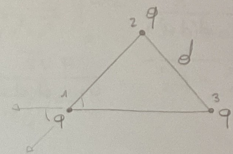
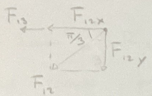
ESERCIZI DI ELETTRICITÀ E MAGNETISMO

FOGLIO EM-A1

$\boxed{1}$ $q = 1\ \mu C$ $n = 3$ $d = 10\ cm$



$$F = \frac{q^2}{4\pi\varepsilon_0 d^2} = 0{,}89\ N$$



$$F_{12x} = F_{12} \cos\frac{\pi}{3} = 0{,}445\ N$$

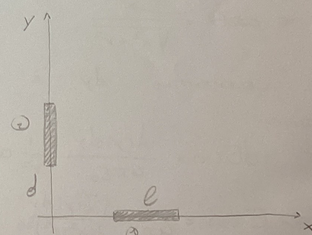
$$F_{12y} = F_{12} \sin\frac{\pi}{3} = 0{,}77\ N$$

$$F = \sqrt{F_x^2 + F_y^2} = 1{,}54\ N$$

$F_{13} + F_{12x}$

$\boxed{2}$ $l = 0{,}5\ m$, $q = 0{,}5\ mC$

$q = \lambda l$



$$E_1 = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda\, dx}{x^2}\hat{x} = \frac{\lambda}{4\pi\varepsilon_0}\hat{x}\int_d^{d+l} x^{-2}\, dx$$

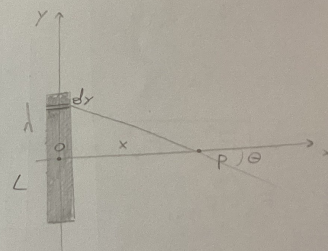
$$= \frac{-\lambda \hat{x}}{4\pi\varepsilon_0} \frac{1}{x}\Big|_d^{d+l} = \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{1}{d+l} - \frac{1}{d}\right)(-\hat{x}) = \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{d-d-l}{d(d+l)}\right)(-\hat{x})$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{\hat{x}}{d(d+l)}$$

$$E_1 = \frac{q}{4\pi\varepsilon_0} \frac{\hat{x}}{d(d+l)} \qquad E_2 = \frac{q}{4\pi\varepsilon_0} \frac{\hat{y}}{d(d+l)}$$

$$E = \sqrt{E_1^2 + E_2^2} = \frac{q\sqrt{2}}{4\pi\varepsilon_0 d(d+l)} = 1{,}06 \cdot 10^2 \ N/C$$

$\boxed{3}$ $\lambda(y) = \alpha|y|$ $\qquad$ $P \equiv (x, 0)$



SE USASSI GAUSS AVREI SOLO COMPONENTE NORMALE USCENTE DALLA SUP. CILINDRICA

$\Rightarrow$ COMPONENTE $E \equiv (E_x, 0)$

NOTO CHE HO SIMMETRIA E SI SOMMANO I CONTRIBUTI SOPRA - SOTTO L'ORIGINE

UN ELEMENTINO dy GENEREREBBE UN CAMPO CON UN CERTO ANGOLO θ, A ME SERVE SOLO LA COMPONENTE x

$$\Rightarrow \cos\theta = \frac{x}{\sqrt{x^2+y^2}}$$

E L'ELEMENTINO dy È A DISTANZA $r = \sqrt{x^2+y^2}$ DAL PUNTO P

QUINDI

$$dE(x) = \frac{\lambda(y)dy}{4\pi\varepsilon_0} \frac{1}{r^2} \cos\theta = \frac{\lambda(y)dy}{4\pi\varepsilon_0} \frac{1}{(x^2+y^2)} \frac{x}{\sqrt{x^2+y^2}}$$

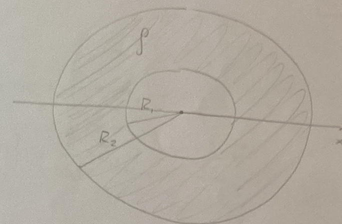
$$\Rightarrow E(x) = \frac{\alpha}{4\pi\varepsilon_0} \int_{-L/2}^{L/2} \frac{x|y|\,dy}{(x^2+y^2)^{3/2}} = \frac{\alpha x}{2\pi\varepsilon_0} \int_0^{L/2} \frac{y}{(x^2+y^2)^{3/2}}\,dy = \frac{\alpha x}{4\pi\varepsilon_0} \int_0^{L/2} \frac{2y}{(x^2+y^2)^{3/2}}\,dy$$

$$= \frac{\alpha x}{4\pi\varepsilon_0}\left(\frac{1}{-\frac{3}{2}+1}(x^2+y^2)^{-\frac{3}{2}+1}\right)\Bigg|_0^{L/2} = \frac{\alpha x}{4\pi\varepsilon_0}\left(-2\frac{1}{\sqrt{x^2+y^2}}\right)\Bigg|_0^{L/2}$$

$$= -\frac{\alpha x}{2\pi\varepsilon_0}\left(\frac{1}{(x^2+\frac{L^2}{4})^{1/2}} - \frac{1}{x}\right) = \frac{\alpha}{2\pi\varepsilon_0}\left(1 - \frac{x}{\sqrt{x^2+L^2/4}}\right)$$

$\boxed{4}$ $R_1 = 2\ \text{cm}$, $R_2 = 9\ \text{cm}$, $Q = 18\ \text{mC}$ UNIF.

DEVO USARE GAUSS IN TUTTE E 3 LE REGIONI



$r < R_1$ NON HO CARICA ALL'INTERNO

$$\phi(E) = 0 \Rightarrow \boxed{E = 0}$$

$R_1 < r < R_2$

$$4\pi r^2 E(r) = \frac{Q(r)}{\varepsilon_0}$$

$$Q(r) = \rho \frac{4}{3}\pi(r^3 - R_1^3) = \frac{Q}{\cancel{\frac{4}{3}\pi}(R_2^3 - R_1^3)}\cancel{\frac{4}{3}\pi}(r^3 - R_1^3)$$

$$= Q\frac{r^3 - R_1^3}{R_2^3 - R_1^3}$$

$$\Rightarrow \boxed{E(r) = \frac{Q}{4\pi\varepsilon_0 r^2}\frac{r^3 - R_1^3}{R_2^3 - R_1^3}}$$

$r > R_2$

$$4\pi r^2 E(r) = \frac{Q}{\varepsilon_0} \rightarrow \boxed{E(r) = \frac{Q}{4\pi\varepsilon_0 r^2}}$$

MASSIMO LO TROVO DA

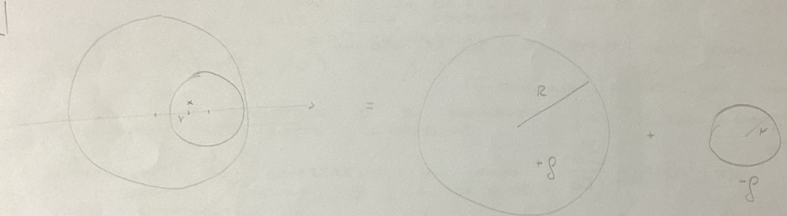
$$E = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r^2}\frac{r^3-R_1^3}{R_2^3-R_1^3}$$

SPARISCE SE $R_2 = r$

FUORI HO $E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$

$\Rightarrow$ IL MAX È AD $r = R_2$, SE $r > R_2$ È DIMINUISCE $E \propto 1/r^2 \to 0$ per $r \to \infty$

$\boxed{5}$



1) SOVRAPP. DEGLI EFFETTI. SE USO GAUSS VEDO CHE HO SOLO CONTRIBUTI RADIALI DI ENTRAMBE LE ρ, PERÒ HANNO DISTANZA $\hat{y}$ DAI CENTRI DELLE SFERE UGUALI E ESSENDO CONTRIBUTI OPPOSTI SI CANCELLANO

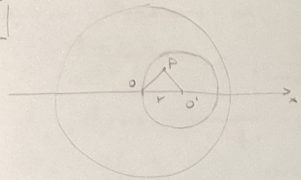
QUINDI HO SOLO COMPONENTE x

+ρ: $4\pi x^2 E(x) = \dfrac{Q}{\frac{4}{3}\pi R^3}\,\dfrac{4}{3}\pi x^3\,\dfrac{1}{\varepsilon_0} \rightsquigarrow \boxed{E_+(x) = \dfrac{\rho x}{3\varepsilon_0}}$

-ρ: $\cancel{4\pi(r-x)^2}\,E_-(x) = \rho\,\dfrac{4}{3}\pi(r-x)^3\,\dfrac{1}{\varepsilon_0} \rightsquigarrow \boxed{E_-(x) = \dfrac{\rho(r-x)}{3\varepsilon_0}}$

$\Rightarrow \boxed{E(x) = \dfrac{\rho r}{3\varepsilon_0}}$

$\boxed{5}$



uso il teorema di sovrapposizione in $P \equiv (b, a, \ldots$

uso Gauss

$$4\pi d^2 E_+(d) = \frac{\rho}{\varepsilon_0}\frac{4}{3}\pi d^3 \qquad \text{con} \quad d = \sqrt{a^2 + b^2}$$

$$\Rightarrow \boxed{E_+(d) = \frac{\rho}{3\varepsilon_0} d\,\hat{e}_+} \qquad \text{con} \quad \hat{e}_+ = \frac{(b, a, 0)}{d}$$

$$4\pi d'^2 E_-(d') = \frac{\rho}{\varepsilon_0}\frac{4}{3}\pi d'^3 \quad , \quad d' = \sqrt{a^2 + (r-b)^2}$$

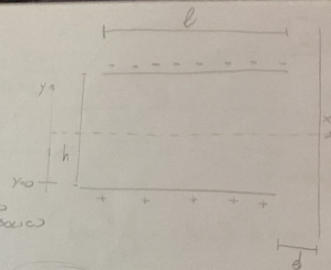
$$\Rightarrow \boxed{E_-(d') = \frac{-\rho}{3\varepsilon_0} d'\,\hat{e}_-} \qquad \text{con} \quad \hat{e}_- = \frac{(-(r-b), a, 0)}{d'}$$

quindi

$$E(P) = \frac{\rho}{3\varepsilon_0}\left(d\hat{e}_+ - d'\hat{e}_-\right) = \frac{\rho}{3\varepsilon_0}\left\{(b, a, 0) - (-(r-b), a, 0)\right\} = \frac{\rho}{3\varepsilon_0}(r, 0, 0) = \frac{\rho}{3\varepsilon_0}\vec{r}$$

$\boxed{6}$ $-|q_e|$, m_e, v_x $t=0$ IN ε_0, μ_0

ℓ, σ, h d SCHERMO



SO $E = \frac{\sigma}{\varepsilon_0}$ $\left(V = \frac{q}{C} = Eh\right)$

a) $F = qE = -\frac{q\sigma}{\varepsilon_0}$ FORZA CHE SENTE e^- $\Rightarrow$ MOTO PARABOLICO

NON URTA LE ARMATURE SE $x(y=-h) > \ell$

MOTO RUA SU y $y(t) = -\frac{1}{2}\frac{q\sigma}{m\varepsilon_0}t^2 + vt + y_0$ $a = -\frac{q\sigma}{m\varepsilon_0}$

IMPONGO $y(t_1) = -h$ $\frac{1}{2}at_1^2 + v_y t_1 = -h$ $\Rightarrow$ $\frac{1}{2}at_1^2 = -h$ $\Rightarrow$ $t_1 = \sqrt{\frac{-\cancel{2}h}{\cancel{2}a}}$

$\Rightarrow t_1 = \sqrt{\frac{mh\varepsilon_0}{q\sigma}}$

HO MRU SU x E VOGLIO $x(t_1) \geq \ell$ $x(t) = vt$

$vt_1 \geq \ell \Rightarrow t_1 \geq \frac{\ell}{v}$ $\frac{mh\varepsilon_0}{q\sigma} \geq \frac{\ell^2}{v^2} \Rightarrow \boxed{\sigma \leq \frac{mv^2h\varepsilon_0}{q\ell^2}}$

b) $\sigma_{MAX} = \frac{mv^2h\varepsilon_0}{q\ell^2} \Rightarrow$ AD $x = \ell$ E METTO $y=0$

HO MRU SU x E y, HO $v_x = v$

v_y LA TROVO DA $y(t) = \frac{1}{2}at^2 + v_y t + y_0 \Rightarrow v_y = at$

$\sigma = \dfrac{v^2 h m \varepsilon_0}{q\ell^2}$ $\qquad a = -\dfrac{q}{m\varepsilon_0}\sigma = -\dfrac{v^2 h}{\ell^2}$

A $\hat{t}$ $y(\hat{t}) = 0$ QUANDO ESCE DAL CONDENSATORE

$v_y = at$ ✿

SU $x(t) = v_x t \Rightarrow x(\hat{t}) = v_x \hat{t} = \ell \Rightarrow \hat{t} = \dfrac{\ell}{v_x} = \dfrac{\ell}{v}$

METTO $\hat{t}$ IN ✿

$v_y = -\dfrac{v^2 h}{\ell^2}\hat{t} \Rightarrow v_y = -\dfrac{v^2 h}{\ell^2}\hat{t} = -\dfrac{v^2 h}{\ell^2}\dfrac{\ell}{v} = -\dfrac{vh}{\ell}$

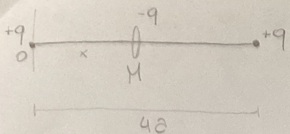
QUINDI $\begin{cases} x(t) = vt \\ y(t) = -v_y t \end{cases}$ ARRIVO ALLO SCHERMO $x(\tilde{t}) = d \Rightarrow \tilde{t} = \dfrac{d}{v}$

QUINDI $y(\tilde{t}) = y_{IMP} = -v_y \tilde{t} = -\dfrac{vh}{\ell}\dfrac{d}{v} = \boxed{-\dfrac{dh}{\ell}}$

c) SE $\sigma \le \sigma_{MAX}$ $\quad t_{IMP} = \hat{t} + \tilde{t} = \dfrac{\ell}{v} + \dfrac{d}{v} = \boxed{\dfrac{\ell + d}{v}}$ NON HO DIPENDENZA DA σ

MA SE $\sigma > \sigma_{MAX}$ $\quad t_{IMP} = \infty$ PERCHÈ e^- VA SULL'ARMATURA

$\boxed{7}$



M SENTE $|F| = \dfrac{q^2}{16\pi\varepsilon_0 a^2}$ DA ENTRAMBE LE CARICHE

È UN EQUILIBRIO INSTABILE PERCHÈ BASTA UNA PICCOLA PERTURBAZIONE E LE DUE FORZE NON SARANNO UGUALI E L'ANELLO SARÀ ATTRATTO DA UNA DELLE DUE q

METTO M A DISTANZA x DA q CHE SENTIRÀ

$$\Delta V(x) = V_1 + V_2 = \frac{q}{4\pi\varepsilon_0 x} + \frac{q}{4\pi\varepsilon_0 (4a-x)} = \frac{q}{4\pi\varepsilon_0}\frac{4a}{x(4a-x)} = \frac{qa}{\pi\varepsilon_0 x(4a-x)}$$

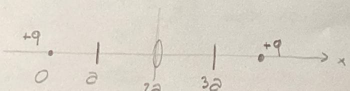
$$U = -qV(x) = -\frac{q^2 a}{\pi\varepsilon_0 x(4a-x)}$$

$$\frac{dU}{dx} = \frac{q^2 a}{\pi\varepsilon_0 x^2}\frac{1}{4a-x} - \frac{q^2 a}{\pi\varepsilon_0 x}\frac{1}{(4a-x)^2} = 0 \Rightarrow \frac{1}{x} - \frac{1}{4a-x} = 0 \Rightarrow \frac{1}{x} = \frac{1}{4a-x}$$

$$\Rightarrow x = 4a - x \Rightarrow \boxed{x = 2a}$$

MA
$$\frac{d^2U}{dx^2} = -\frac{2q^2 a}{\pi\varepsilon_0 x^3}\frac{1}{4a-x} + \frac{q^2 a}{\pi\varepsilon_0 x^2 (4a-x)^2} + \frac{q^2 a}{\pi\varepsilon_0 x^2}\frac{1}{(4a-x)^2} - \frac{2q^2 a}{\pi\varepsilon_0 x}\frac{1}{(4a-x)^3}$$

FIDATI $\dfrac{d^2U}{dx^2} > 0$ ⇒ MASSIMO ⇒ EQ. INSTABILE



PER TROVARE K_F DEVO SFRUTTARE LA CONSERVAZIONE DELL'ENERGIA

AD $x = 2a$ HO $E(2a) = U(2a) + K(2a) = U(2a)$

$$= -\frac{q^2 a}{\pi \varepsilon_0 4 a^2} = -\frac{q^2}{4\pi\varepsilon_0 a}$$

SE POI VA A SBATTERE SUL FERMO HO

$$E(a) = U(a) + K(a) = -\frac{q^2 a}{\pi\varepsilon_0 3 a^2} + K_F = -\frac{q^2}{3\pi\varepsilon_0 a} + K_F$$

QUINDI

$$-\frac{q^2}{4\pi\varepsilon_0 a} = -\frac{q^2}{3\pi\varepsilon_0 a} + K_F \Rightarrow K_F = -\frac{q^2}{\pi\varepsilon_0 a}\left(\frac{1}{4} - \frac{1}{3}\right) = -\frac{q^2}{\pi\varepsilon_0 a}\left(\frac{3-4}{12}\right)$$

$$\Rightarrow \boxed{K_F = \frac{q^2}{12\pi\varepsilon_0 a}}$$