ESERCIZI DI ELETTRICITÀ E MAGNETISMO

$\boxed{\text{FOGLIO EM-15}}$

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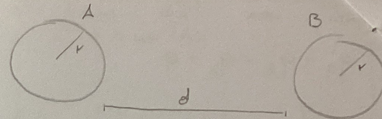
$\boxed{1}$ $r = 1\,mm$ $d = 2\,m$ $V_A = 0\,V$

$q = 1\,mC$ $d \gg r$



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PRIMA DEL COLLEGAMENTO A TERRA HO $V_A(d) = \frac{q}{4\pi\varepsilon_0 r} + \frac{q}{4\pi\varepsilon_0 d}$

SE LA COLLEGO A TERRA $V_A = 0 \Rightarrow V_A = 0 = \frac{q_A}{4\pi\varepsilon_0 r} + \frac{q}{4\pi\varepsilon_0 d}$

$$\Rightarrow q_A = -q\frac{r}{d} = \boxed{-0{,}5 \cdot 10^{-12}\,C}$$

IN UN ISTANTE GENERICO $U_E = \sum \frac{1}{2} q_i V_i = \frac{1}{2} q(V_A + V_B)$

$$\Rightarrow U_E = \frac{1}{2}\frac{q^2}{4\pi\varepsilon_0 r} + \frac{1}{2}\frac{q^2}{4\pi\varepsilon_0 r} + \frac{1}{2}\frac{q^2}{4\pi\varepsilon_0 d} + \frac{1}{2}\frac{q^2}{4\pi\varepsilon_0 d} = \frac{q^2}{4\pi\varepsilon_0 r} + \frac{q^2}{4\pi\varepsilon_0 d}$$

$F = -\frac{\partial U}{\partial d} = +\frac{q_A q_B}{4\pi\varepsilon_0 d^2}$ SE A COLLEGATA $\boxed{F = -\frac{q_A q}{4\pi\varepsilon_0 d^2} = -1{,}1 \cdot 10^{-12}\,N}$?

ESSENDO $d \gg r$ TRATTO LE SFERE COME PUNTIFORMI

$$|F| = \frac{q_A q}{4\pi\varepsilon_0 d^2} = 1{,}12 \cdot 10^{-12}\,N$$

SE LE COLLEGO $V_A' = V_B'$ E AVRÒ $q_A' = q_B'$ MA SO $q_A + q = q_A' + q_B'$

E
$$\begin{cases} V_A' = \frac{q_A'}{4\pi\varepsilon_0 r} + \frac{q_B'}{4\pi\varepsilon_0 d} \\ V_B' = \frac{q_B'}{4\pi\varepsilon_0 r} + \frac{q_A'}{4\pi\varepsilon_0 d} \end{cases} \Rightarrow \frac{q_A'}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{d}\right) = \frac{q_B'}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{d}\right)$$

NDI ORA HO

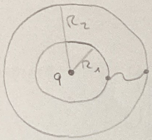
$$\begin{cases} q'_A + q'_B = q_A + q \\ q'_A\left(\frac{1}{r} - \frac{1}{d}\right) = q'_B\left(\frac{1}{r} - \frac{1}{d}\right) \end{cases} \Rightarrow \begin{cases} 2q' = q_A + q \\ q'_A = q'_B \end{cases} \Rightarrow q' = \frac{q_A + q}{2} = \boxed{0{,}49 \cdot 10^{-9}\,C}$$

QUINDI $\quad V'_A = \frac{q'}{4\pi\varepsilon_0}\left(\frac{1}{r} + \frac{1}{d}\right) = 4{,}49 \cdot 10^{3}\ V$

USO $\quad U = \frac{1}{2}\sum q_i V_i = \frac{1}{2} V'_A (q'_A + q'_B) = \frac{1}{2}(q_A + q_B) V'_A$

$\boxed{2}$ $Q = 0 = q_1 + q_2$ $q = 10^{-10}\,C$ $q_0 = q$ IN $d = 25\,cm$ (B)

A, $x_0 = 0.5\,cm$ (A)



$R_1 = 2\,cm$

$R_2 = 5\,cm$

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FILO $\Rightarrow$ STESSO POTENZIALE

PER INDUZIONE COMPLETA $q_1 = -q$, $q_2 = q$

DALLA TEORIA SO CHE HO UNO SCHERMO ELETTROSTATICO $\Rightarrow$ q E LA SFERA R_1 NON SENTONO EFFETTI DA q_0

IL CAMPO DI S_2 IN q_0 $\vec{E} = \frac{q}{4\pi\varepsilon_0 (d+R_2)^2}\hat{x}$ (IN GENERALE $\vec{E}(x) = \frac{q}{4\pi\varepsilon_0 x^2}\hat{x}$)

LA FORZA CHE SENTE q_0 È UGUALE E CONTRARIA A QUELLA SENTITA DAL GUSCIO

$$|\vec{F}| = \frac{q^2}{4\pi\varepsilon_0 (d+R_2)^2} = 0{,}9 \cdot 10^{-9}\,N$$

IL LAVORO $W = -q\,\Delta V$

$$V(B) = \int_{d+R_2}^{\infty} \vec{E}(x)\cdot dx = -\frac{q}{4\pi\varepsilon_0 x}\Big|_{d+R_2}^{\infty} = \frac{q}{4\pi\varepsilon_0 (d+R_2)}$$

SO $V(R_2) = V(R_1) = \frac{q}{4\pi\varepsilon R_2}$

$$V(A) - V(B) = \int_A^B \vec{E}(x) \cdot dx = \int_{x_0}^{R_1} \vec{E}(x) dx + \int_{R_2}^{R_2+d} \vec{E}(x) \cdot dx$$

$$= \frac{q}{4\pi\varepsilon_0} \left\{ \frac{1}{x_0} - \frac{1}{R_1} + \frac{1}{R_2} - \frac{1}{R_2+d} \right\}$$

QUINDI

$$W = -q_0 \Delta V_{AB} = \frac{q^2}{4\pi\varepsilon_0} \left\{ \frac{1}{x_0} - \frac{1}{R_1} + \frac{1}{R_2} - \frac{1}{R_2+d} \right\} = \boxed{1,49 \cdot 10^{-8} \ J}$$

$\boxed{3}$ $C_1 = 5\mu F$ $C_2 = 4\mu F$ $V_1 = 300V$ $V_2 = 250V$

$C_3 = 1\mu F$

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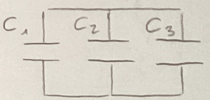
INIZIALMENTE $q_1 = C_1 V_1 = 1{,}5 \cdot 10^{-3} C$ $q_2 = C_2 V_2 = 10^{-3} C$

$U_i = \frac{1}{2} q_1 V_1 + \frac{1}{2} q_2 V_2 = 0{,}35 \text{ J}$



$C_{EQ} = \sum_i C_i = 10\mu F$

VALE $q_3 + q_1 + q_2 = q_1' + q_2' + q_3' \Rightarrow q_1 + q_2 = q_1' + q_2' + q_3'$

$q_i' = C_i V$ $\Rightarrow q_1 + q_2 = V(C_1 + C_2 + C_3) = V C_{EQ}$

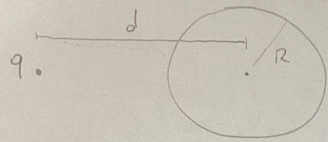
$\Rightarrow V = \frac{q_1 + q_2}{C_{EQ}} = 250 \text{ V}$

$\boxed{q_1' = C_1 V = 1{,}25 \text{ mC}}$ $\boxed{q_2' = C_2 V = 1 \text{ mC}}$ $\boxed{q_3' = 0{,}25 \text{ mC}}$

$U_f = \frac{1}{2} \sum q_i' V = 0{,}3125 \text{ J}$

$\Delta U = -0{,}0375 \text{ J}$

$\boxed{4}$



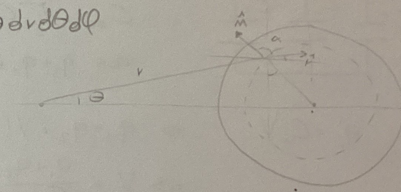
LA CONDIZIONE CHE VOGLIO È CHE

$\vec{E} = 0$ DENTRO LA SFERA

$$\vec{E}_{EXT}(r) = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$$

LA σ SULLA SFERA SARÀ SOLO SULLA SUPERFICIE E NEUTRALIZZERÀ GLI EFFETTI DI q

GENERICO ELEMENTO DI SUPERFICIE $d\Sigma = r^2 \sin\theta \, dr \, d\theta \, d\varphi$



$$\phi_E(\Sigma) = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}\cdot m + E_\sigma(-\hat{m})\cdot\hat{m}$$

GAUSS $\phi_E(\Sigma) = 0$

$$\frac{q}{4\pi\varepsilon_0 r^2}\cos\alpha - E_\sigma = 0 \Rightarrow E_\sigma = \frac{q}{4\pi\varepsilon_0 r^2}\cos\alpha$$

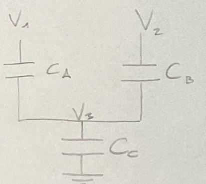
$\boxed{5}$ so $V_1 = \frac{q_1}{4\pi\varepsilon R_1} + \frac{q_2}{4\pi\varepsilon_0 d}$ $\qquad V_2 = \frac{q_1}{4\pi\varepsilon_0 d} + \frac{q_2}{4\pi\varepsilon R_2}$

$R_1, R_2 \ll d \Rightarrow$ CONSIDERO LE DENSITÀ $\sim$ UNIFORMI

$$U = \frac{1}{2}\sum q_i V_i = \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1^2}{R_1} + \frac{q_1 q_2}{d} + \frac{q_1 q_2}{d} + \frac{q_2^2}{R_2}\right)$$

$$\Rightarrow \boxed{U = \frac{1}{8\pi\varepsilon_0}\left(\frac{q_1^2}{R_1} + \frac{q_2^2}{R_2} + \frac{2q_1 q_2}{d}\right)}$$

$\boxed{6}$



$C_A = C \qquad C_B = 2C \qquad C_C = 3C$

$V_1 = 10V \qquad V_2 = 40V \qquad V_C = ?$

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$V_1 - V_3 = q_A / C_A \qquad V_2 - V_3 = q_B / C_B$

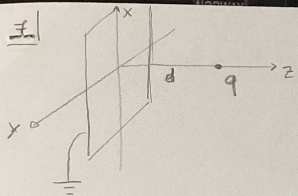
$V_3 = q_C / C_C$

$q_A + q_B = q_C \qquad \frac{q_A}{C_A} + \frac{q_B}{C_B} = \frac{q_A}{C} + \frac{q_B}{2C} = (V_1 - V_3) + (V_2 - V_3)$

$\Rightarrow \frac{1}{3}(V_1 - V_3) + \frac{2}{3}(V_2 - V_3) = \frac{q_A}{3C} + \frac{q_B}{3C} = \frac{q_C}{C_C} = V_3 \Rightarrow V_3 = \frac{1}{3}V_1 + \frac{2}{3}V_2 - V_3$

$\Rightarrow 2V_3 = \frac{V_1}{3} + \frac{2}{3}V_2 \Rightarrow \boxed{V_3 = \frac{1}{2}\left(\frac{V_1}{3} + \frac{2V_2}{3}\right) = 15V}$

7.



$V_P = 0$

$q > 0$, d

CAMPO DI q $\quad E(r) = \dfrac{q}{4\pi\varepsilon_0 r^2} = \dfrac{q}{4\pi\varepsilon_0 \sqrt{(x^2+y^2+z^2)}^2} = \dfrac{q}{4\pi\varepsilon_0 (x^2+y^2+z^2)}$

SUL CONDUTTORE $\quad E(d) = \dfrac{q}{4\pi\varepsilon_0 (x^2+y^2+d^2)}$ $\quad$ (IN UN PUNTO GENERICO x, y)

LA FORZA CHE SENTE q DAL FOGLIO È UGUALE E OPPOSTA A QUELLA SENTITA DAL FOGLIO

IL CAMPO IN UN PUNTO INTERNO DELLA LASTRA (DI SPESSORE h)

$$dE_L(x,y,z) = \frac{\sigma d\Sigma}{4\pi\varepsilon_0 (x^2+y^2+z^2)}$$

CHE SOVRAPPOSTO A E DEV'ESSERE NULLO

$$E_T(x,y,z) = 0 = E(x^i) + E_L(x^i) \Rightarrow -\frac{\sigma d\Sigma}{4\pi\varepsilon_0 (x^2+y^2+z^2)} = \frac{q}{4\pi\varepsilon_0 (x^2+y^2+d^2)}$$