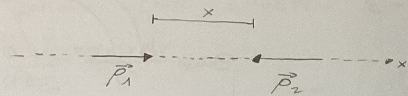
ESERCIZI DI ELETTRICITÀ E MAGNETISMO

FOGLIO EM-A4

$\boxed{1}$ p_1, p_2, x



CALCOLO $E_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{3\vec{p}_1\cdot\vec{x}}{x^5}\vec{x} - \frac{\vec{p}_1}{x^3}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{3\vec{p}_1 x^2}{x^5} - \frac{\vec{p}_1}{x^3}\right) = \frac{2\vec{p}_1}{4\pi\varepsilon_0 x^3} = \frac{\vec{p}_1}{2\pi\varepsilon_0 x^3}$

INIZIALMENTE $U_0 = -\vec{p}_2\cdot\vec{E}_1 = -(\vec{p}_2\cdot\vec{p}_1)\frac{1}{2\pi\varepsilon_0 x^3} = \frac{p_1 p_2}{2\pi\varepsilon_0 x^3}$

SO DALLA TEORIA CHE P TENDE AD ALLINEARSI CON IL CAMPO ESTERNO, QUINDI LA POSIZIONE DI EQUILIBRIO SARÀ CON $p_2 // p_1$

NELLA SITUAZIONE FINALE $U_F = -\vec{p}_2\cdot\vec{E}_1 = -\frac{p_1 p_2}{2\pi\varepsilon_0 x^3}$

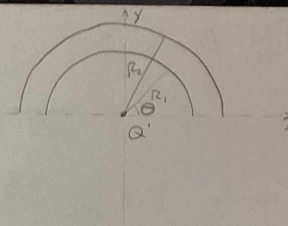
QUINDI $\Delta U = -\frac{p_1 p_2}{2\pi\varepsilon_0 x^3} - \frac{p_1 p_2}{2\pi\varepsilon_0 x^3} = \boxed{-\frac{p_1 p_2}{\pi\varepsilon_0 x^3}}$

$F = \nabla(\vec{p}_2\cdot\vec{E}_1) = \nabla\left(\frac{\vec{p}_1\cdot\vec{p}_2}{2\pi\varepsilon_0 x^3}\right) = \nabla\left(\frac{p_1 p_2}{2\pi\varepsilon_0 x^3}\right) = \frac{p_1 p_2}{2\pi\varepsilon_0}\nabla\left(\frac{1}{x^3}\right) = \boxed{-\frac{3p_1 p_2}{2\pi\varepsilon_0 x^4}}$

$= -\frac{\partial U_{FIN}}{\partial x}$

$\boxed{2}$ $R_1 = 3$ cm $R_2 = 5$ cm $Q = 1{,}8\cdot 10^{-8}$ C

SEMI-CORONA $Q' = -Q$



ESPANSIONE IN MULTIPOLI

$$V(r) = \frac{1}{4\pi\varepsilon_0}\left\{\underbrace{\frac{1}{r}\int d^3r'\,\rho(r')}_{\text{MONOPOLO}} + \underbrace{\frac{1}{r^3}\vec{r}\cdot\int d^3r'\,\rho(r')\,\vec{r}\,' + \dots}_{\text{TERMINE DI DIPOLO}}\right\}$$

? $\int d^3r'\,\rho(r') = 0$ PER NOI

$\hookrightarrow \int d^3r'\,\rho(r')\,\vec{r}\,' = \vec{p}$

→ ORIGINE (SCELTA S.R. ARBITRARIA)

NEL NOSTRO CASO $\vec{p} = \int d^2r'\,\sigma(r')\,\vec{r}\,' + Q'\cdot 0$

$\vec{p}$ AVRÀ COMPONENTE x E y ($r_x = \vec{r}\,'\cos\theta$, $\vec{r}_y = r'\sin\theta$)

$$p_x = \int_{R_1}^{R_2} dr' \int_0^{\pi} d\theta'\, r'\,\sigma(r')\cdot\underbrace{r'\cos\theta}_{r_x} = \int r'^2\sigma(r')\int_0^{\pi} d\theta'\cos\theta = 0$$

$$p_y = \int_{R_1}^{R_2}\int_0^{\pi} dr'\, r'^2\sigma(r')\sin\theta\, d\theta = \frac{2\sigma(r)}{3}(R_2^3 - R_1^3)$$

$\Rightarrow \vec{p} = p_y = \frac{2\sigma}{3}(R_2^3 - R_1^3)\,\hat{u}_y$ CON $\sigma = \frac{Q}{\frac{\pi}{2}(R_2^2 - R_1^2)}$

$$\Rightarrow \vec{p} = \frac{2}{3}(R_2^3 - R_1^3)\frac{2Q}{\pi(R_2^2 - R_1^2)}\hat{y} = \frac{4Q}{3\pi}\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}\hat{y} = \boxed{(4{,}67\cdot 10^{-10})\,\hat{u}_y}$$

LA FORZA SARÀ $\vec{F} = Q'\vec{E}$

$\vec{E}$ DELLA CORONA (PER SIMMETRIA IN O SARÀ SOLO $\hat{y}$)

$$dE_y = -\frac{1}{4\pi\varepsilon_0}\frac{\sigma(r)\,r\,dr}{r^2}\sin\theta$$

$$E_y = -\frac{\sigma}{4\pi\varepsilon_0}\int\frac{dr}{r}\int d\theta\,\sin\theta = -\frac{\sigma}{4\pi\varepsilon_0}\ln\left(\frac{R_2}{R_1}\right)2 = \frac{\sigma}{2\pi\varepsilon_0}\ln\left(\frac{R_1}{R_2}\right) = \frac{Q}{\pi^2\varepsilon_0}\frac{\ln\left(\frac{R_1}{R_2}\right)}{R_2^2-R_1^2}$$

$$\vec{F} = Q'\vec{E}_y = \frac{Q^2}{\pi^2\varepsilon_0}\frac{\ln\left(\frac{R_2}{R_1}\right)}{R_2^2-R_1^2} = \boxed{1{,}18\cdot10^{-3}\ N}$$

$\boxed{3}$ q SUP SFERICA, R E POI VOLUME SFERICO

$r > R$ $\quad 4\pi r^2 E(r) = \frac{q}{\varepsilon_0} \Rightarrow \boxed{\vec{E}(r) = \frac{q\hat{r}}{4\pi\varepsilon_0 r^2}}$ $\qquad \boxed{r<R \quad E(r) = 0}$

$$u_E = \frac{1}{2}\varepsilon_0 E^2(r) = \frac{q^2}{32\pi^2\varepsilon_0 r^4}$$

$$U = \int_\Sigma u_E \, d\Sigma \int_R^\infty \frac{q^2}{32\pi^2\varepsilon_0 r^4} 4\pi r^2 dr = \frac{q^2}{8\pi\varepsilon_0}\left[-\frac{1}{r}\right]_R^\infty = \boxed{\frac{q^2}{8\pi\varepsilon_0 R}}$$

b) $r < R$ $\quad 4\pi r^2 E(r) = \frac{\rho}{\varepsilon_0}\frac{4}{3}\pi r^3 \Rightarrow E(r) = \frac{\rho r}{3\varepsilon_0}$

$$u_E = \frac{\rho^2 r^2}{18\varepsilon_0} \qquad U = \frac{\rho^2}{18\varepsilon_0}\int_0^R r^4 dr \int_0^{2\pi} d\varphi \int_0^\pi \sin\theta \, d\theta = \frac{\rho^2 R^5}{18\varepsilon_0 5} 4\pi = \frac{2\rho^2 \pi R^5}{9\varepsilon_0 5}$$

$r > R$ $\quad 4\pi r^2 E(r) = \frac{\rho}{\varepsilon_0}\frac{4}{3}\pi R^3 \Rightarrow E(r) = \frac{\rho R^3}{3\varepsilon_0 r^2}$

$$u_E = \frac{\rho^2 R^6}{18\varepsilon_0 r^4} \qquad U = \frac{\rho^2 R^6}{18\varepsilon_0}\int_R^\infty \frac{r^2}{r^4} dr \int d\varphi \sin\theta \, d\theta = \frac{\rho^2 R^6}{18\varepsilon_0} 4\pi \left[-\frac{1}{r}\right]_R^\infty = \frac{2\rho^2 R^6 \pi}{9\varepsilon_0 R}$$

SCRIVO MEGLIO

$$\begin{cases} 0 < r < R & U = \frac{2\pi R^5}{5\cdot \cancel{9}\varepsilon_0}\frac{\cancel{9}q^2}{16\pi^2 R^6} = \frac{q^2}{40\pi\varepsilon_0 R} \\ r > R & U = \frac{2R^6\pi}{\cancel{9}\varepsilon_0 R}\frac{\cancel{9}q^2}{16\pi^2 R^6} = \frac{q^2}{8\pi\varepsilon_0 R} \end{cases} \Rightarrow U = \frac{6q^2}{40\pi\varepsilon_0 R}$$

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$R_1 = 30$ cm $\quad Q_1 = 2 \cdot 10^{-8}$ C $\quad I = 0$

$R_2 = 10$ cm $\quad Q_2 = ?$

$D \gg R \sim R_1 \sim R_2$

⇒ mi dice che le densità superficiali sono uniformi

— o —

si collegano $\Rightarrow V_1 = V_2 = V$

so $V_1 = \dfrac{Q_1}{4\pi\varepsilon_0 R_1}$ e $V_2 = \dfrac{Q_2}{4\pi\varepsilon_0 R_2}$ $\Rightarrow$ $\boxed{Q_2 = Q_1 \dfrac{R_2}{R_1} = 0{,}66 \cdot 10^{-8}\ C}$

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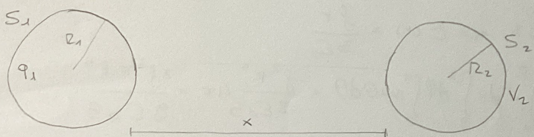

$x \gg R_1, R_2$

$q_1,\ V_2 \quad (V_2(\infty) = 0)$

trova $V_1,\ q_2,\ F$

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sfere conduttrici, quindi $V_1(r \le R_1) = \dfrac{q_1}{4\pi\varepsilon_0 R_1}$, $\quad V_2(r \le R_2) = \dfrac{q_2}{4\pi\varepsilon_0 R_2}$

ma $V_1(x) = V_1(R_1) + V_2(x) = \dfrac{q_1}{4\pi\varepsilon_0 R_1} + \dfrac{q_2}{4\pi\varepsilon_0 x} = \dfrac{q_1}{4\pi\varepsilon_0 R_1} + \dfrac{V_2 R_2}{x}$

poichè al potenziale su S_1 contribuiscono sia S_1 che S_2 a distanza x

ANALOGAMENTE $V_2(x) = \frac{q_2}{4\pi\varepsilon_0 R_2} + \frac{V_1 R_1}{x} = \frac{q_2}{4\pi\varepsilon_0 R_2} + \frac{q_1 \cancel{R_1}}{4\pi\varepsilon_0 \cancel{R_1} x}$

$$V_2 R_2 - \frac{q_1 R_2}{4\pi\varepsilon_0 x} = \frac{q_2}{4\pi\varepsilon_0} \Rightarrow q_2 = V_2 4\pi\varepsilon_0 R_2 - q_1 R_2/x = \boxed{\left(4\pi\varepsilon_0 V_2 - \frac{q_1}{x}\right) R_2}$$

LA FORZA LA TROVO CON IL POTENZIALE

$$U_E = \frac{1}{2} q_1 V_1 + \frac{1}{2} q_2 V_2 = \frac{1}{2}\left(\frac{q_1^2}{4\pi\varepsilon_0 R_1} + \frac{q_1 q_2}{4\pi\varepsilon_0 x} + \frac{q_2^2}{4\pi\varepsilon_0 R_2} + \frac{q_1 q_2}{4\pi\varepsilon_0 x}\right)$$

$$= \frac{1}{2}\frac{q_1^2}{4\pi\varepsilon_0 R_1} + \frac{1}{2}\frac{q_2^2}{4\pi\varepsilon_0 R_2} + \frac{q_1 q_2}{4\pi\varepsilon_0 x}$$

SO $F = \frac{\partial U}{\partial x} = -\frac{q_1 q_2}{4\pi\varepsilon_0 x^2} = -\frac{q_1\left(4\pi\varepsilon_0 V_2 - \frac{q_1}{x}\right) R_2}{4\pi\varepsilon_0 x^2}$

$$\Rightarrow \boxed{|F| = \frac{q_1 (4\pi\varepsilon_0 V_2 - q_1/x) R_2}{4\pi\varepsilon_0 x^2}}$$