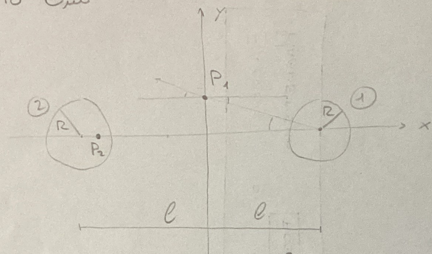
ESERCIZI DI ELETTRICITÀ E MAGNETISMO

FOGLIO EM - A2

1) R . $\rho(r) = kr^2$ $P_1 = (0, \ell/2, 0)$

$R = 5\ cm$ $\ell = 20\ cm$ $P_2 = (\frac{R}{2} - \ell, 0, 0)$

$k = 10^{-3}\ C\,m^{-5}$



IN P_1 AVRÒ LE COMPONENTI x DELLE DUE SFERE UGUALI E CONTRARIE E SI ELIDERANNO MENTRE SI SOMMANO LE COMPONENTI y

SE USO GAUSS FINO P_1

$$E(r)\,4\pi r_1^2 = \frac{k}{\varepsilon_0}\int r^4 \sin\theta \, dr\, d\theta\, d\varphi$$

$$\Rightarrow E = \frac{k}{\varepsilon_0}\frac{1}{4\pi r_1^2}\cancel{4\pi}\frac{R^5}{5} = \frac{kR^5}{5\varepsilon_0 r_1^2}$$ DOVE $r_1 = \sqrt{\frac{\ell^2}{4} + \ell^2} = \frac{\sqrt{5}}{2}\ell$

$$\Rightarrow E = \frac{4kR^5}{25\varepsilon_0 \ell^2} \Rightarrow E_y = \frac{4kR^5}{25\varepsilon_0\ell^2}\,\frac{\ell/2}{\sqrt{5}/2\,\ell} = \frac{4kR^5}{25\sqrt{5}\,\varepsilon_0\ell^2}$$ (CONTRIBUTO DI 1 SFERA)

$$\Rightarrow E_{P_1} = \frac{8kR^5}{25\sqrt{5}\,\varepsilon_0\ell^2} = 126,3\ N/C$$

IN P_2 HO UNO DEI DUE CONTRIBUTI MINORI (ESSENDO IN UNA SFERA)

$$4\pi r_1^2 E(r) = \frac{k}{\varepsilon_0}\int r^4 \sin\theta\, d\theta\, dr\, d\varphi \Rightarrow E_1 = \frac{k}{\cancel{4\pi}\varepsilon_0 r_1^2}\frac{R^5}{5}\cancel{4\pi} = \frac{kR^5}{5\varepsilon_0 r_1^2}, \quad r_1 = 2\ell - \frac{R}{2}$$

$$4\pi r_2^2 E_2(r) = \frac{k}{\varepsilon_0}\int r^4 \sin\theta\, d\theta\, dr\, d\varphi \Rightarrow E_2 = \frac{k}{\varepsilon_0 r_2^2}\frac{r_2^5}{5} = \frac{k r_2^3}{5\varepsilon_0}, \quad r_2 = \frac{R}{2}$$

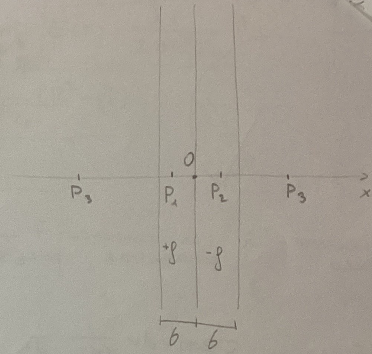
$$E_{P_2} = E_1 + E_2 = \frac{k}{5\varepsilon_0}\left(\frac{R^5}{r_1^2} - r_2^3\right) = \frac{kR^3}{5\varepsilon_0}\left(\frac{R^2}{r_1^2} - \frac{1}{8}\right) = -302,8\ N/C$$

$\boxed{2}$ δ, $+\rho$, $-\rho$

GUARDO $E(x)$ ESSENDO INDEFINITI.

USO GAUSS

$$\pi r^2 E(x) = \frac{1}{\varepsilon_0}\int^{\delta-(\delta-x)} \rho \, dx \int_0^{r} r' dr' \int_0^{2\pi} d\varphi$$



P_1: $\boxed{-\delta \leq x \leq 0}$

$$\pi r^2 E_+(x) = \frac{\rho}{\varepsilon_0}\pi r^2(-x) \Rightarrow E_+(x) = -\frac{\rho}{\varepsilon_0}x$$

$$\pi r^2 E_-(x) = -\frac{\rho}{\varepsilon_0}\pi r^2(\delta) \Rightarrow E_-(x) = -\frac{\rho}{\varepsilon_0}\delta$$

$$\Rightarrow \boxed{E(x) = -\frac{\rho}{\varepsilon_0}(\delta + x)}$$

P_2: $\boxed{0 \leq x \leq \delta}$

$$\pi r^2 E_+(x) = \frac{\rho}{\varepsilon_0}\pi r^2 \delta \Rightarrow E_+(x) = \frac{\rho}{\varepsilon_0}\delta$$

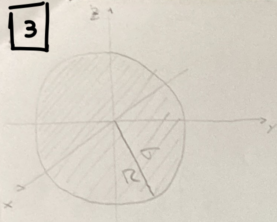
$$\pi r^2 E_-(x) = -\frac{\rho}{\varepsilon_0}\pi r^2 x \Rightarrow E_-(x) = -\frac{\rho}{\varepsilon_0}x$$

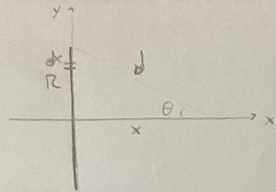
$$\Rightarrow \boxed{E(x) = \frac{\rho}{\varepsilon_0}(\delta - x)}$$

P_3: $\boxed{|x| > \delta}$

$$\pi r^2 E_\pm(x) = \frac{\pm\rho}{\varepsilon_0}\pi r^2 \delta \Rightarrow E_\pm = \pm\frac{\rho}{\varepsilon_0}\delta \Rightarrow \boxed{E(x) = 0}$$

$\boxed{3}$





$$d = \sqrt{R^2 + x^2}$$

SULL'ASSE $\hat{x}$ LE COMPONENTI $\hat{y}$ SI ELIDONO PER SIMMETRIA, QUINDI AVRÒ SOLO COMPONENTE DELL'ELEMENTO dx (SIMMETRICO).
↳ CORONA CIRCOLARE

$$dE(x) = \sigma \cancel{2}\pi r\,dr \frac{1}{\cancel{4}\pi\varepsilon_0}\frac{1}{d^2}\cos\theta\,\hat{x} \quad , \quad \cos\theta = \frac{x}{d} = \frac{x}{\sqrt{r^2+x^2}}$$

$$= \frac{\sigma r\,dr}{2\varepsilon_0}\frac{1}{d^2}\frac{x}{d} = \frac{\sigma x}{\varepsilon_0}\frac{1}{2}\frac{r}{d^3}dr\,\hat{x}$$

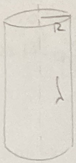
$$\Rightarrow E(x) = \frac{\sigma x}{2\varepsilon_0}\int_0^R \frac{r}{(r^2+x^2)^{3/2}}dr\,\hat{x} = \frac{\sigma x}{2\varepsilon_0}\frac{1}{\cancel{2}}\cancel{2}\left.\frac{-1}{(r^2+x^2)^{1/2}}\right|_0^R = -\frac{\sigma x}{2\varepsilon_0}\left(\frac{1}{\sqrt{R^2+x^2}} - \frac{1}{x}\right)\hat{x}$$

$$\Rightarrow \boxed{E(x) = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{x}{\sqrt{R^2+x^2}}\right)\hat{u}_x} \quad , \quad V(0) - V(x) = \int_0^x \vec{E}(x)\cdot dx \Rightarrow V(x) = -\int_0^x E(x)dx \;,\; V(0)=0$$

$$V(x) = -\int_0^x E\cdot dl = -\frac{\sigma}{2\varepsilon_0}\left(x - \int_0^x \frac{x'}{\sqrt{R^2+x'^2}}dx'\right) = -\frac{\sigma}{2\varepsilon_0}\left(x - \frac{1}{2}2\sqrt{R^2+x^2}\right) = \boxed{\frac{\sigma}{2\varepsilon_0}\left(-x+\sqrt{R^2+x^2}\right)}$$

OPPURE $$V(x) = \frac{1}{4\pi\varepsilon_0}\int_\Sigma \frac{\sigma\,d\Sigma}{d} = \frac{\sigma}{4\pi\varepsilon_0}\int_0^{2\pi}d\theta\int_0^R\frac{r\,dr}{\sqrt{r^2+x^2}} = \frac{\sigma}{4\pi\varepsilon_0}2\pi\left[\frac{1}{2}2\sqrt{r^2+x^2}\right]_0^R = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2+x^2} - x\right)$$

$\boxed{4}$



USO GAUSS IN TUTTE LE REGIONI

$\boxed{r<R}$ $$\cancel{\ell}\, 2\pi r\, E(r) = \frac{\lambda \cancel{\ell}}{\pi R^2 \cancel{\ell}}\, \pi r^2 \cancel{\ell}\, \frac{1}{\varepsilon_0} \Rightarrow \boxed{E(r) = \frac{\lambda r}{2\pi R^2 \varepsilon_0}\,\hat{r}}$$

$$V(r) - V(R) = \int_r^R \frac{\lambda r}{2\pi R^2 \varepsilon_0}\,\hat{r} = \boxed{\frac{\lambda}{2\pi R^2 \varepsilon_0}\left(R^2 - r^2\right)}$$

λ = DENSITÀ DI VOLUME

$\boxed{r>R}$ $$\cancel{\ell}\, 2\pi r\, E(r) = \frac{\lambda \ell}{\pi R^2 \varepsilon_0 \cancel{\ell}}\, \cancel{\pi R^2 \ell} \Rightarrow \boxed{E(r) = \frac{\lambda}{2\pi r \varepsilon_0}\,\hat{r}}$$

$$V(r) - V(R) = \frac{\lambda}{2\pi\varepsilon_0}\int_r^R \frac{dr}{r} = \boxed{\frac{\lambda}{2\pi\varepsilon_0}\log\left(\frac{R}{r}\right)}$$

5 FILO INDEFINITO $-\lambda$

CILINDRO " $R_0 = 2\,cm$, σ

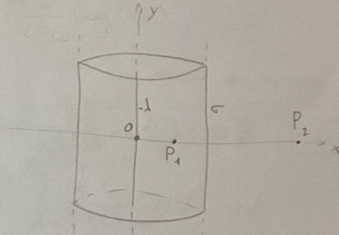
$\Delta V = V(P_1) - V(P_2) = 0$

$P_1 \equiv R_1 = 1cm$, $R_2 = 4\,cm$

TROVA σ/λ

CALCOLO PRIMA IL FILO

USO GAUSS (CILINDRO CON ASSE y)



$$2\pi r E(r) l = -\frac{\lambda l}{\varepsilon_0} \Rightarrow \boxed{E_{FILO}(r) = -\frac{\lambda}{2\pi\varepsilon_0 r}}$$

$$V(0) - V(r) = \int_0^r E_{FILO}(r)\cdot dl \quad , \quad V(0) = 0$$

$$V(r) = \frac{\lambda}{2\pi\varepsilon_0}\int_{R_1}^{R_2}\frac{dr}{r} = \boxed{\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{R_2}{R_1}\right)}$$

CALCOLO PER IL CILINDRO

$r < R_0$ $\quad E(r) = 0 \Rightarrow V = cost \Rightarrow V(R_0) = \frac{\sigma}{4\pi\varepsilon_0}\int\frac{d\Sigma}{r} = \frac{\sigma}{2\varepsilon_0}$

$r > R_0$ $\quad 2\pi r l E(r) = \frac{\sigma}{\varepsilon_0} 2\pi R_0 l \Rightarrow E_{CIL}(r) = \frac{\sigma R_0}{\varepsilon_0 r}$

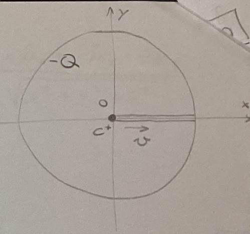
$$V(r) = \int_{R_0}^{R_2} E_{CIL}(r)\cdot dl = \frac{\sigma R_0}{\varepsilon_0}\ln\left(\frac{R_2}{R_0}\right)$$

$$\Delta V = V_{FILO} - V_{CIL} = 0 = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{R_2}{R_1}\right) - \frac{\sigma R_0}{\varepsilon_0}\ln\left(\frac{R_2}{R_0}\right) \Rightarrow \frac{\sigma R_0}{\varepsilon_0}\ln\left(\frac{R_2}{R_0}\right) = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{R_2}{R_1}\right)$$

$$\Rightarrow \frac{\sigma}{\lambda} = \frac{1}{2\pi R_0}\frac{\ln\left(\frac{R_2}{R_1}\right)}{\ln\left(\frac{R_2}{R_0}\right)} = 15{,}92\ ^1/_m$$

$\boxed{6}$ $R = 10\ \text{cm}$, VUOTO , $-Q$

C^+ , $q_{C^+} = q_{PROTONE}$, $v_0 = 3{,}6 \cdot 10^6\ \text{Km}/\text{h}$



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a) USO GAUSS $\quad 4\pi r^2 E(r) = \dfrac{Q^{(INT)}}{\varepsilon_0} = 0 \Rightarrow \boxed{E^{IN} = 0}$

$\Rightarrow V = \text{cost} \Rightarrow \boxed{V(R) = \dfrac{Q}{4\pi\varepsilon_0 R}}$

b) $\boxed{\sigma = \dfrac{Q}{4\pi R^2}}$ MI SERVE Q

FUORI DALLA SFERA $\quad 4\pi r^2 E(r) = -\dfrac{Q}{\varepsilon_0} \Rightarrow E(r) = -\dfrac{Q}{4\pi\varepsilon_0 r^2}$

$$V(R) - V(r) = -\frac{Q}{4\pi\varepsilon_0}\int_r^R \frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{r}\right)$$

ALL'INTERNO CONOSCO V, POSSO TROVARE CON QUALE v ESCE

$U_{IN} + K_{IN} = U_F + K_F \Rightarrow q_{C^+} V(R) + K_{IN} = q_{C^+} V(R) + K_{FIN} \Rightarrow \boxed{v_{OUT_0} = v_0}$

SO $U(\infty) = 0 \quad (V(\infty) = 0)$

$U_{IN} + K_R = \underset{\substack{v = v_0 \\ K_\infty = 0}}{K_\infty} \Rightarrow \dfrac{1}{2} m v_0^2 = -\dfrac{Q q_P}{4\pi\varepsilon_0 R} \Rightarrow \boxed{Q = -6{,}9 \cdot 10^{-7}\ C}$

$\Rightarrow \boxed{\sigma = -5{,}5 \cdot 10^{-6}\ C/m^2}$

c) $V(R) - V(10\,m = P) = \dfrac{Q}{4\pi\varepsilon_0}\left(\dfrac{1}{R} - \dfrac{1}{P}\right)$

$U_{IN} + K_{IN} = \dfrac{1}{2} m v_P^2 + U_P$

$\Rightarrow v_P = \sqrt{\dfrac{2}{m}\left(U_{IN} + \dfrac{m}{2}v_0^2 - U_P\right)} = \sqrt{\dfrac{2}{m}\left(\dfrac{m}{2}v_0^2 + \dfrac{qQ}{4\pi\varepsilon_0}\left(\dfrac{1}{R} - \dfrac{1}{P}\right)\right)} \sim v_0\sqrt{1 + \dfrac{qQ}{4\pi\varepsilon_0}\left(\dfrac{1}{R} - \dfrac{1}{P}\right)\dfrac{1}{v_0^2}} \simeq 0{,}1 v_0$

$\boxed{7}$ q IN $(x,y,z) \equiv (0,0,0)$

HO $E(x,y,z) = \dfrac{q}{4\pi\varepsilon_0 r^2}\,u_r$, $r^2 = (x^2+y^2+z^2)$ $\qquad \hat{u}_r = \dfrac{x\hat{u}_x + y\hat{u}_y + z\hat{u}_z}{\sqrt{x^2+y^2+z^2}}$

$$\vec{\nabla}\cdot\vec{E} = \frac{\partial E}{\partial x} + \frac{\partial E}{\partial y} + \frac{\partial E}{\partial z} = \frac{q}{4\pi\varepsilon_0}\left\{ \nabla \cdot \left(\frac{x\hat{u}_x + y\hat{u}_y + z\hat{u}_z}{(x^2+y^2+z^2)^{3/2}} \right) \right\}$$

$$= \frac{q}{4\pi\varepsilon_0}\left\{ \frac{(x^2+y^2+z^2)^{3/2} - \frac{3}{2}2x^2(x^2+y^2+z^2)^{1/2}}{(x^2+y^2+z^2)^3} + \frac{(r)^3 - 3y^2 r}{r^6} + \frac{r^3 - 3z^2 r}{r^6} \right\}$$

$$= \frac{q}{4\pi\varepsilon_0 r^5}\left\{ (x^2+y^2+z^2)\cdot 3 - 3(x^2+y^2+z^2) \right\} = 0$$

$\boxed{8}$ SFERA R , $\rho(r) = \alpha r$

$$4\pi r^2 E(r) = \frac{\alpha}{\varepsilon_0}\int_0^r r^3 \sin\theta \, dr\, d\theta\, d\varphi$$

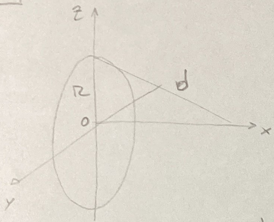
$r<R$ $$E(r) = \frac{1}{4\pi r^2}\frac{\alpha}{\varepsilon_0} 4\pi \frac{r^4}{4} = \frac{\alpha r^2}{4\varepsilon_0}$$

$r>R$ $$E(r) = \frac{1}{4\pi r^2}\frac{\alpha}{\varepsilon_0} 4\pi \frac{R^4}{4} = \frac{\alpha R^4}{4\varepsilon_0 r^2}$$

VALORE MASSIMO SARÀ A $r \equiv R$ POICHÈ $(r<R)$ CRESCE COME r^2 MA FINO A $r = R$ PERCHÈ POI $r>R$ DECRESCE COME $1/r^2$

$$\boxed{E_{MAX} = \frac{\alpha R^2}{4\varepsilon_0}}$$

$\boxed{9}$ R, q



$$dV(x) = \frac{1}{4\pi\varepsilon_0}\frac{dq}{d} \quad , \quad d = \sqrt{R^2+x^2}$$

$$dq = \lambda\, dl \quad , \quad \lambda = \frac{q}{2\pi R}$$

$$dV(x) = \frac{1}{4\pi\varepsilon_0}\frac{q}{2\pi R}\, dl\, \frac{1}{d}$$

$$\Rightarrow V(x) = \frac{1}{4\pi\varepsilon_0}\frac{q}{2\pi R}\frac{1}{d}\int dr\, d\theta = \frac{1}{4\pi\varepsilon_0}\frac{q}{\cancel{2\pi R}}\frac{1}{d}\,\cancel{2\pi R}$$

$$\Rightarrow \boxed{V(x) = \frac{q}{4\pi\varepsilon_0\sqrt{R^2+x^2}}}$$

$$E(x) = -\bar{\nabla} V(x) = -\frac{\partial V}{\partial x} = -\frac{q}{4\pi\varepsilon_0}\frac{\partial}{\partial x}\left\{(R^2+x^2)^{-1/2}\right\} = \cancel{-}\frac{q}{4\pi\varepsilon_0}\left\{\cancel{-}\cancel{\frac{1}{2}}(R^2+x^2)^{-3/2}\,\cancel{2}x\right\}$$

$$\Rightarrow \boxed{E(x) = \frac{q\,x}{4\pi\varepsilon_0 (R^2+x^2)^{3/2}}\,\hat{u}_x}$$

→ PER SIMMETRIA RISPETTO L'ASSE X