_0 \varepsilon_r}$

$\sigma = \frac{Q}{l^2} = 5{,}5 \cdot 10^{-5}\ C/m^2$



QUANDO LO GIRO AVRÒ



$E_1 = \frac{\sigma_1}{\varepsilon_0}$ $E_2 = \frac{\sigma_2}{\varepsilon_0 \varepsilon_r} \Rightarrow E_1 = E_2 \Rightarrow \sigma_2 = \varepsilon_r \sigma_2$

LA CARICA TOTALE $Q = \sigma_1 l(l-x) + \sigma_2 l x$

$\Rightarrow Q = \sigma_1 (l^2 - lx + lx\varepsilon_r) = \sigma_1 (l + x(\varepsilon_r - 1)) l$

$\Rightarrow \sigma_1 = \frac{Q}{l(l + x(\varepsilon_r - 1))}$ COSÌ $\sigma_2 = \frac{\varepsilon_r Q}{l(l + x(\varepsilon_r - 1))}$

$V(x) = E_1 d = E_2 d = \frac{\sigma_1 d}{\varepsilon_0} = \frac{\sigma_2 d}{\varepsilon_r \varepsilon_0} = \frac{Q d}{\varepsilon_0 l(l + x(\varepsilon_r - 1))}$ $E = \frac{Q}{\varepsilon_0 l(l + x(\varepsilon_r - 1))}$

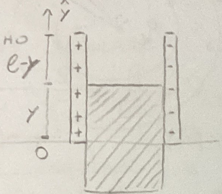
$C(x) = \frac{Q}{V(x)} = \frac{\varepsilon_0 l(l + x(\varepsilon_r - 1))}{d}$

COSÌ CALCOLO $U_e(x) = \frac{Q^2}{2C} = \frac{Q^2 d}{2\varepsilon_0 l(l + x(\varepsilon_r - 1))}$

ORA $F = -\frac{\partial U_e}{\partial x} = \frac{-Q^2 d}{2\varepsilon_0 \ell}\frac{\partial}{\partial x}\left\{\frac{1}{\ell + x(\varepsilon_r-1)}\right\} = -\frac{Q^2 d}{2\varepsilon_0 \ell}\left(-\frac{\ell(\varepsilon_r-1)}{(\ell + x(\varepsilon_r-1))^2}\right)$

$\Rightarrow F = \frac{Q^2 d(\varepsilon_r-1)}{2\varepsilon_0\ell(\ell + x(\varepsilon_r-1))^2}$

ORA HO



CON $F_e(y) = \frac{Q^2 d(\varepsilon_r-1)}{\ell\, 2\varepsilon_0(\ell + y(\varepsilon_r-1))^2}$ (POSITIVA, VERSO L'ALTO)

E AVRÒ $F_G(y) = -mg$

HO EQUILIBRIO $F_e + F_G = 0 \Rightarrow F_e(y) = -F_G(y)$

$\frac{Q^2 d(\varepsilon_r-1)}{2\varepsilon_0\ell(\ell + y(\varepsilon_r-1))^2} = mg \Rightarrow \sqrt{\frac{Q^2 d(\varepsilon_r-1)}{2\varepsilon_0 m g \ell}} = \ell + y(\varepsilon_r-1)$

$\Rightarrow y = \left(\sqrt{\frac{Q^2 d(\varepsilon_r-1)}{2\varepsilon_0 m g \ell}} - \ell\right)\frac{1}{(\varepsilon_r-1)} = \boxed{9{,}1\cdot 10^{-3}\ m = 9{,}1\ mm}$

$\boxed{4}$ $R_0 = 1\,cm$, $R_1 = 5\,cm$ $\varepsilon_r = 4$

$q_L = 10^{-8}\,C$, σ_{p0} , σ_{p1}

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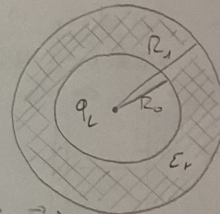


USO GAUSS CON D

$R_0<r<R_1$ $4\pi r^2 D(r) = q_L \Rightarrow \vec{D}(r) = \frac{q_L}{4\pi r^2}\hat{r}$

$D(r) = \varepsilon\vec{E} \rightarrow \vec{E}(r) = \frac{q_L}{4\pi\varepsilon_0\varepsilon_r r^2}\hat{r}$

$\sigma_P = \vec{P}\cdot\hat{r}$ (PERCHÈ $\vec{D}(r) \propto \hat{r}$ e $/\!/ \vec{E} /\!/ \vec{P}$)

$P = \varepsilon_0\chi E = \varepsilon_0(\varepsilon_r - 1)\frac{D}{\varepsilon_0\varepsilon_r} = D\left(1-\frac{1}{\varepsilon_r}\right) = \frac{\varepsilon_r - 1}{\varepsilon_r}D$

$\sigma_p(r) = P(r) = \frac{q_L(\varepsilon_r - 1)}{4\pi\varepsilon_r r^2}$, $\boxed{\sigma_p(R_0) = \sigma_{p0} = -5,97\cdot 10^{-6}\,C/m^2}$

$\rightarrow \vec{P}\cdot\hat{n} = -P$

$\boxed{\sigma_p(R_1) = \sigma_{p1} = 2,39\cdot 10^{-7}\,C/m^2}$

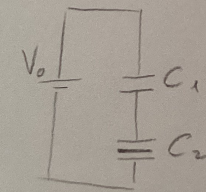
AVRÒ $u_e = \frac{1}{2}\varepsilon_0\varepsilon_r E^2 = \frac{D^2}{2\varepsilon_0\varepsilon_r} \Rightarrow U_e = \int_\tau u_e\,d\tau$

$$U_e = \int_{R_0}^{R_1}\frac{D^2}{2\varepsilon_0\varepsilon_r}4\pi r^2\,dr + \int_{R_1}^{\infty}\frac{D^2}{2\varepsilon_0}4\pi r^2\,dr = \frac{q_L^2}{8\pi\varepsilon_0}\left\{\frac{1}{\varepsilon_r}\int_{R_0}^{R_1}\frac{dr}{r^2} + \int_{R_1}^{\infty}\frac{dr}{r^2}\right\}$$

$$= \frac{q_L^2}{8\pi\varepsilon_0}\left\{\frac{1}{\varepsilon_r}\left(\frac{1}{R_0}-\frac{1}{R_1}\right)+\frac{1}{R_1}\right\} = \boxed{1,79\cdot 10^{-5}\,J}$$

5| $C = 44,25\ pF$, d , $V_0 = 50\ V$, $\varepsilon_r = 10$ $s = d/10$

CALCOLO $q = C_{eq} V_0 = \frac{C_1 C_2}{C_1 + C_2} V_0 = \frac{C}{2} V_0 = 1,1 \cdot 10^{-9}\ C$



POSSO CALCOLARE LE V INIZIALI

$$V_{01} = \frac{q}{C_1} = \frac{q}{C} = V_{02} = 25\ V$$

SE INSERISCO IL DIELETTRICO

$$V = E_0\left(d - \frac{d}{10}\right) + E\frac{d}{10} = E_0 d \frac{9}{10} + \frac{E_0 d}{\varepsilon_r 10} = \frac{E_0 d}{10}\left(9 + \frac{1}{\varepsilon_r}\right) = \frac{E_0 d}{10} \frac{9\varepsilon_r + 1}{\varepsilon_r}$$

$$V_{01} = E_0 d \Rightarrow V = \frac{V_0}{10} \frac{9\varepsilon_r + 1}{\varepsilon_r} = 22,75\ V$$

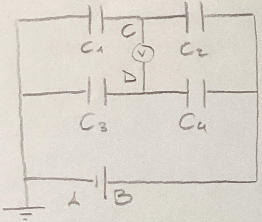
AVRÒ $U_i = \sum \frac{1}{2} qV = qV_{01} = 2,75 \cdot 10^{-8}\ J$

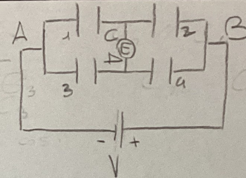
$V_{11} = V_0 - V = 27,25\ V \longrightarrow q_1 = CV_{11} = 1,2 \cdot 10^{-9}\ C$

$$C' = \frac{q_1}{V} = 5,27 \cdot 10^{-11}\ F$$

$U_F = \frac{1}{2} q_1 V_{11} + \frac{1}{2} V q_1 = 1,8 \cdot 10^{-8}\ J$, $\Delta U =$

6





POSSO CALCOLARE

$$*\begin{cases} V_C - V_A = \dfrac{q_1}{C_1} & V_B - V_C = \dfrac{q_2}{C_2} \\ V_D - V_A = \dfrac{q_3}{C_3} & V_B - V_D = \dfrac{q_4}{C_4} \end{cases}$$

PERÒ SE $E=0$ SONO IN EQUILIBRIO $\Rightarrow q_3 = q_4,\ q_1 = q_2$ E

$$V_1 = \frac{q_1}{C_1} = V_3 = \frac{q_3}{C_3}, \quad \frac{q_2}{C_2} = \frac{q_4}{C_4}$$

$$*\begin{cases} V_C = V_A + \dfrac{q_1}{C_1} & V_B = V_C + \dfrac{q_2}{C_2} \\ V_D = V_A + \dfrac{q_3}{C_3} & V_B = V_D + \dfrac{q_4}{C_4} \end{cases}$$

$\Rightarrow$ SE $V_C - V_D = 0$ DEVONO VALERE

QUINDI

$$\frac{q_1}{C_1} = \frac{q_3}{C_3} \rightarrow \frac{C_3}{C_1} = \frac{q_3}{q_1} = \frac{q_4}{q_2} = \frac{C_4}{C_2}$$

$$\Rightarrow \boxed{C_1/C_2 = C_3/C_4}$$