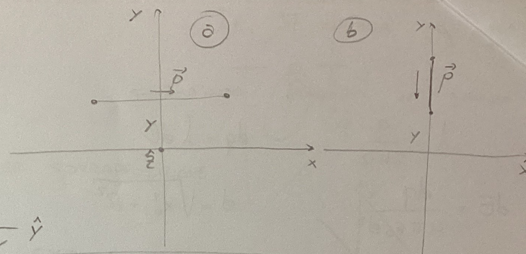
ESERCIZI DI ELETTRICITÀ E MAGNETISMO

FOGLIO EM - A3

$\boxed{1}$ $p = 1,4 \cdot 10^{-8}$ Cm $\quad y = 2,8$ cm

FILO $\lambda = 4\ \mu C/m$



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CALCOLO IL CAMPO DEL FILO

$$\ell\, 2\pi r\, E(r) = \frac{\lambda \ell}{\varepsilon_0} \Rightarrow \vec{E}(r) = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{y}$$

a)

CALCOLO $\boxed{\vec{F} = \vec{\nabla}(\vec{p}\cdot\vec{E}) = 0}$ MA $\boxed{\vec{M} = \vec{p}\times\vec{E} = |p|\frac{\lambda}{2\pi\varepsilon_0 y}(\hat{z}) = 3,59\cdot 10^{-2}\ Nm}$ $\rightarrow |M_a|$

$$b)\quad \vec{F} = \vec{\nabla}(\underbrace{\vec{p}\cdot\vec{E}}_{-\hat{u}_y\cdot\hat{u}_y = -\hat{u}_y}) = \vec{\nabla}\left(-\frac{|p|\lambda}{2\pi\varepsilon_0}\frac{1}{r}\hat{u}_y\right) = -\frac{|p|\lambda}{2\pi\varepsilon_0}\left(-\frac{1}{r^2}\right)\hat{u}_y = \frac{|p|\lambda}{2\pi\varepsilon_0 r^2}\hat{u}_y$$

$$\boxed{|\vec{F}| = \frac{|p|\lambda}{2\pi\varepsilon_0 y^2} = 1,28\ N} \qquad \boxed{M = \vec{p}\times\vec{E} = 0}$$

c) IN UNA POSIZIONE GENERICA HO $M = \frac{|p|\lambda}{2\pi\varepsilon_0 y}\sin\theta\,\hat{u}_z$

$$W = \frac{|p|\lambda}{2\pi\varepsilon_0 y}u_z\int_0^{\pi/2}\sin\theta\, d\theta = -|M_a|\cos\theta\Big|_0^{\pi/2} = \boxed{|M_a| = 3,59\cdot 10^{-2}\ J}$$

OPPURE $W = -\Delta U = -p_{IN}\cdot E - p_{FIN}\cdot E = 0 - \left(-\frac{|p|\lambda}{2\pi\varepsilon_0}\frac{1}{r}\right) = \frac{|p|\lambda}{2\pi\varepsilon_0 r} = 3,59\cdot 10^{-2}\ J$

$\boxed{2}$ p, ANELLO R, $+\lambda$

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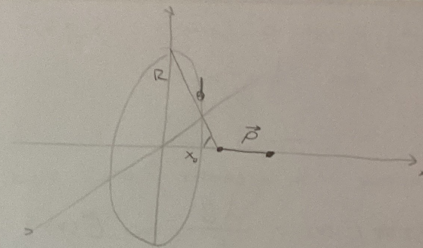

CALCOLO IL CAMPO DELL'ANELLO

$$\lambda = \frac{q}{2\pi R} \quad , \quad dq = \lambda\, dl$$

$$dE = \frac{dq}{4\pi\varepsilon_0 d^2} \qquad d = \sqrt{x_0^2 + R^2}$$

PER SIMMETRIA SU $\hat{x}$ HO SOLO UN CONTRIBUTO $\cos\theta$

$$dE = \frac{\lambda\, dl}{4\pi\varepsilon_0 d^2} \frac{x_0}{d} = \frac{\lambda x_0}{4\pi\varepsilon_0 d^3} dl$$

$$\vec{E} = \frac{\lambda x_0}{4\pi\varepsilon_0 d^3} 2\pi R \Rightarrow \boxed{\vec{E} = \frac{q x_0}{4\pi\varepsilon_0 (x_0^2 + R^2)^{3/2}} \hat{x}}$$

$$\vec{F} = \nabla(\vec{p}\cdot\vec{E}) = \frac{pq\hat{x}}{4\pi\varepsilon_0} \nabla\left(\frac{x}{(x^2+R^2)^{3/2}}\right) = \frac{pq\hat{x}}{4\pi\varepsilon_0}\left\{\frac{1}{d^3} - \frac{3}{2}\frac{2x}{(x^2+R^2)^{5/2}}\right\}$$

$$= \frac{pq\hat{x}}{4\pi\varepsilon_0}\left\{\frac{1}{(x^2+R^2)^{3/2}} - \frac{3x^2}{(x^2+R^2)^{5/2}}\right\}$$

$$\vec{F} = 0 \Rightarrow \frac{1}{(x^2+R^2)^{3/2}} - \frac{3x^2}{(x^2+R^2)^{5/2}} = 0 \Rightarrow \frac{3x^2}{(x^2+R^2)} = 1 \Rightarrow 3x^2 = x^2 + R^2$$

$$\Rightarrow 2x^2 = R^2 \Rightarrow \boxed{x = R/\sqrt{2}}$$

$\boxed{3}$ $\sigma = 10^{-6}\ C/m^2$ $R = 10\ cm$, $x = R$

$m = 10^{-3}\ Kg$, q

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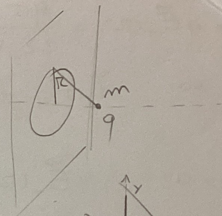
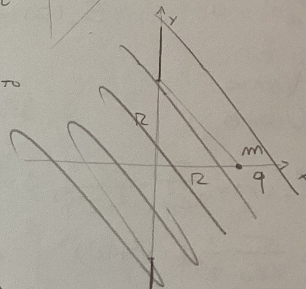
CALCOLO IL CAMPO ELETTRICO PER SOVRAPPOSIZIONE



(FORO) $-\sigma$

$$dE(x) = -\frac{\sigma\, 2\pi r\, dr}{4\pi\varepsilon_0}\frac{1}{d^2} \Rightarrow$$ MA HO SOLO CONTRIBUTO LUNGO $\hat{x}$

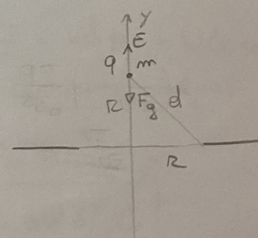


$$dE(x) = \frac{-\sigma\, 2\pi r\, dr}{4\pi\varepsilon_0 d^2}\frac{x}{d} = -\frac{\sigma r x\, dr}{2\varepsilon_0 d^3}$$

$$\vec{E}(x) = -\frac{\sigma x}{2\varepsilon_0}\int_0^R \frac{r\, dr}{(x^2+r^2)^{3/2}} = -\frac{\sigma x}{2\varepsilon_0}\left(-2\frac{1}{(x^2+r^2)^{1/2}}\frac{1}{2}\right)_0^R$$

$$= -\frac{\sigma x}{2\varepsilon_0}\left(\frac{1}{x} - \frac{1}{(x^2+R^2)^{1/2}}\right) = -\frac{\sigma}{2\varepsilon_0}\left(1 - \frac{x}{(x^2+R^2)^{1/2}}\right)\hat{x}$$

$$\vec{E}(R) = -\frac{\sigma}{2\varepsilon_0}\left(1 - \frac{R}{\sqrt{2}R}\right)\hat{x} = -\frac{\sigma}{2\varepsilon_0}\left(1 - \frac{1}{\sqrt{2}}\right) = \frac{\sigma}{2\varepsilon_0}\frac{1-\sqrt{2}}{\sqrt{2}}\hat{x}$$



(PIANO) σ , $2\pi r^2 \vec{E}(r) = \frac{\pi r^2 \sigma}{\varepsilon_0} \rightsquigarrow \boxed{\vec{E}(x) = \frac{\sigma}{2\varepsilon_0}\hat{x}}$

$$\vec{E}_T(R) = \frac{\sigma}{2\varepsilon_0}\left(1 + \frac{1-\sqrt{2}}{\sqrt{2}}\right) = \frac{\sigma}{2\varepsilon_0}\frac{\sqrt{2}+1-\sqrt{2}}{\sqrt{2}} = \frac{\sigma}{2\sqrt{2}\varepsilon_0}\hat{x}$$

$$= q\vec{E}_+(R) = \frac{q\sigma}{2\varepsilon_0\sqrt{2}}\hat{y}$$ poi ho $$F_g = -mg\,\hat{y}$$

e so $$\sum F_i = 0 \Rightarrow \frac{q\sigma}{2\varepsilon_0\sqrt{2}} = mg \Rightarrow \boxed{q = \frac{mg}{\sigma}2\varepsilon_0\sqrt{2} = 2{,}45\cdot 10^{-7}\,C}$$

$$\vec{E}_+(0) = 0 \qquad \vec{E}_+(R) = \frac{\sigma}{2\sqrt{2}\varepsilon_0}\hat{y}$$

$$\Downarrow$$

$$V(0)=0$$

$$U(0)=0$$

$$\vec{E}_+(y) = \frac{\sigma}{2\varepsilon_0}\frac{y}{(y^2+R^2)^{1/2}}\hat{y} \qquad -\Delta V = V(0)-V(R) = \int_0^R \frac{\sigma}{2\varepsilon_0}\frac{y}{(y^2+R^2)^{1/2}}\,dy$$

$$= \frac{\sigma}{2\varepsilon_0}\frac{1}{\cancel{2}}\cancel{2}(y^2+R^2)^{1/2}\Big|_0^R = \frac{\sigma R}{2\varepsilon_0}(\sqrt{2}-1)$$

$$\Rightarrow \Delta U = -q\Delta V = \frac{\sigma q R}{2\varepsilon_0}(\sqrt{2}-1) \Rightarrow \boxed{W = -\Delta U = \frac{\sigma q R}{2\varepsilon_0}(1-\sqrt{2}) = -5{,}73\cdot 10^{-4}\,J}$$

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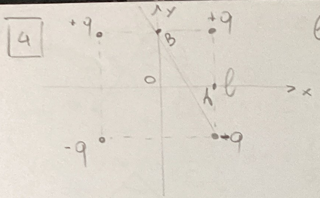

$\ell = 14,1\ cm$

$U = -3.6 \cdot 10^{-5}\ J$

$A = \left(\frac{\ell}{2}, 0\right)$ $\qquad$ $B = \left(0, \frac{\ell}{2}\right)$

$W = 2.8 \cdot 10^{-7}\ J$

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SO $U = \frac{1}{4\pi\varepsilon_0} \sum_{i,j} \frac{q_i q_j}{r_{ij}} = -\frac{1}{4\pi\varepsilon_0} q^2 \left(\cancel{-\frac{1}{\ell}} - \frac{1}{\sqrt{2}\ell} \cancel{+\frac{1}{\ell}} - \frac{1}{\sqrt{2}\ell} \cancel{-\frac{1}{\ell}} \cancel{+\frac{1}{\ell}} \right)$

$= -\frac{1}{4\pi\varepsilon_0} q^2 \frac{2}{\sqrt{2}\ell} = \frac{\sqrt{2}}{4\pi\varepsilon_0} q^2 \frac{1}{\ell}$

$\Rightarrow q = \sqrt{\frac{-4U\pi\varepsilon_0\sqrt{2}\ell}{\sqrt{2}}} = \boxed{1,99 \cdot 10^{-8}\ C}$

$\Delta U = \Delta(U + U_{q^*}) = \underbrace{\Delta U}_{=0} + \Delta U_{q^*} = \Delta U_{q^*} = -W$

$W = -\Delta U = -q^* \Delta V = q^*(V(P_A) - V(P_B))$

$V_-(P_B) = \frac{-q}{4\pi\varepsilon_0} \frac{2}{\sqrt{5}\ell}$ $\qquad$ $V_+(P_B) = \frac{q}{4\pi\varepsilon_0} \frac{2}{\ell}$ $\qquad$ (2 CONTRIBUTI OGNUNO)

$V(P_B) = \frac{q}{\pi\varepsilon_0 \ell}\left(1 - \frac{1}{\sqrt{5}}\right) = 2820\ V$

$V(P_A) = \frac{q}{4\pi\varepsilon_0}\left(-\frac{1}{\ell/2} + \frac{1}{\ell/2} - \frac{2}{\sqrt{5}\ell} + \frac{2}{\sqrt{5}\ell}\right) = 0$

$\Rightarrow \boxed{q^* = \frac{W}{V(P_B)} = 9,93 \cdot 10^{-11}\ C}$

5 $m = 10\,g$, $l = 5\,cm$, Q , $\theta = 150°$

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CAMPO DI UNA SFERETTA $E(r) = \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$

$r = 2x = 2l\cos\alpha = 9{,}6\ cm$

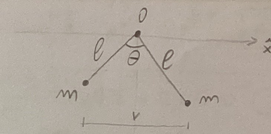
OGNI SFERETTA SENTE $\vec{F} = Q\vec{E}(r) = \dfrac{Q^2}{4\pi\varepsilon_0 r^2}\hat{u}_x$

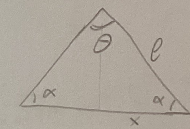
$\vec{F}_G = -mg\,\hat{y}$

so $\sum F_i = 0 \Rightarrow \dfrac{Q^2}{4\pi\varepsilon_0 r^2}\hat{x} + mg\hat{y} + \vec{T} = 0$

$$\begin{cases} T\sin\alpha = mg \\ T\cos\alpha = \dfrac{Q^2}{4\pi\varepsilon_0 r^2} \end{cases} \Rightarrow T = \frac{mg}{\sin\alpha}$$

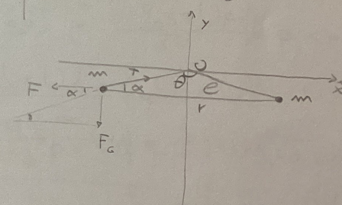
$$\Rightarrow Q = \sqrt{\frac{4\pi\varepsilon_0 r^2 mg}{tg\,\alpha}} = \boxed{6{,}13\cdot 10^{-7}\ C}$$



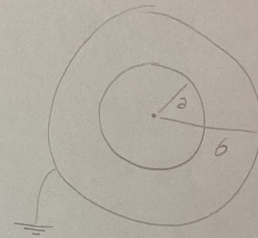


$\pi = \theta + 2\alpha$

$\Rightarrow \alpha = \dfrac{\pi - \theta}{2} = 15°$



$\boxed{6}$ SFERA a, $b > a$ $\quad V_b = 0 \quad V_A = V_0 \quad$ TROVA ρ_a



CAMPO

$r < a$ $\quad 4\pi r^2 E(r) = \dfrac{\rho \frac{4}{3}\pi r^3}{\varepsilon_0} \Rightarrow \boxed{E(r) = \dfrac{\rho r}{3\varepsilon_0}}$

$$V(0) - V(r) = \frac{\rho}{3\varepsilon_0}\int_0^r r\,dr = \frac{\rho r^2}{6\varepsilon_0}$$

$$\Rightarrow \boxed{V(r) = -\frac{\rho r^2}{6\varepsilon_0} + V_0}$$

$b > r > a$ $\quad 4\pi r^2 E(r) = \dfrac{\rho \frac{4}{3}\pi a^3}{\varepsilon_0}$

$$\Rightarrow \boxed{E(r) = \frac{\rho a^3}{3\varepsilon_0 r^2}}$$

$$V(a) - V(b) = \frac{\rho a^3}{3\varepsilon_0}\int_a^b \frac{dr}{r^2} = \frac{\rho a^3}{3\varepsilon_0}\left(-\frac{1}{r}\right) \Rightarrow V(b) = \frac{\rho a^3}{3\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right) + V_a$$

$$V(a) = -\frac{\rho a^2}{6\varepsilon_0} + V_0$$

$$V(b) = 0 = \frac{\rho a^3}{3\varepsilon_0}\frac{a-b}{ab} - \frac{\rho a^2}{6\varepsilon_0} + V_0 = \frac{\rho a^2}{3\varepsilon_0}\left(\frac{a-b}{b} - \frac{1}{2}\right) + V_0 = \frac{\rho a^2}{3\varepsilon_0}\left(\frac{2a-2b-b}{2b}\right) + V_0$$

$$\Rightarrow V_0 = \frac{\rho a^2}{3\varepsilon_0}\,\frac{3b-2a}{2b} \Rightarrow \boxed{\rho = \frac{6b\varepsilon_0 V_0}{a^2(3b-2a)}}$$

RICALCOLO IL POTENZIALE

$0 < r < a \qquad E = \dfrac{\rho r}{3\varepsilon_0}$

$a < r < b \qquad E = \dfrac{\rho a^3}{3\varepsilon_0 r^2}$

$$V(0) - V(b) = \int_0^b \vec{E} \cdot d\vec{l} = \int_0^a E(0<r<a) \cdot dl + \int_a^b E(a<r<b) \cdot dl$$

$$= \frac{\rho}{3\varepsilon_0} \int_0^a r\,dr + \frac{\rho a^3}{3\varepsilon_0} \int_a^b \frac{1}{r^2}\,dr = \frac{\rho a^2}{6\varepsilon_0} + \frac{\rho a^3}{3\varepsilon_0} \left(-\frac{1}{r} \right) \Bigg|_a^b = \frac{\rho a^2}{6\varepsilon_0} + \frac{\rho a^3}{3\varepsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right)$$

$$= \frac{\rho a^2}{6\varepsilon_0} + \frac{\rho a^3}{3\varepsilon_0} \frac{b-a}{ab}$$

$$\Rightarrow V(b) = V_0 - \frac{\rho a^2}{6\varepsilon_0} + \frac{\rho a^2 (a-b)}{3b\varepsilon_0}$$

$$V(b) = 0 \quad \Rightarrow V_0 = \frac{\rho a^2}{6\varepsilon_0} + \frac{\rho a^2 (b-a)}{3b\varepsilon_0} = \frac{\rho a^2}{3\varepsilon_0} \left(\frac{1}{2} + \frac{b-a}{b} \right) = \frac{\rho a^2}{3\varepsilon_0} \left(\frac{b + 2b - 2a}{2b} \right)$$

$$\Rightarrow \frac{3\varepsilon_0 V_0}{a^2} = \left(\frac{3b - 2a}{2b} \right) \rho \quad \Rightarrow \boxed{\rho = \frac{6b\varepsilon_0 V_0}{a^2 (3b - 2a)}}$$